Long/short portfolio behavior with barriers

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Outline

This presentation is based on a Morgan Stanley Investment Management working paper written by Thomas Hewett and Kenneth Winston.

Part 1

The modeling problem

Long/short portfolios

•Assume we have a long/short portfolio composed of two long-only sides

•L(T) is the long side's value at time T, S(T) the short side, and the portfolio's value is L(T)-S(T).

•To start, we assume each side follows standard correlated geometric Brownian motion (GBM):

$$\frac{dL}{L} = a_L dt + s_L dZ_L \quad \frac{dS}{S} = a_S dt + s_S dZ_S \quad dZ_L dZ_S = r dt$$

$$Z_L, Z_S \sim N(0,1)$$

Lognormals & blowups

- Linear combinations of lognormals (GBM) are not lognormal. In fact, they are not anything tractable
- For large numbers of well-behaved GBM’s, assuming the (long-only) sum is lognormal may not be too far off
- For a small number (e.g. 2) of possibly ill-behaved GBM’s, can’t get away with this
- In particular, the value of the short side can exceed the value of the long side, commonly called a blowup or a meltdown
- If this happens
  1. A GBM does not suffice (can’t take on negative values)
  2. The risk manager may get fired

- When we realized #2 above, we immediately began studying the problem intensely

Using ratios to avoid RMFC

- The risk-manager-firing-condition (“RMFC”) L(T)-S(T)<0 is equivalent to L(T)/S(T)<1.
- We can avoid RMFC by keeping the ratio above one
  - (Not really. If L(0)=$200,000,000 and S(0)=$100,000,000, and at some time T, L(T)=$0.02 and S(T)=$0.01, the ratio is OK but for all practical purposes we have attained RMFC. Will return to this later.)
- Can use Itô’s lemma on the ratio f=L/S to obtain the GBM

$$\frac{df}{f} = A\,dt + \Sigma dZ$$

$$A = a_L - a_S + s_S^2 - r s_L s_S \qquad \Sigma^2 = s_L^2 + s_S^2 - 2 r s_L s_S$$

$$\Sigma dZ = s_L dZ_L - s_S dZ_S$$

The terminal distribution

- The RMFC at time T can be expressed more generally as Pr(L(T)/S(T)=r), where r is some critical ratio (bankruptcy is r=1).
- Can solve for this exactly as a normal distribution cumulant
  - Pr(L(T)/S(T) =r)=N(D_1), where

$$D_1 = \frac{\ln\left(\frac{rS(0)}{L(0)}\right) - (A - \Sigma^2/2)T}{\Sigma\sqrt{T}}$$

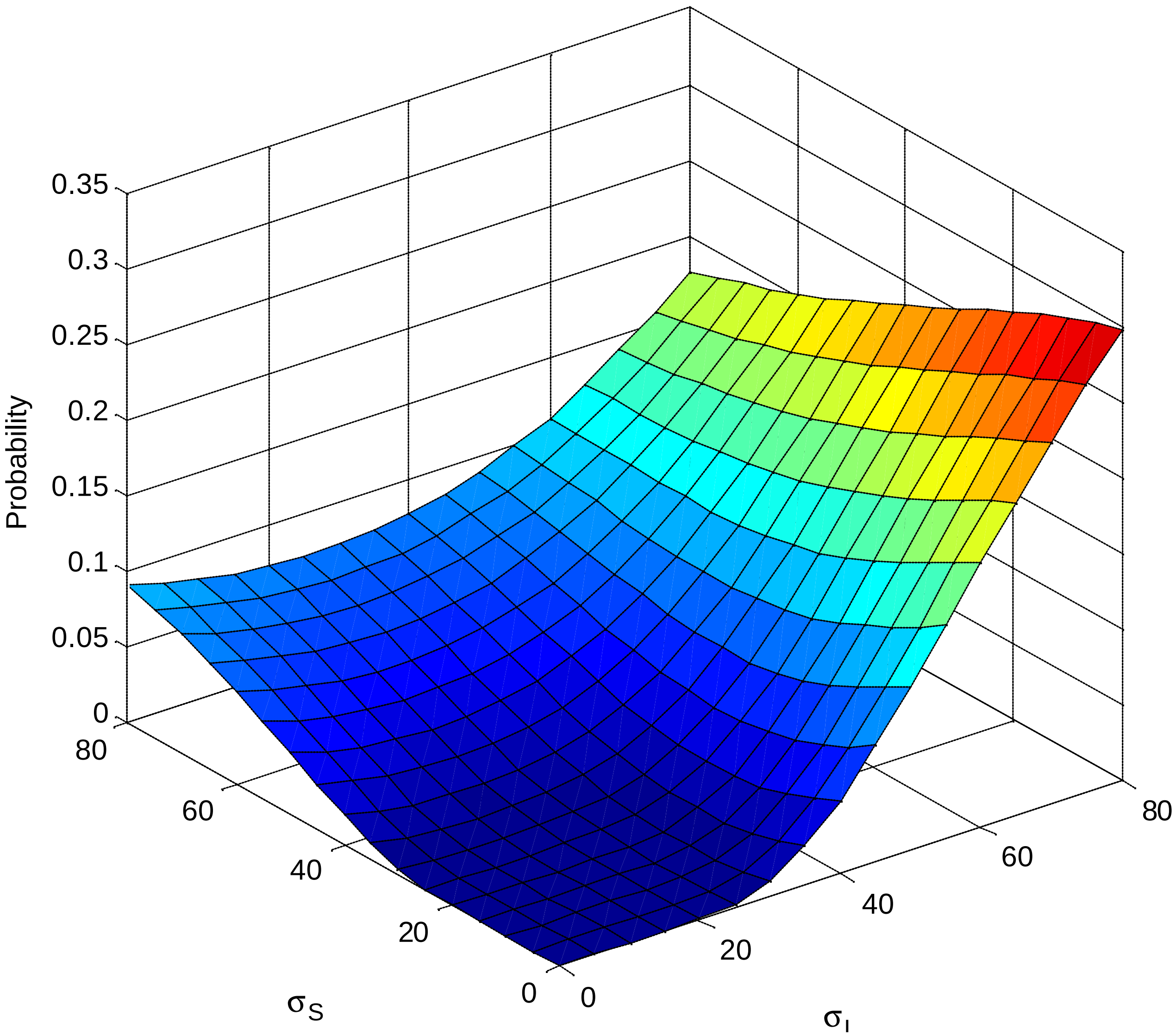
Parameter	Description	Value
$\mathbf{a}_L$	Long side drift	3%
$\mathbf{s}_L$	Long side volatility	20%
$\mathbf{a}_S$	Short side drift	-2%
$\mathbf{s}_S$	Short side volatility	15%

Parameter	Description	Value
?	Long/short correlation	.5
L(0)	Initial long	2
S(0)	Initial short	1

We use these parameters for our examples unless otherwise noted

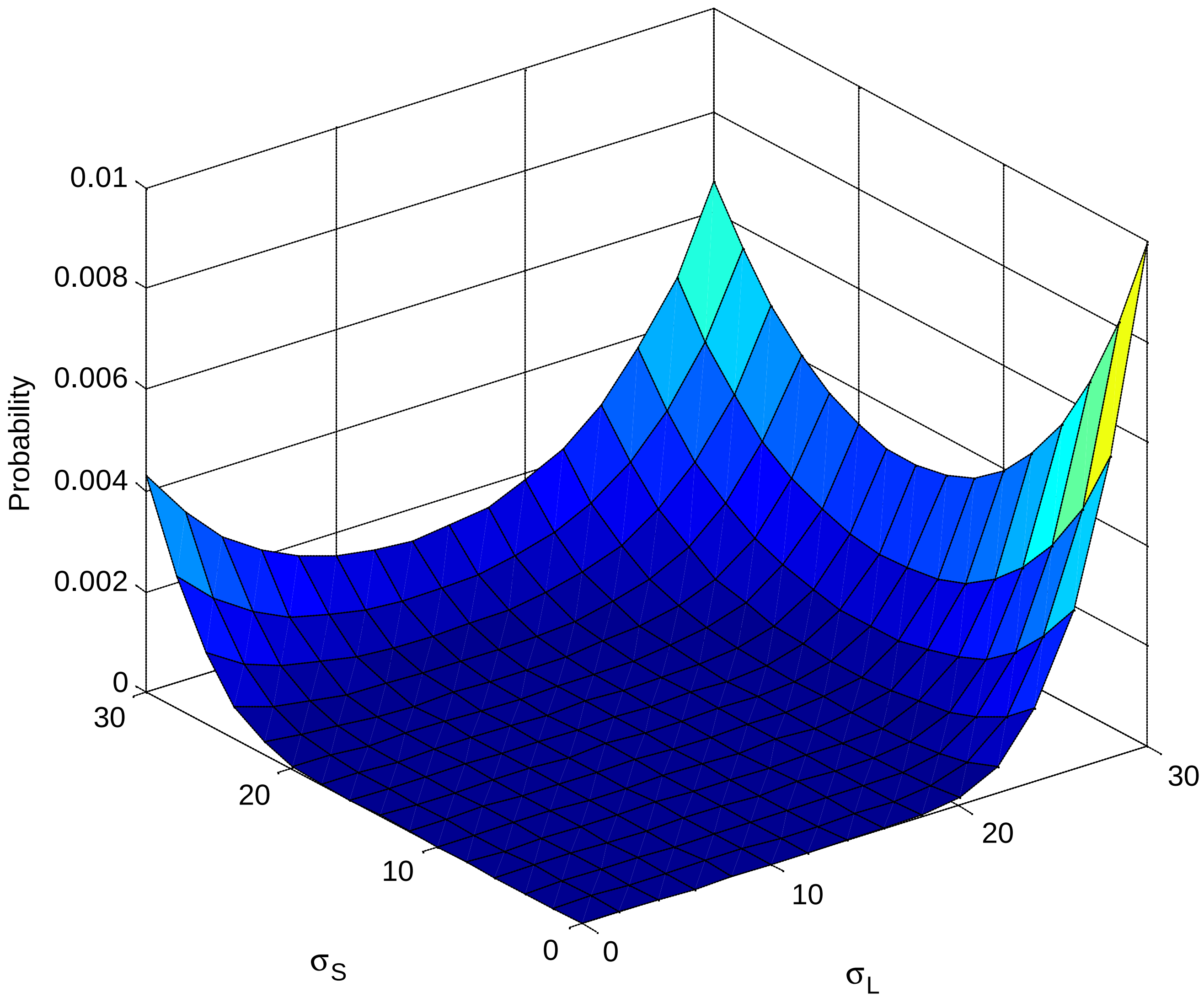
Failure (bankruptcy, r=1) surface (high vols)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation



Failure (bankruptcy, r=1) surface (vols < 30)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation



Stopping time

- The GBM for the ratio allows paths where L(t)/S(t)<r while L(T)/S(T)>r, 0<t<T. In practice, this would not be allowed.
- This is an absorbing barrier problem. We define

$$t_r = Inf\{t : t > 0, L(t)/S(t) \le r\}$$

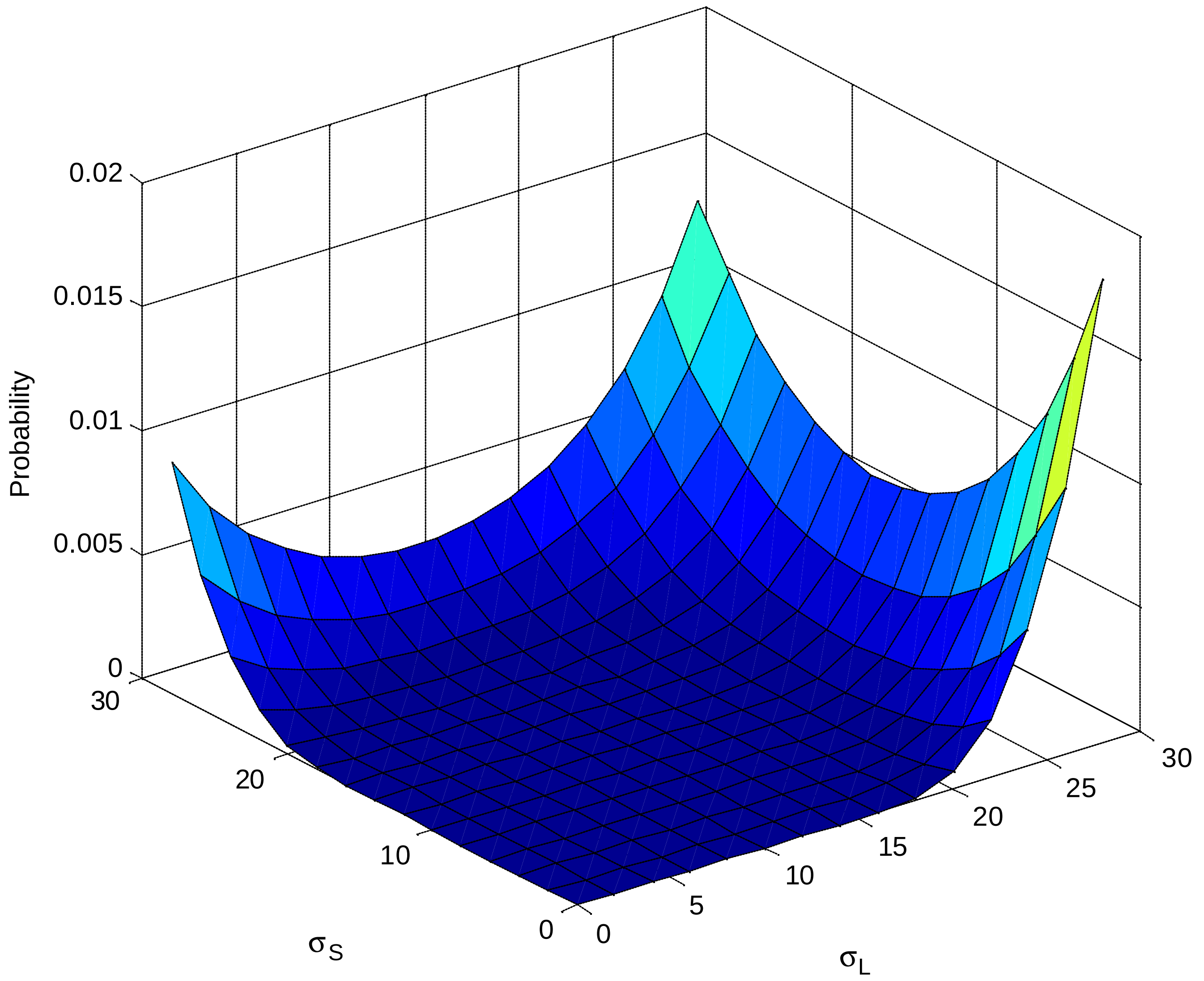
- We compute the probability that the first stopping time t_r is after time T – that is, that we survive until at least time T:

$$\Pr(t_r \le T) = N(D_1) + \left(\frac{L(0)}{rS(0)}\right)^{1-\frac{2A}{\Sigma^2}} N(D_2) \qquad D_2 = D_1 + 2\frac{\left(A - \Sigma^2/2\right)\sqrt{T}}{\Sigma}$$

- Note this is the same as the terminal probability shown previously, plus an additional term that is always positive.

Combined failure probability (vols < 30)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation



Part 2

Difference analysis – analytic approximations

The Kirk approximation

- As noted above, while ratios give an exact answer for the case of bankruptcy (r=1), they can be misleading.
- Assume an initial budget constaint L(0)-S(0)=1. We really want Pr(L(T)-S(T)=k)=Pr(L(T)/(S(T)+k) =1)=Pr((L(T)-k)/S(T) =1) for values of k=0.
- L/(S+k) is not a GBM. Analogous to the problem of pricing an option on the spread of two GBM-distributed securities – a problem for which there is no known solution.
- [Kirk 1995] used an approximation in a similar problem pricing barrier options:

$$\frac{d(S+k)}{S+k} = \left(\frac{S}{S+k}\right)\frac{dS}{S} = \frac{S}{S+k}\left(a_S dt + s_S dZ_S\right)$$

$$\approx \frac{S(0)}{S(0)+k}\left(a_S dt + s_S dZ_S\right)$$

Translation of previous results to differences

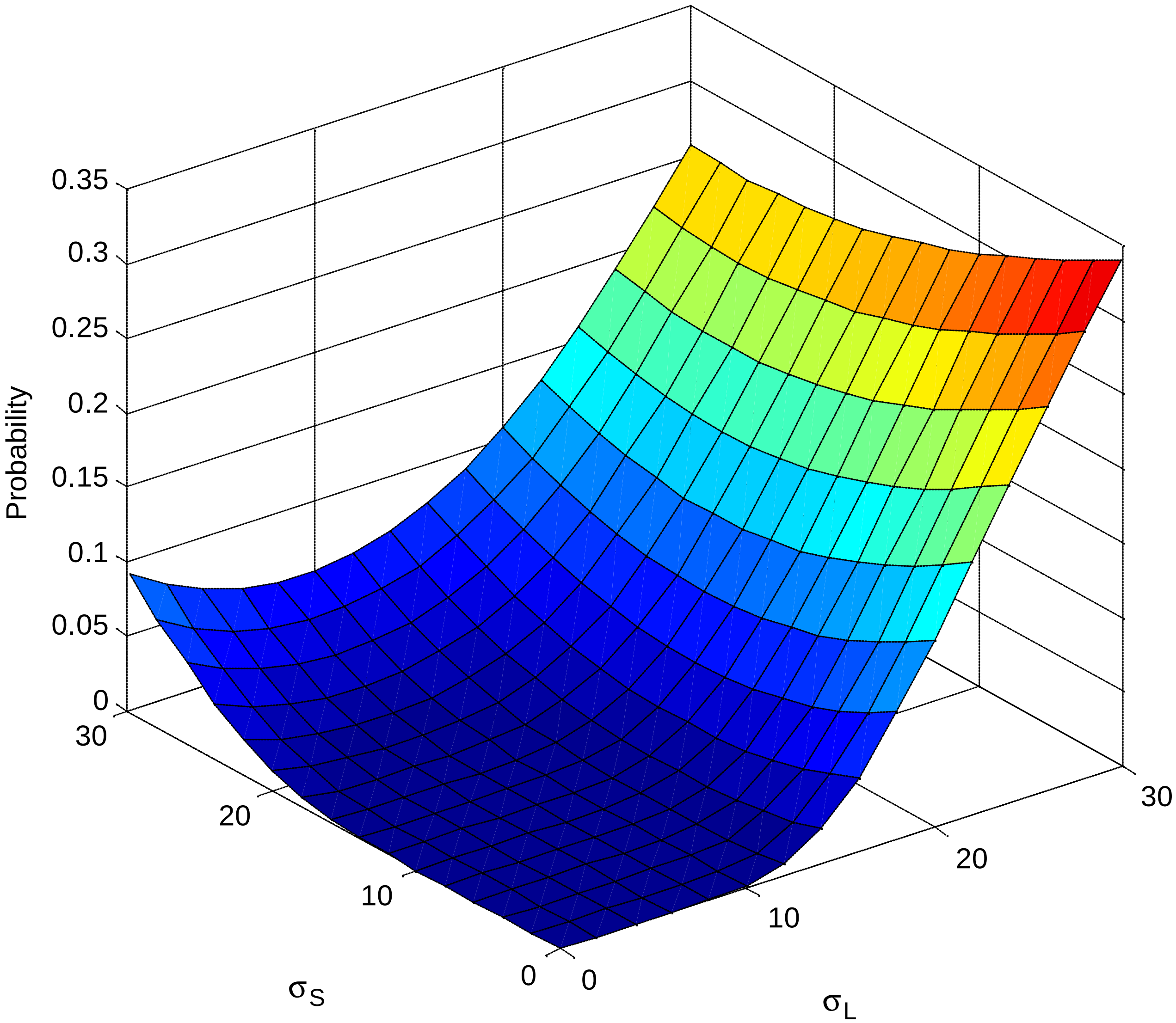
- Using the Kirk approximation, we can translate the ratio results into results of similar form, obtaining approximate expressions for
  - Pr(L(T)-S(T)=k)=Pr(L(T)/(S(T)+k) =1)=Pr((L(T)-k)/(S(T) =1).
- Define the first stopping time for the difference as

$$t_k = Inf\{t : t > 0, L(t) - S(t) \leq k\}$$

- We can similarly obtain an approximate expression for Pr(t_k=T)
- Monte Carlo analysis over a wide variety of parameters confirms that the approximations are very accurate

Failure probability (vols < 30)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation
- 50% drawdown absorbing barrier



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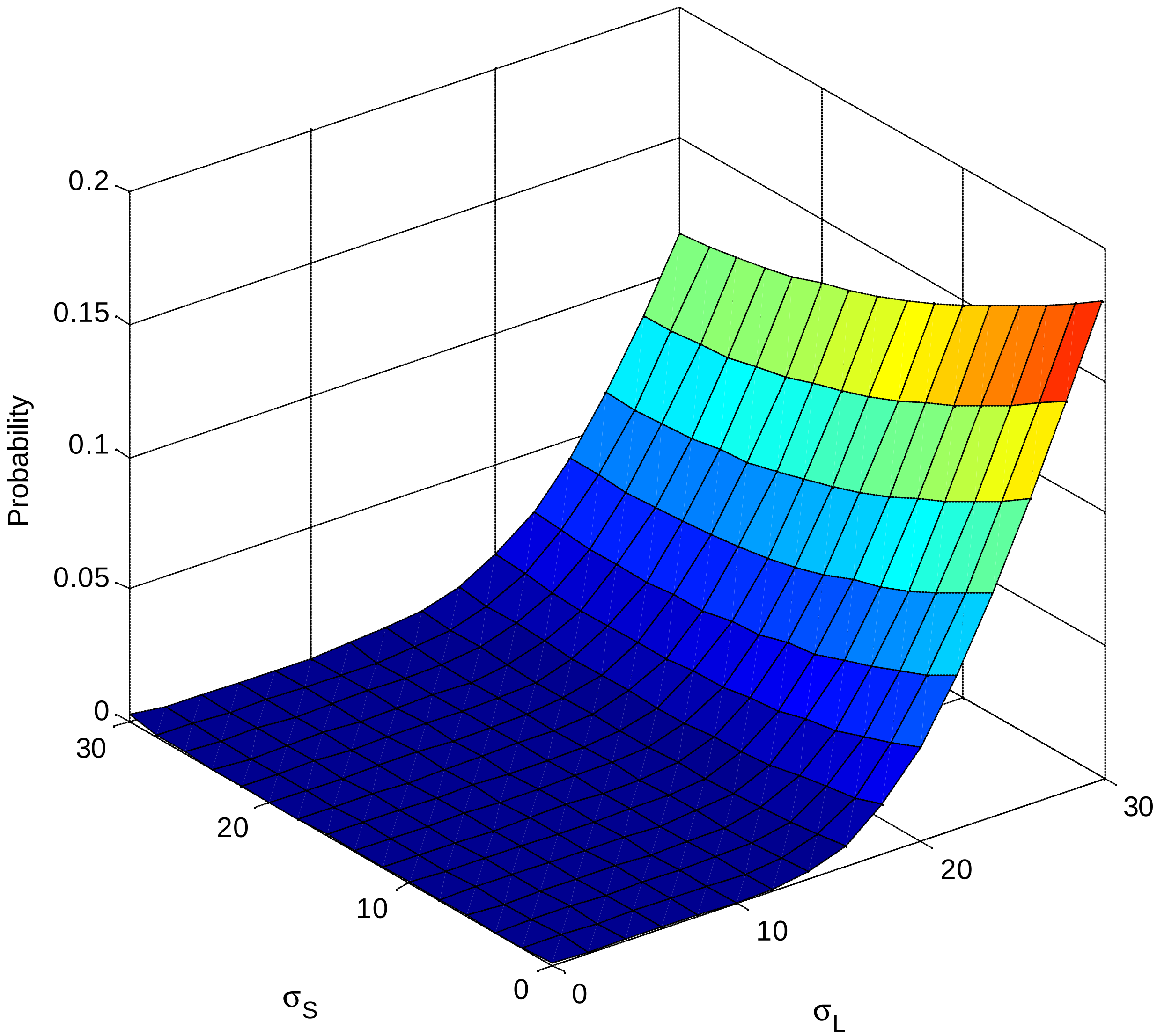
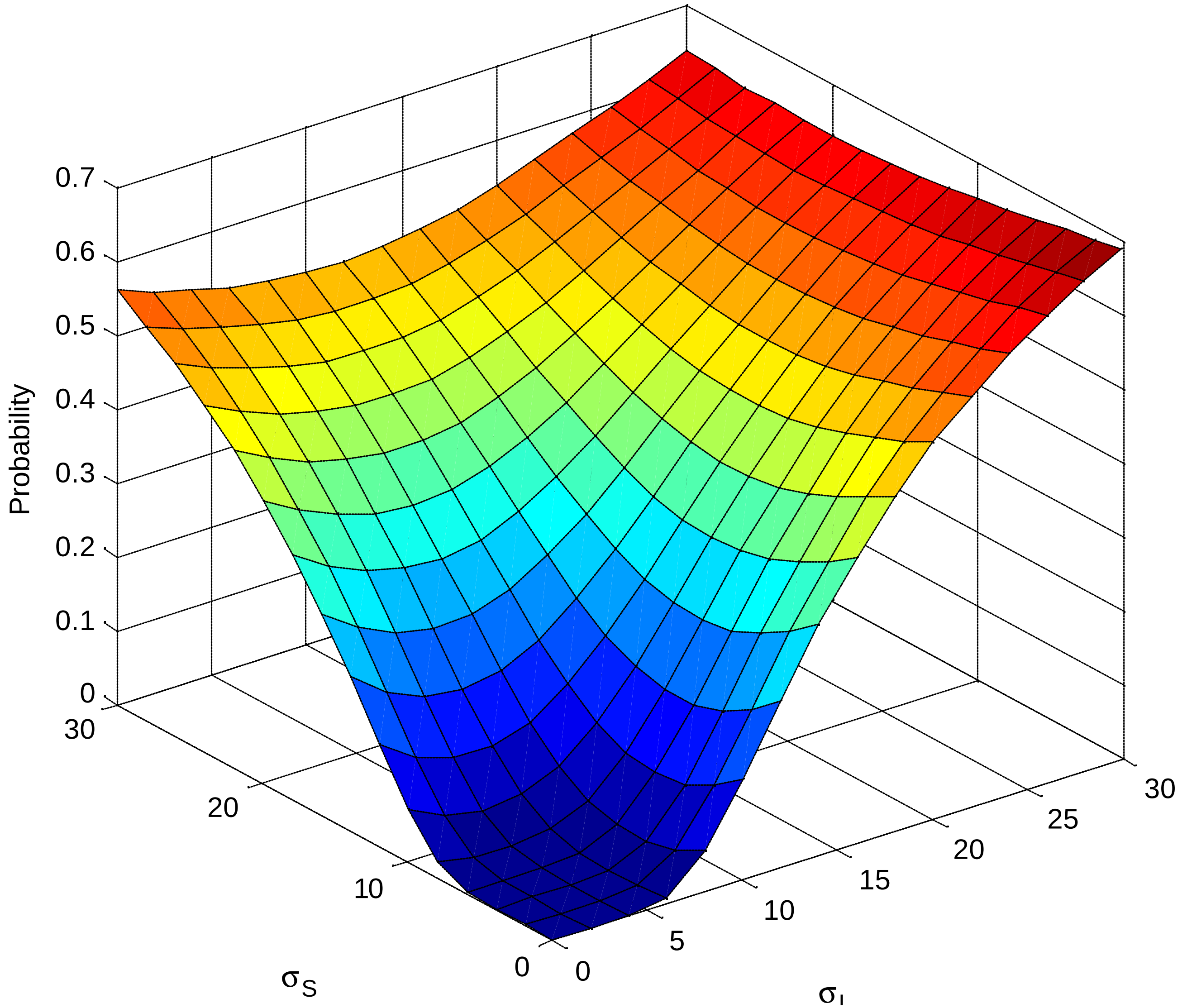
Failure probability (lower leverage)

- Initial leverage=2 (long=1.5, short=.5)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation
- 50% drawdown absorbing barrier



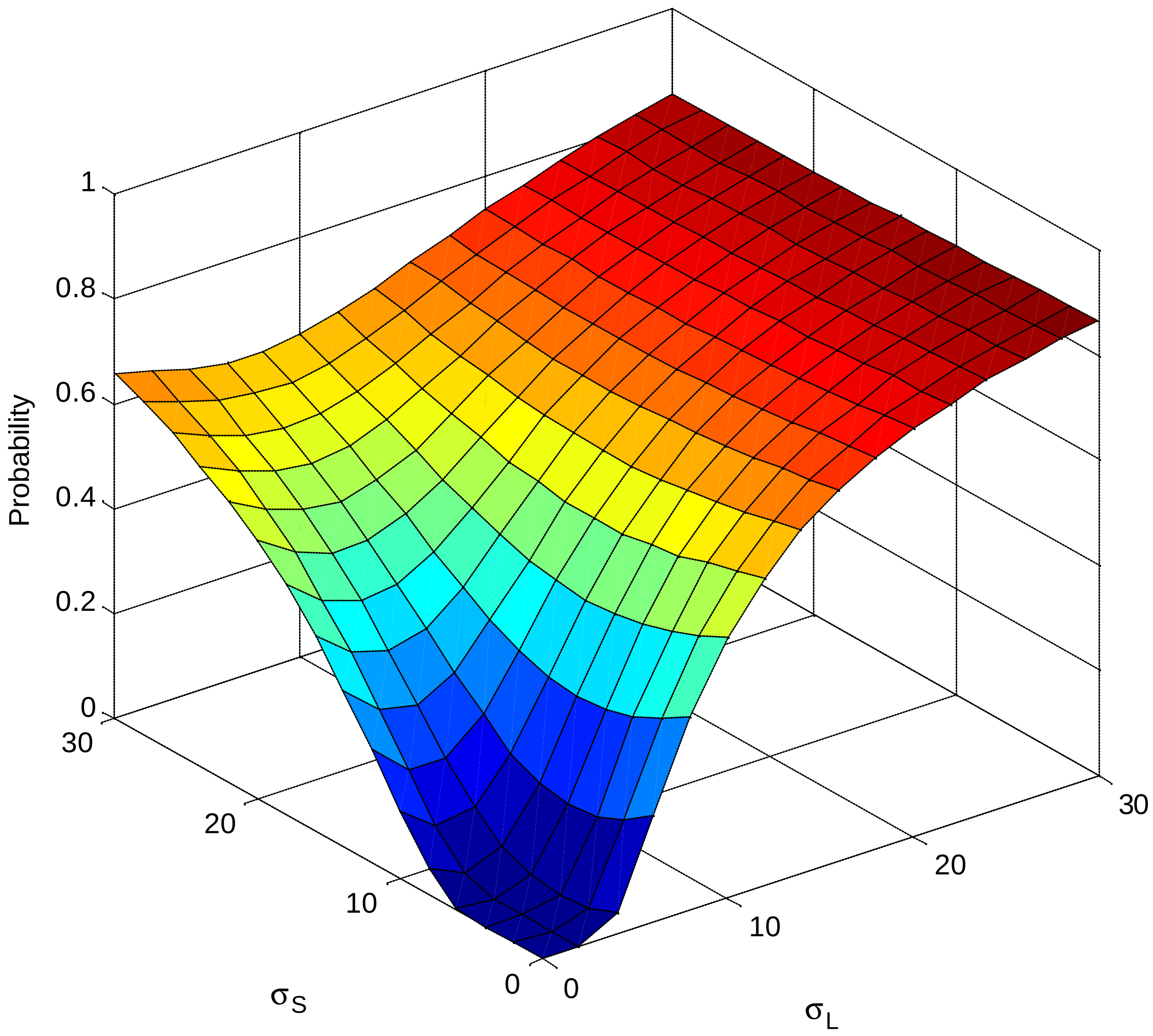
Failure probability (higher leverage)

- Initial leverage=8 (long=4.5, short=3.5)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- 5 correlation
- 50% drawdown absorbing barrier



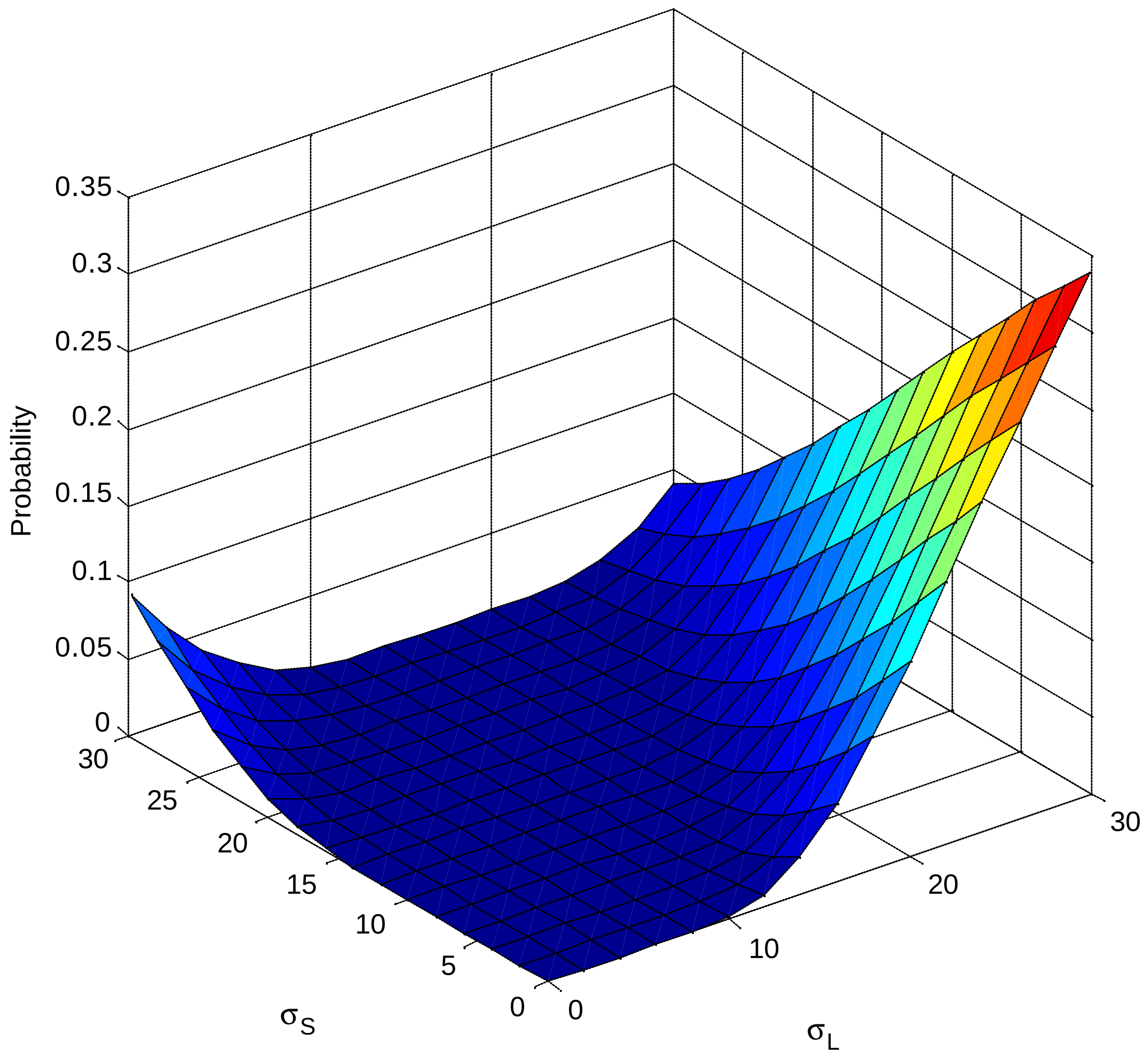
Failure probability (less drawdown allowed)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation
- 90% drawdown absorbing barrier



Failure probability (high correlation)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .9 correlation
- 50% drawdown absorbing barrier



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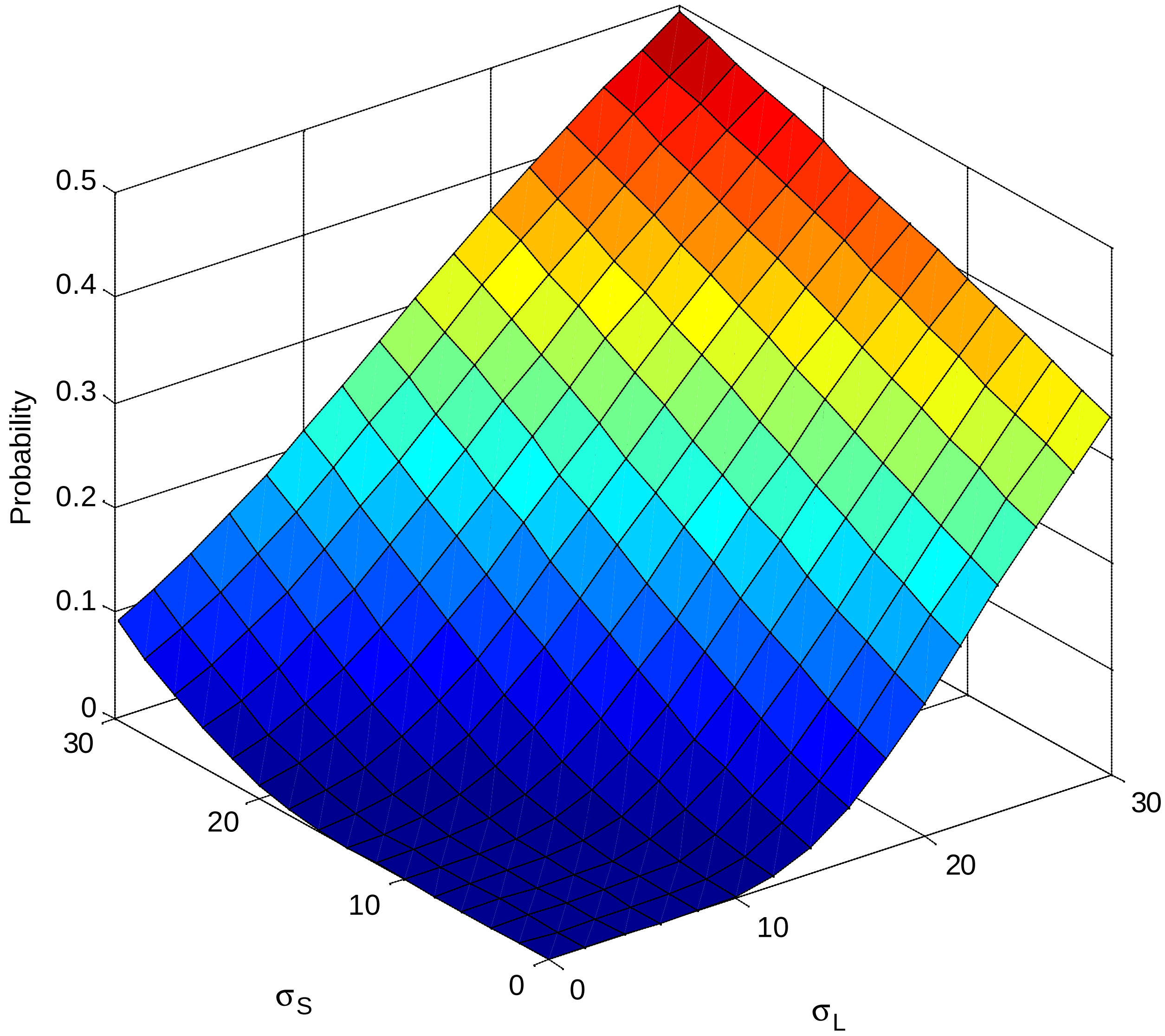
Failure probability (negative correlation)

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- -.5 correlation
- 50% drawdown absorbing barrier



Part 3

Success surfaces

Combining success metric with failure avoidance

- Previous calculations have focused on failure
- Focusing only on avoiding failure leads to excessive risk avoidance.
- We compute the joint probability of success (portfolio value reaches K) given that we avoid any interim failure (portfolio value below k). The cumulative "win" probability is

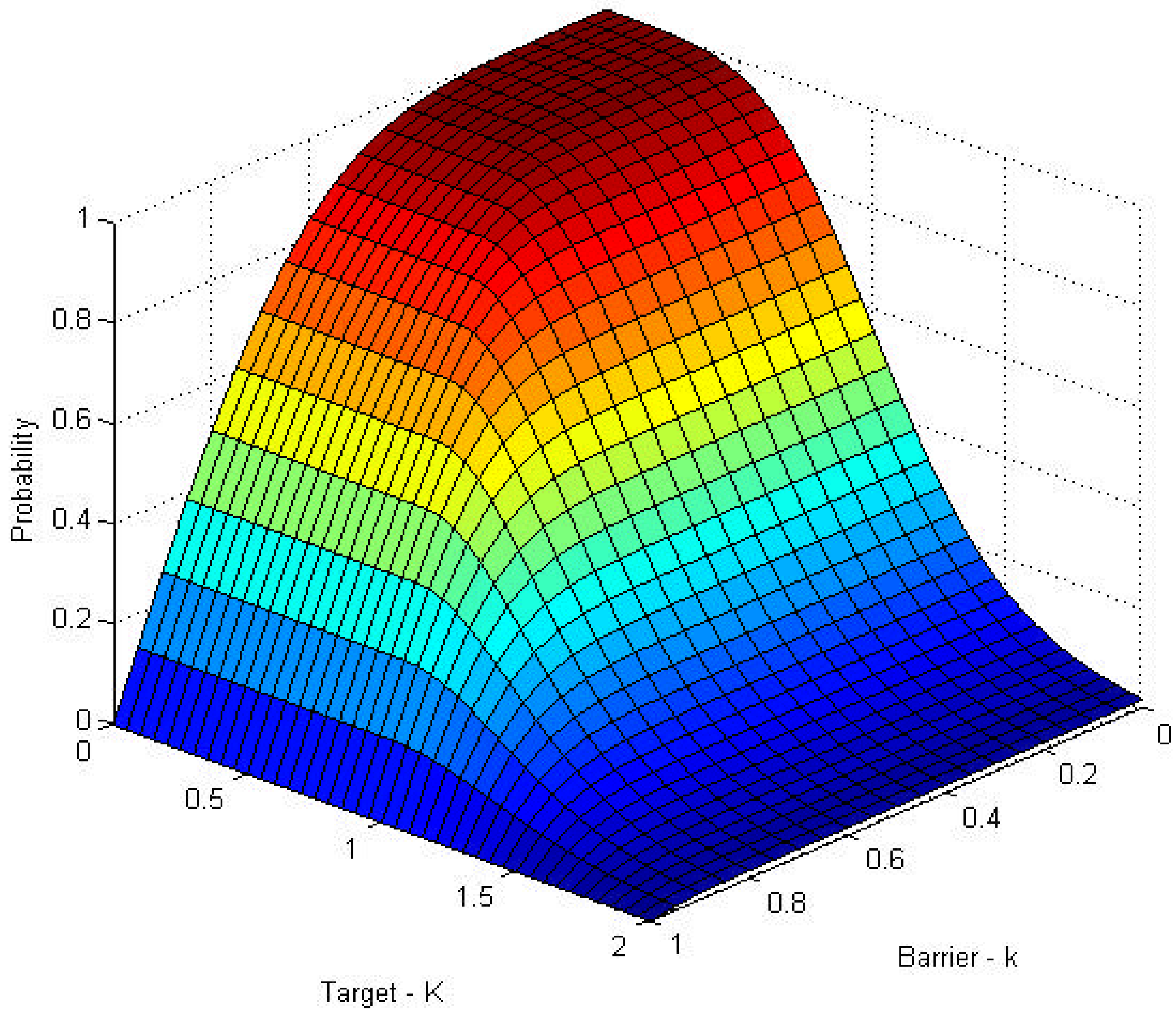
$$W(K,k) = \Pr(L(T) - S(T) \geq K, \\ \forall t \in [0,T]: L(t) - S(t) > k)$$

- [Lee 2004] priced barrier options using a trivariate normal distribution function. We started with the Lee formula and adapted it to our case. Fortunately it reduced to a bivariate normal.
- Clearly the mathematics of our problem is similar to the mathematics of barrier option pricing. <u>CONJECTURE: there is a portfolio including barrier options that maps 1-1 to our problem.</u>

Success surface – basic parameters

W(K,k)

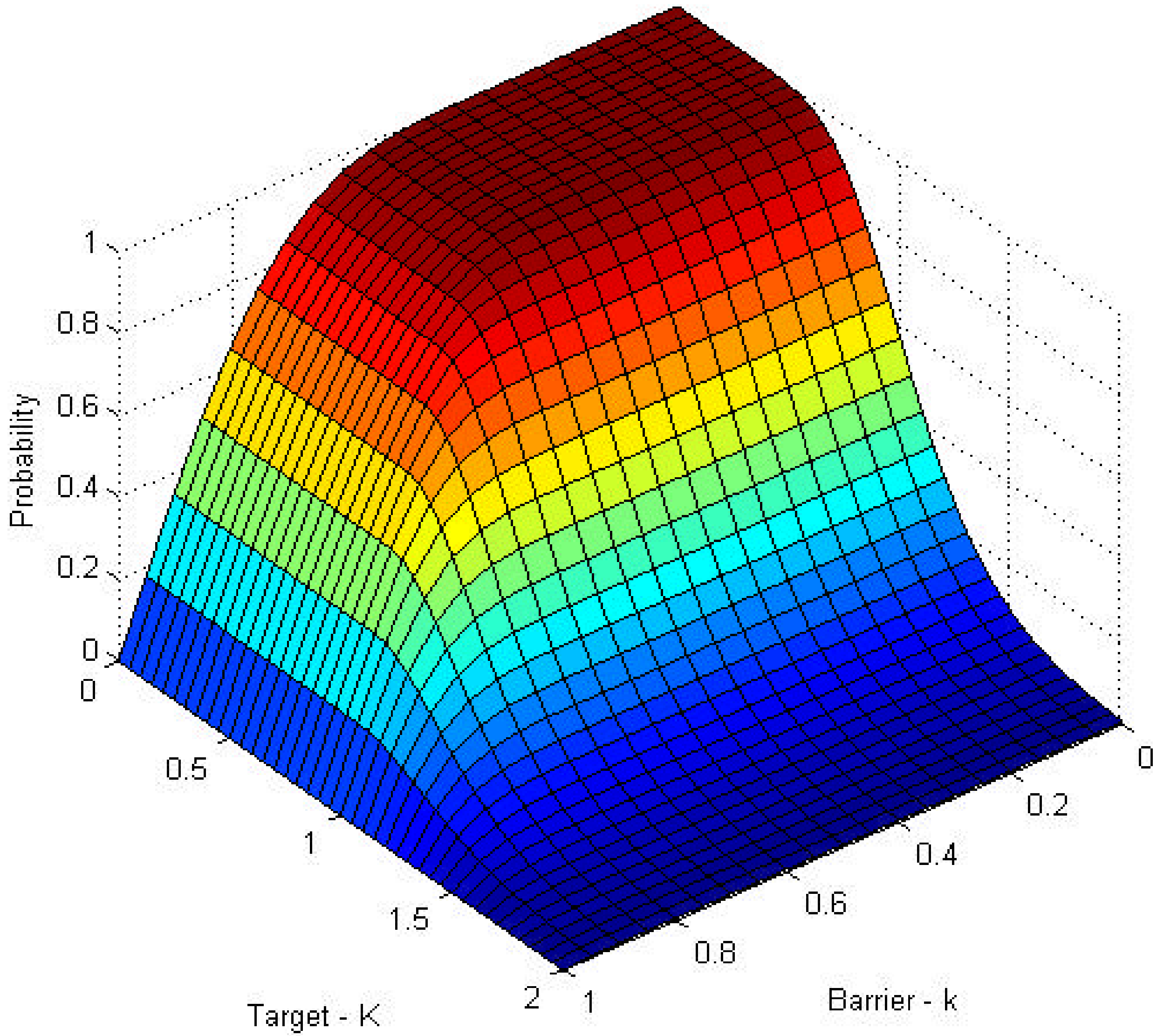
where we have 300bps skill on long side (a_L=3%), 200bps skill on short side (a_S=-2%), long vol s $_L$=20%, short vol s $_S$=15%, 50% correlation (?=.5), initial leverage 3 (L(0)=2, S(0)=1), 1 year.



Success surface – high correlation

W(K,k)

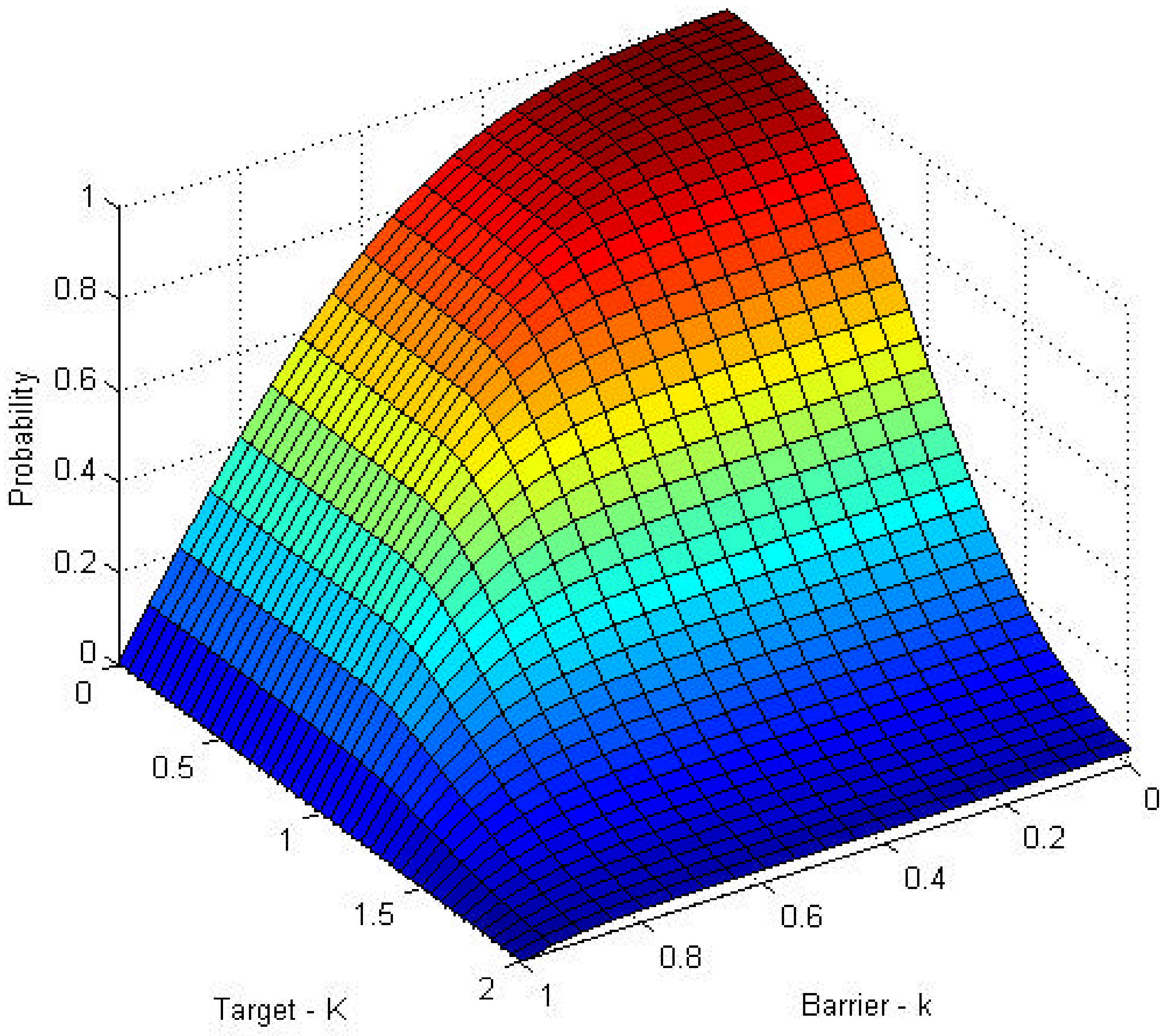
where we have 300bps skill on long side (a_L=3%), 200bps skill on short side (a_S=-2%), long vol s $_L$=20%, short vol s $_S$=15%, high correlation (?=.9), initial leverage 3 (L(0)=2, S(0)=1), 1 year.



Success surface – no correlation

W(K,k)

where we have 300bps skill on long side (a_L=3%), 200bps skill on short side (a_S=-2%), long vol s $_L$=20%, short vol s $_S$=15%, no correlation (?=0), initial leverage 3 (L(0)=2, S(0)=1), 1 year.



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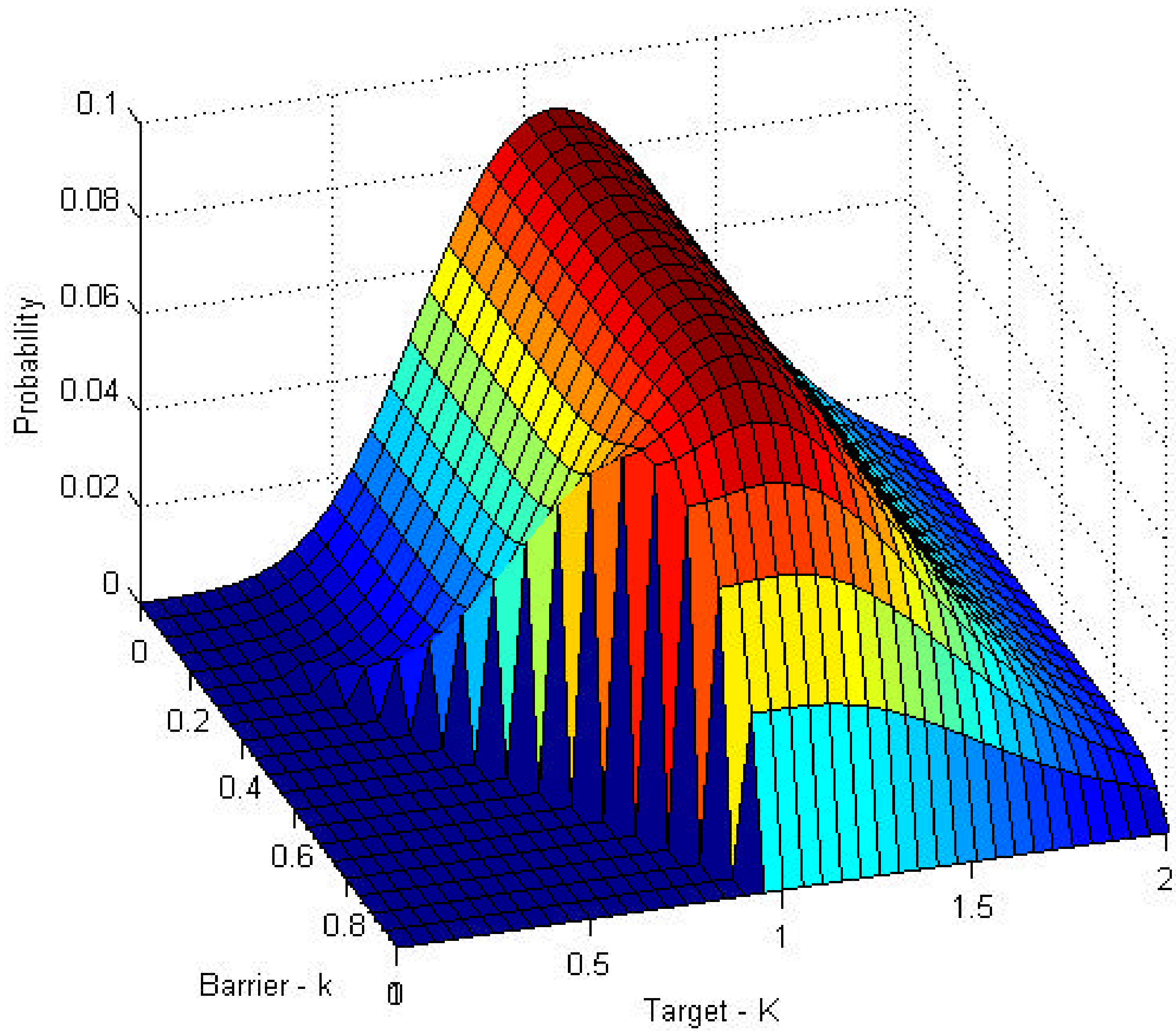
Incremental success surface – double skill

W(K,k)

where we have 600bps skill on long side (a_L=6%), 400bps skill on short side (a_S=-4%), long vol s $_L$=20%, short vol s $_S$=15%, 50% correlation (?=.5), initial leverage 3 (L(0)=2, S(0)=1), 1 year.



Part 4

Risk management techniques

Repositioning

•So far we have seen that unmanaged long/short portfolios usually have large chances of unacceptable drawdowns

•Thus we can't leave them unmanaged for long – in our examples, we used one year

•We need to shorten the examination period, thus reducing the unacceptable drawdown risk. At the end of each of the shorter periods, we have to decide if and how to reposition the portfolio

•What repositioning – that is, what risk management techniques – maximize success while minimizing the chance of an unacceptable drawdown?

Some techniques

- Maintain constant leverage
- Set VaR limits
- Set overall portfolio volatility limits
- Stop losses – various forms of put replication
- Maintain large cash balances
- Lower long volatility or increase long/short correlation

Constant leverage

- We can solve constant leverage ? analytically; it is a GBM:

$$\frac{d(L-S)}{L-S} = a_l\,dt + s_l\,dZ$$

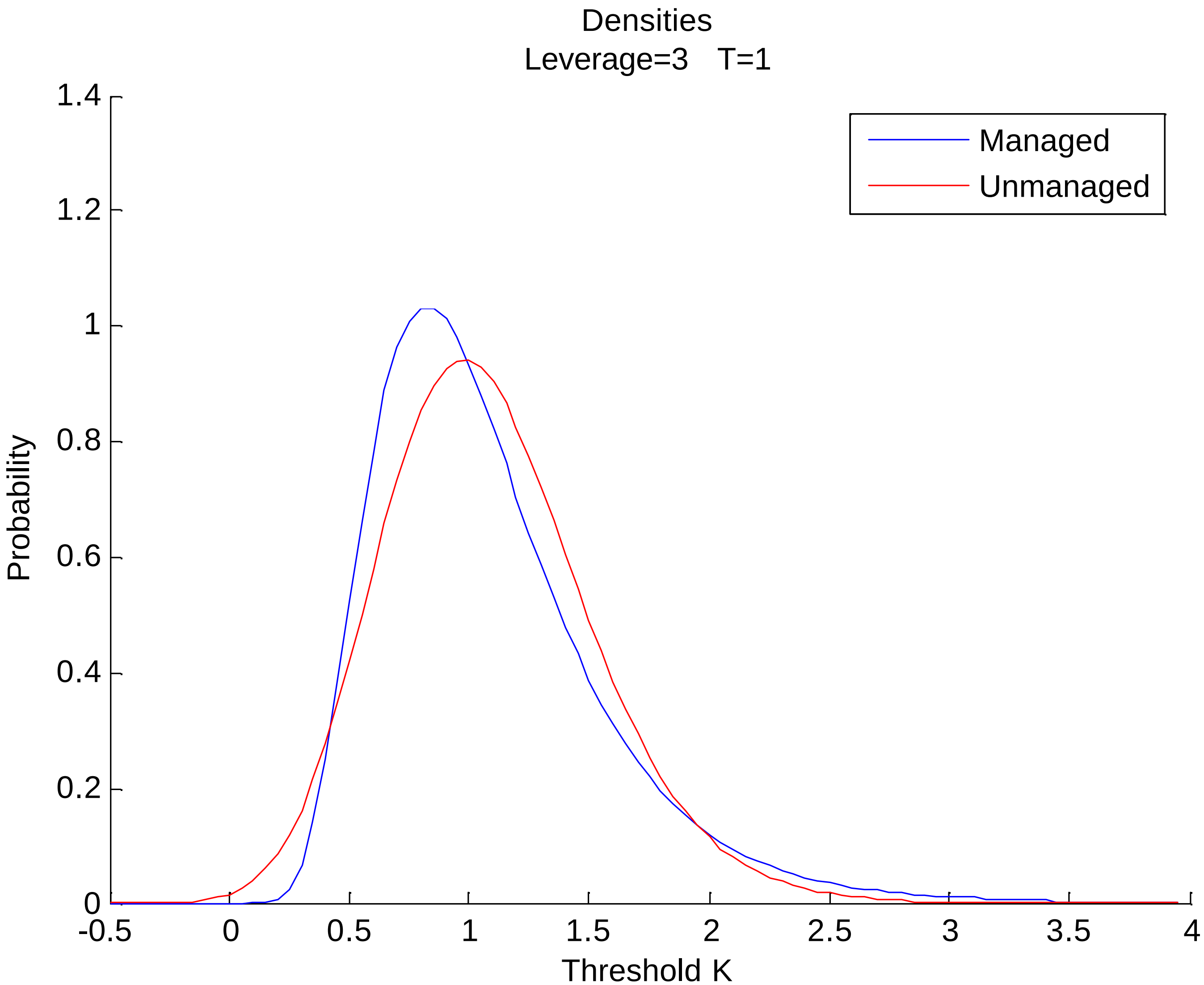
$$a_l = a_L \frac{l+1}{2} - a_S \frac{l-1}{2} \qquad s_l^2 = \frac{1}{4}\left((l+1)^2 s_L^2 - 2(l^2-1)\,r\,s_L s_S + (l-1)^2 s_S^2\right)$$

- From this we can compute the probability of hitting a drawdown absorbing barrier k with constant leverage. For k=0, the probability is zero, so this is an effective form of risk management in this sense

Terminal density function

Constant leverage versus unmanaged

Interaction between managed (constant leverage) and unmanaged is complex. The lines cross several times.



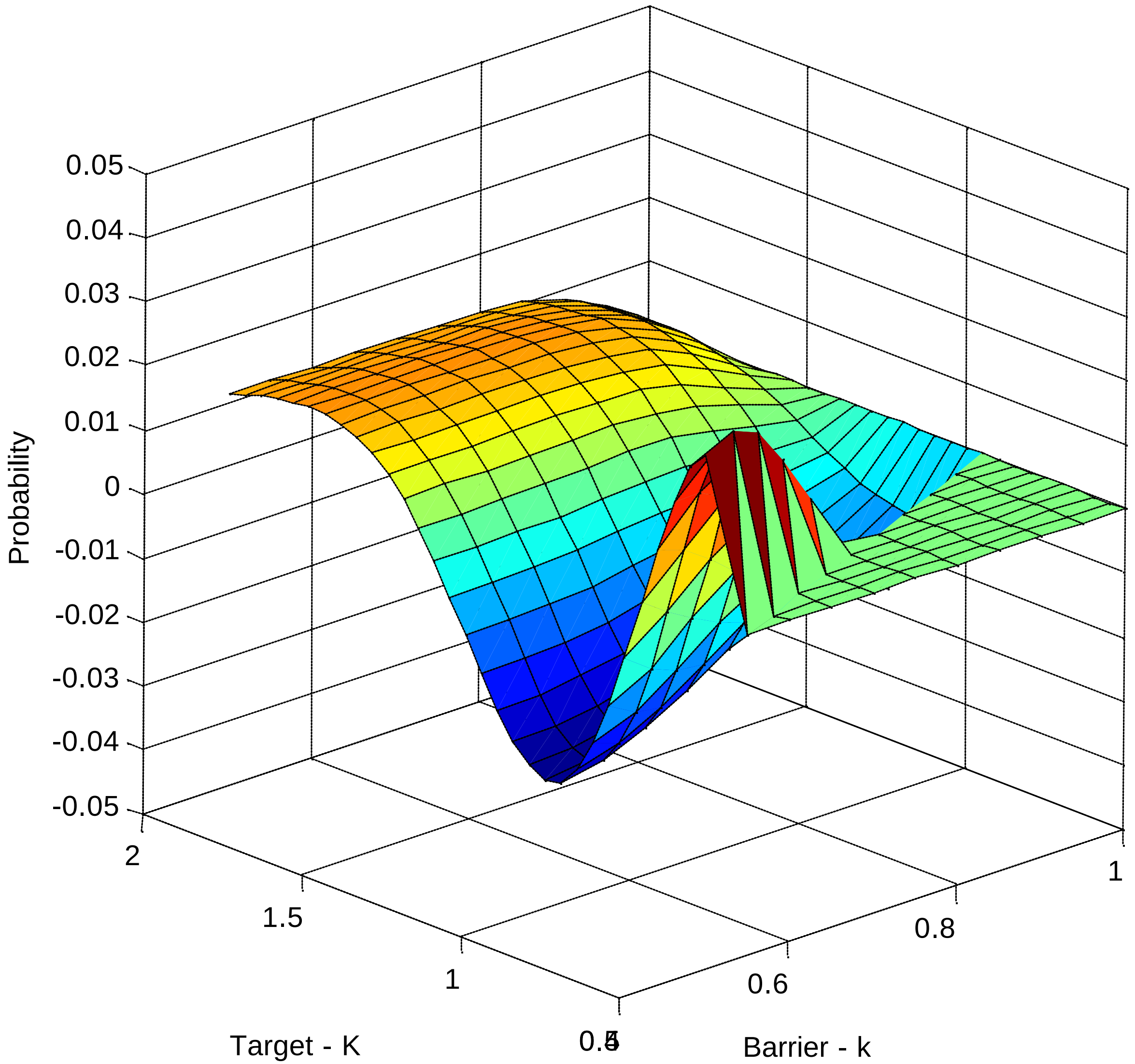
Success probability differences

Constant leverage minus unmanaged

W(K,k)

Constant lev-unmanaged

where we have 300bps skill on long side (a_L=3%), 200bps skill on short side (a_S=-2%), long vol s $_L$=20%, short vol s $_S$=15%, 50% correlation (?=.5), initial leverage 3 (L(0)=2, S(0)=1), 1 year.



Conclusions

•Intuition developed on long-only portfolios may fail to be a good guide for long/short portfolios

•Long/short portfolios that may look innocuous from a traditional (VaR or standard deviation) view can have unacceptably high chances of hitting an drawdown barrier if left unmanaged

•High correlation, low long volatility good

•Short side volatility can be good

•Skill not particularly important in avoiding drawdown barriers

•Effect of constant leverage risk management is complex