Parameter Estimation Error in Portfolio Optimization

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Main Points for Today

- While portfolio optimization procedures continue to grow in popularity, important caveats remain
  - Appropriateness of the objective function
  - Estimation error in parameters
  - Assumptions of a single period model
- In this presentation, we'll address only the first two issues. The last will be addressed separately.
  - Consider the three major approaches to dealing with estimation error, and their respective strengths and weaknesses
- Its not black magic, its using common sense

Optimization Objective Function

- Almost all commercial portfolio optimizers use the mean-variance objective function described in Levy and Markowitz (1979)

  $U = A - (S^2 / RAP) - C$

  This just says that investor's objective is to maximize risk- adjusted returns, net of costs. Portfolio return variance is the proper measure of risk because the difference between the arithmetic average rate of return and the geometric average rate of return is proportional to the variance (see Messmore, 1995)

Objective Function Issues

- Some people have argued that investors don't mind upside surprises, so the objective should focus on some form of "downside risk" or "fat tail" risks
- Numerous studies have shown that the mean-variance objective function is correct over a broad range of asset management problems
  - Cremers, Kritzman and Page (2003)
- Either a diversified portfolio, or an objective that can be approximated by quadratic utility, is sufficient. Both are not simultaneously needed

Downside Risk

- There are cases where downside risk matters
  - An investor with a concentrated portfolio and a bilinear utility function
  - For example, consider the manager (not the investor) of a hedge fund that is leveraged. Once it loses enough to shut down, the manager doesn't really care how much it loses
  - Another case is a trading desk. Once you go to negative net worth, you're done
- The failure to consider "downside risk" and "fat tail" risk is of minimal consequence in traditional asset management cases
  - The loss of utility is less than one tenth as important as the potential losses from parameter estimation error

Parameter Estimation Error

- In 1952 Markowitz introduces "Modern Portfolio Theory"
  - It says that if you <u>know exactly</u> the parameters (mean, standard deviation, correlation) of the distributions of asset returns, you can form a portfolio that provides either the highest level of return for a given level of risk, or the lowest level of risk for a given level of return
  - It's brilliant and exactly right using the stated assumptions
  - Unfortunately, in the real world we never <u>know</u> this information. We only have estimates of this information for the uncertain future
- Every portfolio optimization problem faces two sources of risk, not one
  - the risks inherent in markets and securities
  - the risk of being wrong in our expectations

Estimation Risk is Not a New Issue

- Lots of literature discusses the problem
  - Stein (1955) shows that traditional sample statistics are not appropriate for multivariate problems
  - A series of papers describe the problem in detail: Barry (1974), Michaud (1989)
  - Empirical tests by Chopra and Ziemba (1993) show that errors in return estimates are more important that errors in risk estimates
  - Jorion (1992) and Broadie (1993) use Monte Carlo simulations to estimate the magnitude of the problem
- Optimizing without consideration of estimation error can be worse (a lot worse!) than not optimizing at all
- The issue is how to take estimation risk into account

A Brute Force Approach

- Use constraints to force the optimized portfolio to “look right”
  - Constrain maximum active position sizes to force the portfolio to be diversified
  - Constrain portfolio attributes to control things like average market cap, P/E or other security properties to “acceptable ranges”
- Portfolio constraints often get in the way of good portfolio construction and limit portfolio performance even when stock selection is good
  - Transfer coefficient from Clarke, daSilva and Thorley (2002)
  - A set of alphas always exists that will cause your portfolio to fall within a set of arbitrary constraints, but *if those alphas and your alphas don’t match you are wasting your predictive power*

Three Better Approaches

- Changing the objective function to explicitly include estimation risk. One form of this approach is often called “robust” optimization
- Bayesian rescaling of the input parameters to certainty-equivalent values
- Use of Monte Carlo simulations or resampling methods to find a range of optimal portfolios, and pick the one you like best

Incorporating Estimation Risk Directly into the Objective Function

- Lets extend our objective function to explicitly include estimation risks

  $U = A - ((S^2 + E^2) / RAP) - C$

  The E^2 term is the incremental risk of estimation error
- Unfortunately if you consider that there are four sources of uncertainty (security risks, estimation error in returns, estimation error in risks, and single period distortion, there are ten separate terms that E^2 has to include
- The problem is how to estimate the right value of E

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Robust Optimization Forms

- Find the allocation that maximizes likely performance under a "worst case" of estimation error
  - Implicit single period assumption. Being prepared for the worst makes sense if we only get one chance
  - If we're likely to survive, why condition on actions in each period on an assumption of the worst case scenario?
  - *Was Pascal right? Or is asset management like football*?
- Assume certain covariance data with ellipsoidal uncertainty for returns
  - Ceria and Stubbs (2004)
- Assume min/max bounds on returns and ellipsoidal uncertainty for covariances
  - Goldfarb and Inyegar (2003)
- Known factor covariances; exposures subject to error
  - Risk error and return errors are othogonal
    Assume min/max bounds on returns and covariances
  - Halldorsson and Tutuncu (2003)

Possible Concerns Adding Stuff to the Objective Function

- Assuming min/max bounds on parameters may not help very much in practice since we're not saying anything about the distribution of the parameters within the boundaries
- For asset allocation, the number of assets for which we need to estimate parameters is usually small relative to the number of data observations that we can observe
- For equity cases, the number of assets is far larger than the number of observations. This can lead to the "covariance matrix of estimation errors" not being positive semi-definite. People resort to a variety of simplifying assumptions
  - Same problem in Black-Litterman
  - Idzorek (2003)
  - Build a factor model of your own estimation errors?

Short Cuts on Estimation Error

- Many use simplifying assumptions
  - The risk of making bad return estimates is proportional to the risk of the stock. This is the mathematical equivalent of changing the value of RAP to be more conservative
- We may have to assume that estimation risks are random (uncorrelated across stocks) If we knew what kind of errors we were going to make, we would avoid making them
  - The risk of making bad return estimates is proportional to the asset specific risk of the stock. This is the math equivalent of changing the asset specific portion of RAP
  - Assume that the risk of making bad return estimates is equal for all stocks. This is the math equivalent of adding an incremental asset specific risk to each stock

A Really Cheap Approximation for the Confidence Interval on Tracking Error

- When you compute a tracking error between a portfolio and a benchmark, you get an R^2 as an output
  - The square root of R^2 is R
- The confidence interval on a correlation coefficient is approximately:

$$(1-R^2) / (n-2)^{.5}$$

  - If we assume that estimation error in tracking error arises from errors in correlation, not the absolute volatility of the portfolio and benchmark, then you can work the algebra backward for a confidence interval on tracking error
  - This is a reasonable approximation for small tracking errors since any bias in the model is likely to be offset as the portfolio and benchmark volatility will be of very similar magnitude
- Empirical results for our risk models are consistent with Richard Young's presentations at recent UBS/Alpha Strategies events

Bayesian Rescaling Methods

- Traditional statistics use samples of data to evidence whether a particular hypothesis is true or false
- Bayesian statistics try to come up with the most "efficient" estimate
  - Start with a "common sense" prior belief
  - Examine the sample data
  - Weight your prior belief and the sample data in inverse proportion to their dispersion
  - Remember "Pascal's Conjecture" on the existence of God?

A Simple Example

- Fama and French (1992) wrote a famous paper that challenged the CAPM. This lead to lots of academic papers with titles like “Is Beta Dead?” and “Is Beta Dead, Again?”
  - Their argument was that the US equity risk premium was about 3% with a standard error of 3% and not statistically significantly different from zero. Therefore it did not exist
- Others countered using Bayesian arguments
  - Its irrational to assume investors would take equity risk for no expected gain
  - Most prior studies had estimated the US equity risk premium was around 6% per year
  - The Fama-French data wasn’t statistically different from 6% either, so the best answer is a compromise between 3% and 6%

Another Example

- You have three assets, A, B, and C for which you have formed return and risk expectations, and from which you need to form a portfolio
  - We know the Markowitz answer is right if all of our parameters are exactly right
- But what if your favorite supreme being shows up and tells you that your estimates of the future are just totally wrong
  - If you have no information, all three assets are equally good
  - The common sense answer is to equal weight the portfolios
- In the real world, our estimates aren't perfect, but they aren't worthless either. The true optimal portfolio is somewhere between the Markowitz solution and equal weighted

Bayesian Approaches

- Jorion (1985, 1986) applies Bayesian rescaling to quantify the preceding problem to estimates from historical sample data
  - It assumes that historical risk data is good, but historical return data is not sufficient
  - The common sense prior is to hold the minimum risk portfolio of risky assets, our starting point is to assume that each asset will have the historic average return across all assets
  - Weight the historic return for each asset, with the average for all
- Black and Litterman (1991)
  - Assume the global markets are efficient. The common sense prior is to hold the world wealth portfolio
  - We can therefore back out the "implied returns" from global asset class weights and weight those in with our forecasts
  - Full use requires the covariance matrix of estimation errors

Bayesian Estimators of Security Returns

- Grinold (1994) introduces an “alpha scaling” rule of thumb for stock alphas. But beware the fine print
  - Grinold assumes that alphas are uncorrelated across stocks. This may be true for true “bottom up” stock picking, but mathematically cannot be true for almost all quantitative strategies that rely on favorable common characteristics across stocks
- Bulsing, Sefton and Scowcroft (2004) create a generalized framework in which both Grinold and Black-Litterman are special cases
  - The asset specific portion of expected alpha should be scaled to asset specific risk. Common factor related portions of expected alpha should be scaled to the volatility of the relevant factor

Bayesian Methods Can Also Be Applied to Risk

- Multi-group risk models are a sort of "stealth" Bayesian approach. You're assuming that every member of the group has the average characteristics of all members.
  - Elton, Gruber and Padberg (1977)
- A Bayesian estimation for security covariance is presented in Ledoit and Wolf (2004)
  - Basically shrinks the differences in correlation and volatilities across securities

Resampling: A Monte Carlo Method

- Bey, Burgess and Cook (1990) introduce resampling in optimization
  - Take all your sample data and randomize it using bootstrap resampling. Recalculate all problem parameters
  - Find the optimal portfolio for each different data scenario
  - Do this lots of times, eliminate outlier portfolios and pick the "optimal" portfolio you like best
  - Addresses errors in both returns and risks
- diBartolomeo (1993) illustrates resampling in Northfield asset allocation procedures
- Gold (1995) uses Northfield resampling optimization to deal with lack of liquidity in real estate portfolios
  - You can't sell part of a building to rebalance your portfolio
  - *Portfolio weights can be thought of in "sensible ranges"*

Parametric Resampling

- Michaud (1998) introduces parametric resampling
  - Convert all input parameters into a multivariate normal distribution
  - Take random draws from the multivariate normal to generate new “scenarios” of data
  - Form an optimal portfolio for each scenario
  - Michaud’s firm has patented a method to average across all optimal portfolios to find a good compromise
  - Scherer and Martin (2005) argue that the averaging process can create biases due to simple constraints such as “long only portfolios”, and is ineffective on long/short portfolios
  - Cardinality constraints (max # of assets) can also be problematic

Peculiarities of Resampling

- Remember the issue of single period assumptions?
  - Markowitz and Van Dyjk (2003) argue that the best way to deal with this is to realize that if parameters are only approximate, we may be indifferent among similar portfolios.
  - We rebalance the portfolio only if the change between the initial portfolio and the new optimal is large enough to be material.
  - They advocate testing for indifference, but say its mathematically very difficult (in closed form)
- This is comparable to the Bey, Burgess, Cook procedure to eliminate outliers from the set of scenario optimal portfolios
- Very beneficial to control of trading costs
- Much more numerically intensive than other methods

Summary

- Except for rare cases, misspecification of the objective function is not a major concern for portfolio optimization
- Dealing rigorously with estimation error is a critical step in the use of portfolio optimization
  - We have three methods currently available to address estimation errors
  - Used properly, all three will produce relatively similar results in terms of portfolio return errors
  - Resampling methods have additional benefits related to trading costs
- The Northfield Open Optimizer will soon incorporate our own version of the alpha rescaling ala Bulsing, Sefton and Scowcroft, and risk adjustment vaguely related to Ledoit and Wolf.
  - We'll also introduce a entirely new method for dealing with the distortions caused by the single period assumption, to particularly address trading costs

References

- Levy, H. and H. M. Markowitz. "Approximating Expected Utility By A Function Of Mean And Variance," American Economic Review, 1979, v69(3), 308-317.
- Messmore, Tom. "Variance Drain," Journal of Portfolio Management, 1995, v21(4), 104-110.
- Cremers, Jan, Mark Kritzman and Sebastien Page. "Portfolio Formation with Higher Moments and Plausible Utility", Revere Street Working Paper Series 272-12, November. 2003
- Markowitz, Harry. "Portfolio Selection," Journal of Finance, 1952, v7(1), 77-91.
- Barry, Christopher B. "Portfolio Analysis Under Uncertain Means, Variances, And Covariances," Journal of Finance, 1974, v29(2), 515-522.
- Michaud, Richard O. "The Markowitz Optimization Enigma: Is 'Optimized' Optimal?," Financial Analyst Journal, 1989, v45(1), 31-42.

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References

- Chopra, Vijay K. and William T. Ziemba. "The Effect Of Errors In Means, Variances, And Covariances On Optimal Portfolio Choice," Journal of Portfolio Management, 1993, v19(2), 6-12.
- Jorion, Philippe. "Portfolio Optimization In Practice," Financial Analyst Journal, 1992, v48(1), 68-74.
- Clarke, Roger, Harindra de Silva and Steven Thorley. "Portfolio Constraints And The Fundamental Law Of Active Management," Financial Analyst Journal, 2002, v58(5,Sep/Oct), 48-66.
- Fama, Eugene F. and Kenneth R. French. "The Cross-Section Of Expected Stock Returns," Journal of Finance, 1992, v47(2), 427-466.
- Jorion, Philippe. "International Portfolio Diversification With Estimation Risk," Journal of Business, 1985, v58(3), 259-278.
- Jorion, Philippe. "Bayes-Stein Estimation For Portfolio Analysis," Journal of Financial and Quantitative Analysis, 1986, v21(3), 279-292.

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References

- Black, Fischer and Robert Litterman. "Asset Allocation: Combining Investor Views With Market Equilibrium," Journal of Fixed Income, 1991, v1(2), 7-18.
- Grinold, Richard C. "Alpha Is Volatility Times IC Times Score," Journal of Portfolio Management, 1994, v20(4), 9-16.
- Gold, Richard B. "Why The Efficient Frontier For Real Estate Is 'Fuzzy'," Journal of Real Estate Portfolio Management, 1995, v1(1), 59-66.
- Stein, Charles. “Inadmissability of the Usual Estimator for the Mean of Multivariate Normal Distribution”, Proceedings of the 3rd Berkeley Symposium on Probability and Statistics, 1955.
- Broadie, Mark. “Computing Efficient Frontiers with Estimated Parameters”, Annals of Operations Research, 1993.
- Bulsing, Mark, James Sefton and Alan Scowcroft, “Understanding Forecasting: A Unified Framework for both Analyst and Strategy Forecasts”, UBS Working Paper, 2003

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References

- Bey, Roger, Richard Burgess and Peyton Cook, “Measurement of Estimation Risk in Markowitz Portfolios”, University of Tulsa Working Paper, 1990
- diBartolomeo, Dan, “Portfolio Optimization: The Robust Solution”, Northfield Working Paper, 1993
- Michaud, Richard. 1998. “Efficient Asset Management”, Cambridge, MA, Harvard Business School Press.
- Sherer, Bernt and R. Douglas Martin. 2005. “Introduction to Portfolio Optimization with NUOPT and S-PLUS”, New York, Springer.
- Elton, Edwin J., Martin J. Gruber and Manfred W. Padberg. "Simple Rules For Optimal Portfolio Selection: The Multi Group Case," Journal of Financial and Quantitative Analysis, 1977, v12(3), 329-345.

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References

- Idzorek, Thomas. "A Step-by-Step Guide to the Black-Litterman Model", *Zephyr Associates Working Paper*, 2003.
- Ceria, S. and Robert Stubbs. "Incorporating Estimation Error in Portfolio Optimization: Robust Efficient Frontiers", *Axioma Inc. Working Paper,* 2004.
- Ledoit, Olivier and Michael Wolf, "Honey, I Shrunk the Sample Covariance Matrix", *Journal of Portfolio Management*, 2004.
- Goldfarb, D. and G. Iyengar. "Robust Portfolio Selection Problems", *Mathematics of Operations Research*, 2003.
- Halldorsson, B. and R.H. Tutuncu. "An Interior Point Method for a Class of Saddle Point Problems", *Journal of Optimization Theory and Applications*", 2003.

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