Convertible Bond Risks in “Everything Everywhere”

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Structure

We need to answer the following questions

- What is a convertible bond?
- How can we sensibly price it in a consistent fashion?
- How can we decompose it into “atomic” risks?
- How does this fit into our “Everything Everywhere” model?

What is a convertible bond?

Literature Review I

- Frankle, A. W. and C. A. Hawkins. "Beta Coefficients For Convertible Bonds," Journal of Finance, 1975, v30(1), 207-210.
- Ingersoll, Jonathan E., Jr. "A Contingent-Claims Valuation Of Convertible Securities," Journal of Financial Economics, 1977, v4(3), 289-321.
- Brennan, Michael J. and Eduardo S. Schwartz. "Analyzing Convertible Bonds," Journal of Financial and Quantitative Analysis, 1980, v15(4), 907-929
- Beatty, Randolph P. "Estimation Of Convertible Security Systematic Risk: The Marginal Effect Of Time, Price, Premium Over Bond Value, And Conversion Value/Call Price," Advances in Financial Planning and Forecasting, 1987, v2(1), 135-154.

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Literature Review II

- Beatty, Randolph P., Cheng L. Lee and K. C. Chen. "On The Nonstationarity Of Convertible Bond Betas: Theory And Evidence," Quarterly Review of Economics and Business, 1988, v28(3), 15-27.
- Altman, Edward I. "The Convertible Debt Market: Are Returns Worth The Risk?," Financial Analyst Journal, 1989, v45(4), 23-31.
- Brooks, Robert and Bill Attinger. "Using Duration And Convexity In The Analysis Of Callable Convertible Bonds," Financial Analyst Journal, 1992, v48(4), 74-77.
- Ferguson, Robert, Robert E. Butman, Hans L. Erickson and Steven Rossiello. "An Intuitive Procedure To Approximate Convertible Bond Hedge Ratios And Durations," Journal of Portfolio Management, 1995, v22(1), 103-111.

Literature Review III

- Sparaggis, Takis D. "Factor And Spread Analysis Of The Convertible Securities Market," Financial Analyst Journal, 1995, v51(5), 68-73.
- Carayannopoulos, Peter. "Valuing Convertible Bonds Under Assumption Of Stochastic Interest Rates: An Empirical Investigation," Quarterly Journal of Business and Economics, 1996, v35(3,Summer), 17-31.
- Datta, Sudip and Mai Iskandar-Datta. "New Evidence On The Valuation Effects Of Convertible Bond Calls," Journal of Financial and Quantitative Analysis, 1996, v31(2,Jun), 295-307.
- Ho, Thomas S. Y. and David M. Pfeffer. "Convertible Bonds: Model, Value Attribution, And Analytics," Financial Analyst Journal, 1996, v52(5,Sep-Oct), 35-44.
- Tsiveriotis, Kostas and Chris Fernandes. "Valuing Convertible Bonds With Credit Risk," Journal of Fixed Income, 1998, v8(2,Sep), 95-102.

Literature Review IV

- Ma, Ronald and Cecilia Lambert. "In Praise Of Occam's Razor: A Critique Of The Decomposition Approach In IAS 32 To Accounting For Convertible Debt," Accounting and Business Research, 1998, v28(2,Spring), 145-153.
- Sarkar, Sudipto. "Duration And Convexity Of Zero-Coupon Convertible Bonds," Journal of Economics and Business, 1999, v51(2,Mar/Apr), 175-192.
- Epstein, David, Richard Haber and Paul Wilmott. "Pricing And Hedging Convertible Bonds Under Non-Probabilistic Interest Rates," Journal of Derivatives, 2000, v7(4,Summer), 31-40.
- Kariya, Takeaki and Hiroshi Tsuda. "CB - Time Dependent Markov Model For Pricing Convertible Bonds," Asian-Pacific Financial Markets, 2000, v7(3,Sep), 239-259.
- Mayers, David. "Convertible Bonds: Matching Financial And Real Options," Journal of Applied Corporate Finance, 2000, v13(1,Spring), 8-21.

What Are Convertible Bonds?

Simple yet Complex!

A bond that may be converted at some time in the future at the bond holders' discretion into N shares of stock X at a price P.

Key Common Features:

- call schedule
- put schedule
- sinking fund or other redemption features
- convertible into issuing (or another) companies stock
  - may be convertible at any time
  - may be convertible at a fixed price or a time-dependent price

A Bond plus a Warrant plus Embedded Options?

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How Can We Price A Convertible Bond?

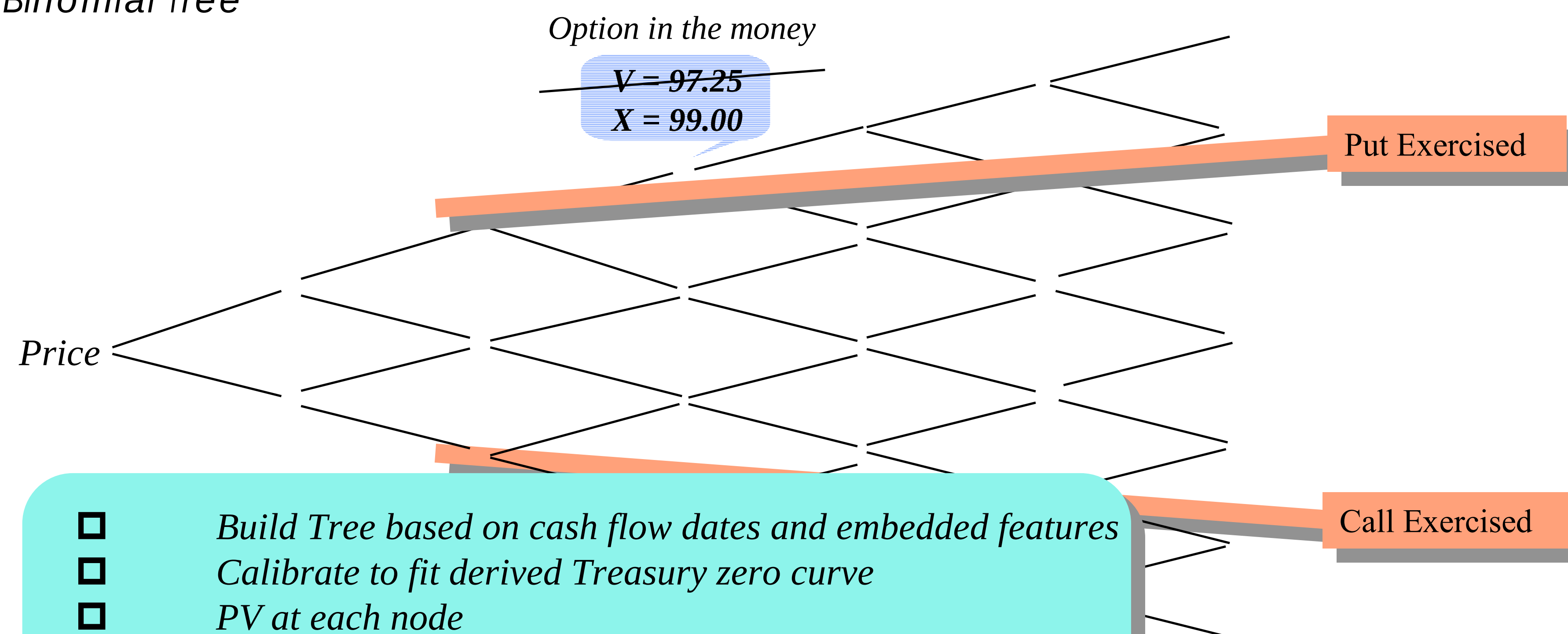
Break it into pieces and price those…

- Choose an approach for bonds with embedded options
- Choose a *consistent* approach for stock options
- Add the pieces together

Important: the equity model must be consistent with the bond model

How Do We Price a Bond?

Binomial Tree



- *Build Tree based on cash flow dates and embedded features*
- *Calibrate to fit derived Treasury zero curve*
- *PV at each node*
- *If PV>Option strike, assume exercised => Value = Strike*
- *Continue back down tree*
- *PV at time 0 (now)*

Tree accommodates Calls, Puts and Sinking Funds

How Could We Price a Stock Option or Warrant?

It must be fast, useable in production environment, and consistent with bond pricing model

Black-Scholes	-	easy to use, but assumes interest rates are constant!
Finite Difference Methods	-	hard to set up constraints, intractable in production system
Binomial Tree	**-**	**easy and fast to use but not very accurate?**
Trinomial Tree	-	much slower and more difficult but better accuracy?

Choose the Right Model – *consistent* with Bond Pricing Model

How to Connect Them and Price Convert in One Go?

Is there an easy way?

Standard Practitioner Choice as Proposed by Many Textbooks:

- ❑ Black-Scholes for stock option part of convertible
  (ignores inconvenient BS assumptions about constant interest rates)

Other Academic or Textbook Answers:

- ❑ Finite Difference Methods to Solve PDE directly
  (difficult to run as large-scale production process)
- ❑ Binomial tree for stock prices, determine at each node whether instrument is stock or bond. Use risk-free rate (fixed) to discount stock, use risky rate (fixed) to discount bond.*
  (ignores term-structure of interest rates entirely)

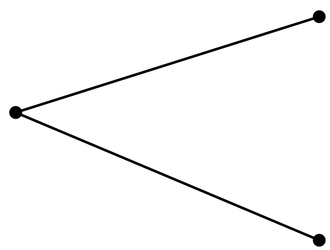
** Suggested by Goldman Sachs: "Valuing Convertible Bonds as Derivatives," Quantitative Strategy Research Notes, November 1994*

No easy way unless you make horrible assumptions

Binomial Tree Approach – More Detail

Could We Adapt This Approach?

Use a binomial tree for stock prices, determine at each node whether instrument is stock or bond. Use risk-free rate (fixed) to discount stock, use risky rate (fixed) to discount bond.



At this node the convert value > bond value
The bond is converted. Since it is now an equity we should discount it at the risk-free rate.

At this node the convert value < bond value
The bond is not converted. Since it is still a risky bond we should discount it at the risky rate.

*The value at the target node is reached by discounting the value at the two future nodes by the **average** of the risky and risk-free rates.*

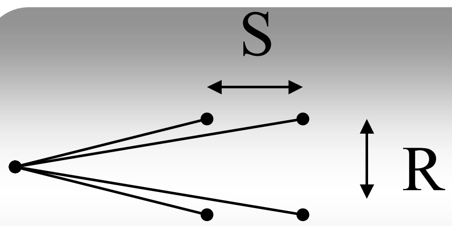
*With this approach the **same** two rates (one risky, one risk-free) are used throughout the tree.*

Can we adapt this to include stochastic interest rates?

Our Approach

Two trees – two stochastic processes

- We use ***two combined*** trees at the same time.
- The interest rate tree allows us to model the short-rate diffusion
- The stock price tree allows us to model the stock price diffusion
- We discount value back at each node based on the average rate
  - if it's still a bond at some node we should use risky rate
  - if it's an equity at some node we should use risk-free rate
- Given what we determine it to be at each future node we can discount the value back to the preceding level.



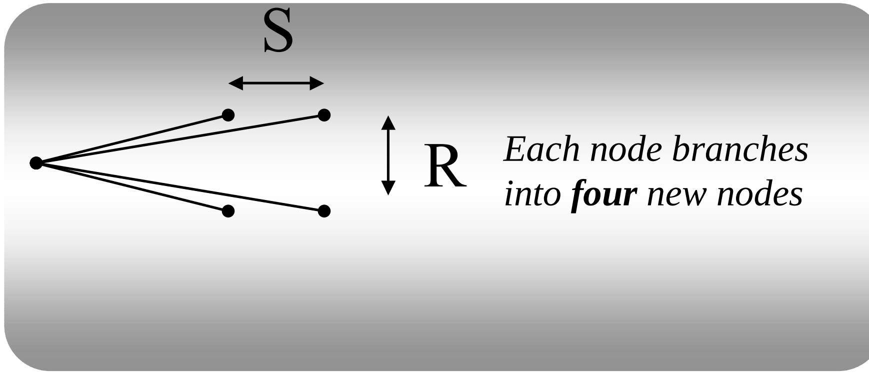
Each node branches into ***four*** *new nodes*

Diffusion processes for both stock and interest rates

Branching and Dividends

Other Features of the Stock Diffusion Tree

- Most stocks pay dividends
- What about volatility?
- What about accuracy?



- Branching for stock tree **includes** dividends based on constant annual dividend rate of q

  $$u = \exp\{ (r-q-\sigma^2/2)\Delta t + \sigma \sqrt{(\Delta t)} \}$$
  $$d = \exp\{ (r-q-\sigma^2/2)\Delta t - \sigma \sqrt{(\Delta t)} \}$$

- Stock tree branching based on volatility – can be either **one** volatility or a **term-structure** of volatility. (Ours is one for now…)

- Add many extra nodes at the short end to ensure accuracy – 50 additional levels added over first five years.

Diffusion processes for both stock and interest rates

Valuation

An attempt at clarity...

For each of Four Nodes Branching from Target Node:

- Compare Value V_N with convert value
- Compare (if not converted) with Call / Put strike
- If called, compare *again* with convert value (so-called "forced" convert)
- Final result: a new value V_N^*, and a status (Bond or Equity)
- Final final result: a discount rate D_N:
  - appropriate *risky* rate if Bond
  - or appropriate *risk free* rate if Equity

Use the Average Rate (average of D_1 to D_4) to discount V_1^* to V_4^* back to Target Node

Continue Rolling Back through Tree

Semi-Final Result: Price of Instrument

Stochastic Processes and Bond features Modeled Fully

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Correlation Between Stock Price Process and Interest Rates

- Stock prices are correlated to interest rates. So we have to compute the state price densities for evaluating the equity warrant have to be computed conditionally on the covariance matrix of the equity factors and the term structure factors. We "bend" the stock pricing tree to fit expected returns, given the term structure state

- Margrabe, William. "The Value Of An Option To Exchange One Asset For Another," Journal of Finance, 1978, v33(1), 177-186.
- Hull, John and Alan White. "Numerical Procedures For Implementing Term Structure Models II: Two-Factor Models," Journal of Derivatives, 1994, v2(2,Winter), 37-48.
- Hull, John and Alan White. "Valuing Derivative Securities Using The Explicit Finite Difference Method," Journal of Financial and Quantitative Analysis, 1990, v25(1), 87-100.

Capturing Equity Related Covariance

- To get the equity factor exposures arising from the warrant, we can observe fraction of terminal states of the combined tree that reflect conversion into equity. We can then treat the equity factor exposure today as the present value of the expectation of the future equity factor exposure.
- Its like computing a more sophisticated version of a "delta neutral" underlying equity exposure.
- Using the tree pricing procedure captures the interaction of multiple option features
  - Conversion to equity effectively reduces bond maturity
  - Call or put options on the bond effectively shorten the expiration date of the warrant

Risk in the "Everything Everywhere" Model

Can we model converts with the same risk factors?

- 19 Factors, plus currency covariance matrix
  - *5 geographic regions*
  - *6 aggregate industry-sectors*
  - *Interest rates*
  - *Energy cost (an inflation proxy)*
  - *Investor confidence #1: large cap – small cap spread*
  - *Investor confidence #2: emerging - developed spread*
  - *Dividend yield: a proxy for growth / value in equities*
  - *Three-Factor Model of Term Structure Movements*
  - *Currencies*

Sources of Risk in Convertible Bonds can be modeled by EE Factor Set

How Can We Treat These in Our EE Risk Model?

Can we model converts with the same risk factors?

- We capture interest-rate risk by varying our three *term-structure* factors and re-pricing under term-structure changes
- We capture *credit risk* as a duration-weighted exposure to a credit-synthetic
- We capture *embedded options* (calls, puts etc) *explicitly* in the pricing process
- We capture the *convert* risk explicitly in the pricing process. Equity risk is captured with a fancy version of a delta-neutral underlying
- We capture *currency risk* explicitly as factors in the risk model

Sources of Risk in Convertible Bonds can be modeled by EE Factor Set

Conclusions

- We can both price and evaluate the risks of convertible bonds using a three dimension tree approach to adapted to our EE risk model

- We believe this model is a substantial advance over methods that rely on seriously flawed simplifying assumptions.

- Empirical evidence is encouraging. Our model prices are in excellent agreement with reported trading prices