Optimal Algorithmic Trading

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May 2005

Topics for Today

- An apology to those who attend this seminar regularly
  - I've talked about this topic in some form for the last three years. Thank you for your patience. We're closing in on something fun
- An overview of the trading problem
- Existing algorithmic trading methods
- The Northfield algorithm with Instinet
  - A full utility function in discrete time
  - Market impact model
  - Market impact interaction across securities and across time

The Trading Problem

- Trading is the implementation of our portfolio decisions
- The most popular way to measure trading effectiveness is *implementation shortfall* as defined in Perold (1988)
  - Once we've decided that we want to change our portfolio, we measure the performance of our actual portfolio, against the hypothetical portfolio we now want to hold (assuming we could trade instantly at no cost)
- More simplistic measures are often substituted, and easily gamed by traders
  - Measure the "cost" of a buy as the average price (including commissions) against VWAP for that day
  - Simple way to win: wait until late in the day, if the expected trade price is greater than VWAP, don't trade until tomorrow

Explicit and Implicit Costs

- Most people see trading costs as having several components
  - Agency costs
  - Bid/Asked Spread
  - Market Impact (my trade moves the price)
  - Trend Costs (other people's trades move the price, maybe in my favor)
- Often overlooked ingredients
  - My large concurrent trades (my trade in Ford impacts the price of GM)
  - If I expect a stock to go up, I want to buy it before, not after it goes up. If I expect a stock to go down, I want to sell it before, not after it goes down
  - If the stock price moves too much before I trade, I may decide to cancel the trade altogether. If I was right about my return expectation, this is very costly

A Simple Framework Captures It All

- The implementation shortfall is the cumulative return on a long/short portfolio that we are trying to liquidate to cash
  - We are long stocks we have and don't want (e.g. an undone sell order)
  - We are short stocks we want and don't have (e.g. an undone buy order)
- Using this framework, we can use a fairly traditional utility function
  - Sensibly determine trade-offs between opportunity costs (short term alpha), risk and explicit trading costs
- The Northfield trading algorithm is an optimization in discrete time over the parameters of this utility function

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Evolution of Trading Algorithms

- Bertsimas and Lo (1998) and Bertsimas, Hummel and Lo (1999) propose a dynamic programming solution that trades opportunity costs against market impact
  - Set up as a complex set of differential equations requiring Bellman equation methods to solve
- Almgren and Chriss (2001, 2001)
  - Add risk to the problem
  - Form a “cost versus risk” efficient frontier
  - Still computationally complex
- In 2004, I worked with an MIT student group to develop a microstructure model of market impact
  - Included a closed form solution to the optimal schedule for a single stock

Some Algorithms are Really Simple

- Use VWAP
  - Schedule to trade some constant percentage of expected volume
  - Randomize timing of execution “firing” to reduce front-running
- Use Modified Dollar Cost Averaging
  - Start with a constant percentage of volume
  - Establish the “arrival” price when the trader gets order
  - Adjust the speed over time based on something like:

    POV = POV(VWAP) + M * (Arrival Price/Current Price-1)

  - Will generally produce a favorable looking result for traders
  - Risk of large implementation shortfalls is huge if the stock price trends against you, and you eventually have to trade at a very disadvantageous price just to finish
- Use Risk Adjusted Dollar Cost Averaging
  - Make the M coefficient an inverse a function of security volatility

Our Algorithm

- Using the long/short portfolio framework for implementation shortfall, we begin with the single period mean-variance utility function from Markowitz and Levy (1979)

  $U = A - S^2/T - C*A$

- Remember "alphas" are turned around so we're trying to liquidate the portfolio
  - We're short stocks we want to buy (i.e. we think will go up)
  - We're long stocks we went to sell (i.e. we think will go down)
- In the absence of short term alphas, trade urgency is a function of the marginal risk contribution to the long/short portfolio
  - We then trade risk off against explicit transaction costs

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Now Lets Go to Discrete Time

- Lets break up the trading horizon into N periods of unequal length that will each contain 100/N% of the expected daily volume

- We want to maximize the summation of utility of the N discrete periods

  $$U = \Sigma_{v=1 \text{ to } n}(Av - Sv^2/T - Cv*A)$$

  - The Av term is your short term alpha forecasts for each stock. These can be revised at each time V
  - The Sv term is the volatility of the remaining portion of the long/short portfolio at time V
  - T is your marginal rate of substitution for risk/return
  - Cv is all explicit trading costs to be incurred in period V
  - A is the aggressiveness level of trading
  - Need to reconcile “clock time” with “volume time”

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We Do Risk Models for a Living

- We can compute the risk term (S^2) from our short term risk models

- We've had a Short Term US Model for a long time. See (diBartolomeo and Warrick, Chapter 12, *Linear Factor Models in Finance*) for details

- We have a prototype of a Short Term Global Model up and running. Testing is ongoing

- The security volatility and correlation estimates are also inputs to our market impact model

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How about Short Term Alphas?

- Many people ignore this as too difficult to estimate
- Most popular approach is to establish a “target” price for the end of the period. Alpha is just the percentage price difference times some expected information coefficient
  - This will add some of the same directional tendencies as the simpler algorithms
  - If a stock you expect to go up drops in price, its alpha will increase making you want to trade it faster
  - If a stock you expect to go down increases in price, its alpha will decrease making you want to trade it faster
- Short-term reversal strategies could be as easily accommodated

Our Chronology on Trading Costs

- Despite a lot of research, we have had reservations regarding the usefulness of market impact models
- Concurrent trades have a large influence on expected market impact of trades
- We incorporated user definable market impact functions that could handle cross-impacts of concurrent trades into our Optimizer in March 2003
  - Not many clients used these functions and some that did had trouble getting reasonable parameters for market impact of large trades.
- Since then we've created a market impact model that relies on a simple economic model and boundary conditions

Trading Cost Estimation

- Agency Costs are essentially known in advance
- Bid/Asked Spreads: Some time variation but reasonably stable
- Market Impact: Lots of models exist
  - Underlying factors are highly significant
  - Explanatory power is typically low
  - Often estimated on empirical data sets that do not contain really large trades because traders know that liquidity is insufficient to undertake them
- Trend Costs: This is the “risk” piece of our function
  - Other people’s trades can move the price for or against us
  - Ex-post often the largest absolute part of the costs. Pretty darn random.

Transaction Cost Functional Form

- Lets consider a simple model of direct trading costs. Lots of models look like this. Costs of trading increase with trade size at a decreasing rate

  $M = a + bX + c(\text{abs}(X^{1/2}))$

  M is the expected cost to trade one share

  X is the number of shares to be traded

  a is the fixed costs per share

  b,c are coefficients expressing the liquidity of the stock (estimated from fundamentals and trading data)

- Empirical literature suggests is somewhere between linear with trade size and the square root of size
  - Using units of percentage of expected volume clarifies things

Market Impact Model Problems

- Most market impact models do not deal effectively with very large trades.
  - Traders know they can’t do these trades so they break them up into a series of small trades. As no empirical data is available, models don’t deal with the steep increase in costs at liquidity limits

- Our solution is to add another term to the cost equation

  $d(\max(X_t-L_t,0))^2$

  L_t is one sided volume in t periods that will cause serious liquidity breakdown

  d is not estimated from empirical data but from assumption of optimal trade break up

A Bit Fancier on Direct Costs

$$M = a + bX_T + c(abs(X_T^{1/2})) + d(max((Xt-L)^2,0)) + Z_t$$

M is the expected cost to trade one share

X_t is the absolute value of shares to be traded in t periods

a is the fixed costs per share

b,c d are coefficients expressing the liquidity of the stock

U_t = expected short term trend of stock return (including covariance with other stocks with predicted trends

Z_t = expected influence due to the covariance of this stock with the market impact of my concurrent trades

L = is the biggest trade we think the market can handle with normal liquidity (empirical evidence suggests between 10% and 50% ADV)

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But in Discrete Time We Need One More Thing

- We need to address that the market impact caused by trades in one period may impact prices in subsequent periods
  - We call this “stickiness”
  - The magnitudes depend on how long your time periods are

$$M_t = a + [\ bX_t + c\,(\text{abs}\,(X_t^{\ 1/2}))\ ^+ d(\max((X_t-L)^2,0))\ ] + Z_t$$

$$+ \Sigma_{\ v=1 \text{ to } t-1}(K^{t-v}\ [\ bX_v + c(\text{abs}(X_v^{1/2}\ )) + Z_v\ ^+ d(\max((X_v-L)^2,0))\ ])$$

  - K is a coefficient between zero and one

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Lets Talk about the Z term

$$Z_{it} = \text{Sum}_{\ j = 1 \text{ to } m}\ (bX_{jt} + c(\text{abs}\ (X_{jt}{}^{.5})) + d * (\max\ [X_{jt} - Li, 0]^\wedge 2) * (P_{ij} * Q_{ij})$$

For all i <> j

Pij = the correlation between stocks i and j derived from risk model and adjusted for volume fluctuation correlation [see Domowitz, Hansch and Wang (2005)]

Qij = 1 if [Change in Shares (i) * Change in Shares (j)] > 0
Qij = -1 if [Change in Shares(i) * Change in Shares(j)] < 0

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Until Now Users Estimated b,c,d

- Clients are accustomed to only estimating the value of A, the basic cost per share
- Current market impact models estimate B and C, but typically from empirical analysis of small trades
  - Large trades (i. e. bigger than L) don't show up in historical databases because traders know they are too big to execute
  - Tick by tick trade and quote data not available for many markets
- Initial client parameterizations have been mixed
  - 150% transaction costs on a sell?
- At this seminar last year, I talked about using boundary conditions to ensure rational parameters
  - Objective is to minimize mean squared error of estimates

What Might Reasonable Bounds Be?

- How about assuming the maximum market impact would be equal to the premium paid in typical hostile takeovers?
  - Academic studies and M&A databases (Dealmarker's Journal) show mean premium from 37 to 50% with standard deviation of around 30%
- Where does the distribution of observed bid and offer sizes top out from existing databases (e.g. TAQ)?
  - Well below one half day average trading volume
- If there is an imbalance between buyers and sellers (think October 19, 1987) how much can prices move?
  - Market averages dropped about 25% in October 1987

Formulating Market Impact

- Assume there is a liquidity provider on the other side of the trade
- They will take the other side of your trade, then slowly feed the position back into the market to avoid market impact (e.g. 1% of ADV per day)
- The liquidity provider needs to cover the cost of the money they use to finance the trade, and a return on capital they hold in reserve against possible losses. This will be a function VAR caused by the asset specific risk of the security
- As the trade gets larger, an increasing number of liquidity providers will compete to participate, driving down the rate of return on reserved capital

Lets Look at the Math

$$Mi(X) = Pi * (.5 * G(X)/365) * W + [Pi * (2 * Si / 250^{.5})] * R(X) * (.5 * G(X) /365)$$

X is the size of the trade

W is the financing cost in % per annum

G(X) is the number of days it will take the liquidity provider to get rid of the position at no market impact

Si is the asset specific annual volatility of stock i

R(X) is the annual % rate of return that the liquidity provider can earn on a trade of size X

G(X) is linear in X, R(X) is a decreasing function of X so M is a less than linear function of X

We’ve got the last piece in the puzzle

- Calibrate the R(x) function from reasonable estimates, empirical data and boundary conditions, given the functional form
  – Use market capitalization as a proxy for the “visibility” of a trade, so for bigger firms, R(X) drops faster
  – Thanks to Instinet, we have access to a very rich database of anonymous but detailed trade information

- Select a reasonable value of L
  – Somewhere between 10% and 50% of ADV

- Convert the R(x) function into the b,c, and d coefficients for each stock

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So Now What

- For any trade, or set of trades the Northfield/Instinet algorithm can compute the optimal schedule of transactions that will best balance opportunity costs, risk and direct trading cost over any chosen number of discrete periods

- Recalculate the schedule at any point to reflect unexecuted trades, desired new trades, changes in security prices, or changes in market conditions

- Traders can order automatic, electronic executions in accordance with the schedule

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Conclusions

- Algorithmic trading is an important contributor to trading efficiency
- The use of trading algorithms is new, and some of the methodologies are very simple
- Northfield has developed an algorithmic trading technique that encompasses all important aspect of the problem
- We believe that this algorithm represents a robust and practical solution to optimal trade scheduling over a wide range of trade sizes
- The implementation of methodology on Instinet will be the first fully transparent algorithmic trading solution

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