Risk Budgeting: Concept, Interpretation and Applications

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Eddie Qian, PhD, CFA
Senior Portfolio Manager

260 Franklin Street
Boston, MA 02110
(617) 439-6327

PanAgora

Risk Contribution

- **Risk contribution – attribution of total risk to individual underlying components of a portfolio in percentage terms**
- **Examples**
  - Fixed income portfolio: sector risk, yield curve risk, …
  - Equity portfolio: systematic risk (risk indices/industries), specific risk, …
  - Asset allocation: TAA risk (stock/bond, cap rotation, …), sleeve active risk, …
  - Manager selection, strategy allocation
  - Asset allocation portfolio: beta risk from stocks, bonds, commodities, …
    - Parity portfolios

An Example

- **A simple illustration**
  - Two active strategies – strategy A at 1% active risk, strategy B at 2% active risk, two sources uncorrelated
- **Total active risk**
  - Equals $\sqrt{1\%^2 + 2\%^2} = 2.24\%$
  - What is the risk contribution from strategy A and B?
  - Answer – A contributes 20% and B at 80%
  - Contribution to variance
    - Strategy A $\frac{1}{1+4}$, Strategy B $\frac{4}{1+4}$
- **Contribution based upon variances and covariances**

Mathematical Definition

- **Marginal contribution -** $\frac{\partial s}{\partial w_i}$
- **Risk contribution -** $w_i \frac{\partial s}{\partial w_i}$
- **VaR contribution -** $w_i \frac{\partial \mathrm{VaR}}{\partial w_i}$

- **Well defined in financial engineering terms**
- **But objected by some financial economists for a lack of economic interpretation**

Objections

- Risk is not additive in terms of standard deviation or VaR
- You can only add risk when returns are uncorrelated
- A mathematical decomposition of risk is not itself a risk decomposition
- Risk budgeting only make sense if considered from mean-variance optimization perspective
- Marginal contribution to risk is sensible, but not risk contribution

Loss Contribution

- *Question*: **For a given loss of a portfolio, what are the likely contribution from the underlying components?**
- *Answer*: **The contribution to loss is close to risk contribution**

- **Risk contribution can be interpreted as loss contribution**
- **Risk budgeting ~ loss budgeting**
- ***Risk budgets do add up***

An Asset Allocation Example

- **Balanced benchmark**
  - 60% stocks, 40% bonds
  - Is it really balanced?
- **Risk contribution**
  - Stock volatilities at 15%, bond volatilities at 5%
  - Stocks are actually ***9 times riskier than bonds***
  - Stocks' contribution is roughly 95% and bonds at 5%

$$\frac{6^2 \cdot 3^2}{4^2 \cdot 1^2 + 6^2 \cdot 3^2} = 95\%$$

- **The balanced fund – a misnomer**

Loss Contribution of a 60/40 Portfolio

Loss >	Stocks	Bonds
2%	95.6%	4.4%
3%	100.1%	-0.1%
4%	101.9%	-1.9%

- **Stocks' contribution is almost 100%**
- **It is greater than the theoretical value of 95%, because stocks have greater tail risk**

Mathematical Proof

- ***Question*: For a given loss of a portfolio, what are the likely contribution from the underlying components?**
- ***Answer*: Conditional expectation**

Total risk: $s = \sqrt{w_1^2 s_1^2 + w_2^2 s_2^2 + 2 r w_1 w_2 s_1 s_2}$

Risk contribution:

$$p_1 = \left(w_1 \frac{\partial s}{\partial w_1} \right) \Big/ s = \frac{w_1^2 s_1^2 + r w_1 w_2 s_1 s_2}{s^2}$$

$$p_2 = \left(w_2 \frac{\partial s}{\partial w_2} \right) \Big/ s = \frac{w_2^2 s_2^2 + r w_1 w_2 s_1 s_2}{s^2}$$

Loss contribution? $c_i = E\left(w_i r_i \mid w_1 r_1 + w_2 r_2 = L \right) / L$

Mathematical Proof

- **The loss contribution is approximately risk contribution**
- **Cases where they are identical**
  - Zero expected returns
  - Large losses
  - Mean-variance optimal weights

$$c_1 = p_1 + \frac{p_2 w_1 m_1 - p_1 w_2 m_2}{L} \approx p_1 + \frac{D_1}{L}$$

$$c_2 = p_2 + \frac{p_1 w_2 m_2 - p_2 w_1 m_1}{L} \approx p_2 + \frac{D_2}{L}$$

Mean-variance Optimal

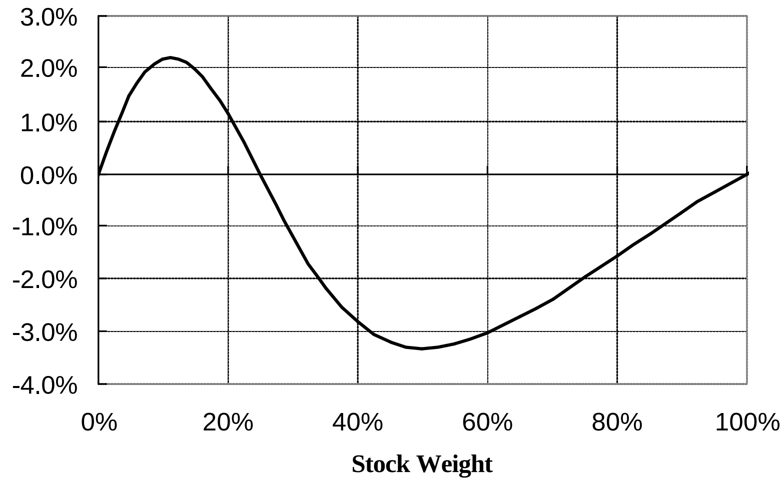
- **Risk contribution is equivalent to loss contribution**
- **Risk contribution is also equivalent to expected return contribution**

$$\frac{w_1 m_1}{p_1} = \frac{w_2 m_2}{p_2} = l$$

- **The interpretation is valid and very close even if the portfolio is not MV optimal**

Loss Contribution

Figure 1 The value of D_1 over standard deviation for asset allocation portfolios



- **The difference between risk contribution and loss contribution is quite small**

Loss Contribution of a 60/40 Portfolio

Loss	Predicted c_1	Realized c_1	N	Predicted Std	Realized Std
-4% to -3%	93.5%	89.8%	45	28.0%	26.1%
-5% to -4%	92.8%	92.7%	23	21.0%	20.7%
-6% to -5%	92.3%	88.1%	11	16.8%	16.1%
-7% to -6%	92.0%	99.5%	9	14.0%	18.7%
-8% to -7%	91.8%	90.1%	8	12.0%	18.6%
-19% to -8%	91.3%	102.4%	12	10.5%	12.3%

- **Realized loss contribution from stocks increases as losses grow**
- **It could be due to higher tail risks of stocks**

Fat Tails

	S&P 500	US LT Gvt	60/40 Portfolio
Avg Return	0.98%	0.46%	0.78%
Stdev	5.61%	2.27%	3.61%
Skewness	0.39	0.66	0.40
Kurtosis	9.58	5.09	7.64
Corr w/ S&P 500	1.00	0.14	0.97

- Ibbotson 1929 - 2004

- **Risk contribution in terms of standard deviation assumes normal distribution**
- **We need to extend the risk contribution to non-normal return portfolio**
  - Hedge funds

Hedge Fund Returns

	Convertible Arbitrage	Dedicated Short	Emerging Markets	Equity Mkt Neutral	Distressed	E.D. Multi-Strategy	Risk Arbitrage	Fixed Income Arb	Global Macro	Long/Short Equity	Managed Futures	Multi-Strategy
Average	8.94	-0.87	8.83	9.80	13.00	10.31	7.82	6.64	13.70	11.77	6.89	9.13
Stdev	4.70	17.66	16.88	2.99	6.64	6.13	4.30	3.79	11.46	10.52	12.20	4.35
IR	1.90	-0.05	0.52	3.28	1.96	1.68	1.82	1.75	1.20	1.12	0.57	2.10
Skewness	-1.36	0.86	-0.62	0.30	-2.85	-2.64	-1.29	-3.26	0.01	0.24	0.04	-1.30
Kurtosis	3.38	2.03	4.30	0.33	17.82	17.57	6.41	17.33	2.50	3.75	0.43	3.76

- CSFB Tremont 01/1994 – 03/2005

- **Some strategies have high IR, but also high negative skewness and high positive kurtosis**

Same Interpretation

- **Value at Risk**
  - Maximum loss at a given probability
  - 95% VaR, 99% VaR
- **Contribution to VaR**
  - Interpretation: the expected contribution to a portfolio loss equaling the size of VaR
  - It changes with different level of VaR
  - It is not easy to calculate because analytic formula is rarely available for VaR

Cornish-Fisher Approximation

- **Cornish-Fisher approximation for VaR**

$$\mathrm{VaR} = m + \tilde{z}_a s$$

$$\tilde{z}_a \approx z_a + \frac{1}{6}\left(z_a^2 - 1\right)s + \frac{1}{24}\left(z_a^3 - 3z_a\right)k - \frac{1}{36}\left(2z_a^3 - 5z_a\right)s^2$$

- **Example**
  - 99% VaR for the 60/40 portfolio
  - Normal assumption VaR at 99% is -7.6%, $z_a = -2.33$
  - Approximate VaR is -13%, $\tilde{z}_a = -3.81$

Cornish-Fisher Approximation

- **Approximation for VaR contribution**
- **We have an analytic expression of VaR**
- **Algebraic function, linear, quadratic, cubic and quartic polynomials**
- **We can then calculate** $w_i \frac{\partial \text{VaR}}{\partial w_i}$

Application to 60/40 Portfolio

Loss	Predicted VaR %	Predicted c_1	Realized c_1
-3.50%	84.90%	93.5%	89.8%
-4.50%	90.50%	92.8%	92.7%
-5.50%	94.20%	92.3%	88.1%
-6.50%	97.10%	92.0%	99.5%
-7.50%	99.20%	91.8%	90.1%
-8.50%	100.90%	91.3%	102.4%

- **Stocks' VaR contribution increases as VaR increases**
- **Capture the high tail risks**
- **Contribution to standard deviation shows declining instead**

Other Applications

- **Need to calculate skewness and kurtosis from historical returns**
- **Risk management and risk budgeting**
    - Asset allocation with traditional assets and hedge funds
        - High moments strategies: event driven, distressed securities, fixed income arbitrage, etc
    - Strategy allocation among managers with non-normal alphas

Risk Budgets Do Add Up

- **Financial interpretation of risk contribution**
  - Loss contribution
  - It applies to both standard deviation and VaR
  - Cornish-Fisher approximation can be used to calculate VaR contribution
  - It should clear up some doubts about the concept
- **Applications**
  - Risk budgeting for active risks
  - Risk budgeting for beta portfolios – parity portfolios
  - Efficient method for risk budgeting of hedge funds