Global Market Impact: What we know, what we don’t know

Dan diBartolomeo

Northfield Information Services, Inc.

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Bait and Switch

- The bait
  - Our abstract for this presentation in the agenda included discussion of empirical results for our new global market impact model. Those results are not ready yet.
- The switch:
  - We're going to describe the history of our work in market impact and our view of the key issue
  - Describe new theoretical work on market impact
  - Make the distinction between estimating market impact for a portfolio optimization and doing it for trade scheduling analysis
  - Present very preliminary results on the estimation of market impact over four important global markets. This bit includes finding "the dog that didn't bark"
- The reward
  - New information from the 1.2 million trade Instinet database

What did we think at the start of this research in 2002?

- Despite a lot of research, we have had reservations regarding the usefulness of market impact models.
- Trading risk is the volatility of a long/short portfolio consisting of the differences between your pre-trading and post-trading portfolios
- Concurrent trades have a large influence on expected market impact of trades
- Using market impact in optimization requires accounting for cross effects in market impact. It also improves market impact estimation for each trade
- Optimal “break-up” of large trades also requires understanding the effects of concurrent trades

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What have we gotten done since 2002?

- Developed methods of assigning trade urgency as a function of risk contributions to the "portfolio" of undone trades
- Incorporating user definable market impact and cross-market impact of trades into optimization procedures
- Identified boundary conditions for reasonable market impact estimation
- Completed a discrete time optimization algorithm for trade "scheduling"
  - Recently went "live" via Instinet

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The Problem of Portfolio Rebalancing

- We want to minimize the loss of value in getting our portfolio from being what it is, to being what we want it to be
  - Trading N shares of one stock
  - Trading N shares of each of M stocks
  - Trading N shares of M stocks in T periods
- We have to consider both direct costs and implicit costs from our aversion to risk
- Where is the balance? What is optimal T for each trade?

The Usual Way of Looking At Trading Costs

- Most people see trading costs as having several components
  - Agency costs
  - Bid/Asked Spread
  - Market Impact (my trade moves the price)
  - Trend Costs (other people's trades move the price, maybe in my favor)
- Often overlooked ingredients
  - My large concurrent trades (my trade in Ford impacts the price of GM)
  - The implicit cost of waiting

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Let's See What We Can Reasonably Estimate

- Agency Costs are essentially known in advance
- Bid/Asked Spreads: Some time variation but reasonably stable
- Market Impact: Lots of models exist. Underlying factors are highly significant, but explanatory power is typically quite low
- Trend Costs: They can move the price for or against us. Ex-post often the largest part of the costs. Pretty darn random. Or so it seems.
- Market impact and trend costs are hard to disentangle so maybe the market impact models work better than we think

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In the Portfolio Optimization Context

- Most people who use an optimizer for rebalancing portfolios use some form of transaction cost estimate
- Many people use some form of non-linear transaction cost function that includes a market impact component
- If interdependencies between market impacts are not accounted for, large trades will be incorrectly specified
  - Consider two sets of orders
  - Buy 5 Million shares of Ford, Buy 5 Million shares of GM
  - Buy 5 Million shares of Ford, Sell 5 Million shares of GM
  - Are expected market impacts the same?

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Transaction Cost Functional Form

- Lets consider a simple model of direct trading costs. Lots of models look like this. Costs of trading increase with trade size at a decreasing rate

  $M = a + [\ bX + c(abs(X))^{Y}\ ]$

  M is the expected cost to trade one share

  X is the number of shares to be traded

  $Y > 0, Y < 1$

  a is the fixed costs per share

  b,c are coefficients expressing the liquidity of the stock (estimated from fundamentals and trading data)

- Empirical literature varies on the value of Y but the most widely observed values are around .5

A Fancier Specification

$$M = a + [\, bX_T + c(\mathrm{abs}(X_T))^{1/2}\,] + u_t + Z_t + (s_t^2/RAP)$$

M is the expected cost to trade one share

X_t is the absolute value of shares to be traded in t periods

a is the fixed costs per share

b,d are coefficients expressing the liquidity of the stock and trading skill (… optimal break up of large trades?)

U_t = expected short term trend of stock return (including covariance with other stocks with predicted trends

Z_t = expected influence due to the covariance of this stock with the market impact of my concurrent trades

RAP = risk tolerance

S_t^2 = return variance associated with waiting t periods to complete the trade including covariance with other stocks traded

But What Are We Solving For?

$$M = a + [\ bX_T + c(\mathrm{abs}(X_T))^{1/2}\ ] + u_t + Z_t + (s_t^2/RAP)$$

- Ideally we're solving for X and T simultaneously, such that M is minimized
  - St comes straight from our short horizon risk models. Be careful to adjust for kurtosis observed in short horizon returns
  - Estimate u_t from somewhere. It's the long term upward drift of the market . If we had a better estimate we'd trade on it
  - q_t is estimated from the covariance with the market impact of other concurrent trades we have open. This has to be estimated as a simultaneous process.
  - B, C come from your favorite models of market impact and trade break up.

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Market Impact Model Problems

- There are lots of market impact models around
  - Low explanatory power (they work well on average over thousands of trades but are very weak at estimation of a given trade in a given stock on a given day
  - Our simultaneous estimation of the market impacts over the portfolio of undone trades helps with the first issue. We care about getting the impact estimate right for the entire package
- Since the effect of the covariance between market impacts is dependent on the other stocks we are going to trade concurrently, it must be computed during a portfolio optimization process.
  - Luckily it converges and our optimizer can handle the “on the fly” changes in the specification of the problem

Market Impact Model Problems

- Most market impact models do not deal effectively with very large trades.
  - Traders know they can't do these trades so they break them up into a series of small trades. As no empirical data is available, models don't deal with the steep increase in costs at liquidity limits

- Our solution is to add another term to the cost equation

  $d(\max(\text{abs}(X_t - L_t),0)^2$

  L_t is one sided volume in t periods that will cause serious liquidity breakdown

  d is not estimated from empirical data but from assumption of optimal trade break up

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So Where Are We Now?

- Our optimizer now has a flexible functional form built into the objective function that takes cross-market impact and liquidity limits into account. It looks like:

$$Ci = a + [\ bXi + c(abs(Xi))^{1/2}\ ] + d(max(abs(Xi\text{-}L),0)^2 + Zi$$

Zi= (Sum [j = 1 to m] (BX(j) + CX(j)^.5 + D * Max[X(j) - L(j),0]^2) * Pij * Qij) * (2 / m-1)

for all i <> j

Pij = the correlation between stocks i, j derived from risk model

Qij = 1 if [Change in Shares (i) * Change in Shares (j)] > 0
Qij = -1 if [Change in Shares(i) * Change in Shares(j)] < 0

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But We've Left It to Users to Assign Parameters

- Clients are accustomed to only estimating the value of A, the basic cost per share
- Current market impact models estimate B and C, but typically from empirical analysis of small trades
  - Large trades (i. e. bigger than L) don't show up in historical databases because traders know they are too big to execute
  - Tick by tick trade and quote data not available for many markets
- Initial client parameterizations have been mixed
  - 150% transaction costs?
- Use boundary conditions to ensure rational parameters
  - Objective is to minimize <u>value weighted</u> mean squared error of estimates

What Might Reasonable Bounds Be?

- How about assuming the maximum market impact would be equal to the premium paid in typical hostile takeovers?
  - Academic studies and M&A databases (Dealmarker's Journal) show mean premium from 37 to 50% with standard deviation of around 30%
- Where does the distribution of observed bid and offer sizes top out from existing databases?
  - 99% of orders are within half an average days volume
- If there is an imbalance between buyers and sellers (think October 19, 1987) how much can prices move?
  - Market averages dropped about 25% in October 1987
  - Incomplete sell orders outnumbered incomplete buy orders 9 to 1 on that day

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New and Interesting Stuff About Market Impact

- In 2004, I worked with a group of MIT students to build a theoretical market impact model
  - The resulting complex function was monotonically increasing in size, but was approximately a square root over small trades but grew logarithmically after that (Brown, et. al., 2004)
- A number of studies have argued that costs for buying and selling are different
  - When the upward drift in the market is accounted for, no meaningful difference is observed in the costs of buys and sells (Hu, 2005)
- If order sizes have a Poisson distribution, you can derive the exponent on the market impact function to be .5

Amortization Functions

- In a portfolio optimization process, total estimated costs must be traded off against expected alpha and risk
  - Amortize trading costs over the expected holding period
  - Adjust the amortization rate to reflect “the probability of realization” which is less than one for finite holding periods
- For small transaction costs, arithmetic amortization is sufficient, but if costs are large we need to consider compounding
- Assume a trade with 20% trading cost and an expected holding period of one year.
  - We can get an expected alpha improvement of 20%. But if we give up 20% of our money now, and invest at 20% for one period, we only end up with 96% of the money we have now.

What have we learned so far from the Instinet data

- So far we've looked at data from four countries (US, Japan, Canada, Australia)
- We have both order sizes and fill sizes, so we can accumulate costs over the entire order
- There is a big right skew to trade sizes. About 1% of orders are really far out in the right tail
- The impact to size relationship:
  - Is positive and significant assuming a square root function.
  - Is positive and significant assuming a linear function
  - If you have a multiple regression with both square root and linear terms, one changes sign. This may arise from multi-collinearity or may indicate that an exponent less than .5 is needed.

Optimal Trade Scheduling

- If we know the urgency of trades, and the likely impact, we can create optimal trade “schedules” to break up large block trades into a series of smaller trades
  - We still need an assumption about the extent that market impact in one period is a permanent move in the price and how much is transient. We assume exponential decay
- Once we have that, the problem becomes a dynamic optimization requiring a Bellman equation solution
  - Our formulation uses a nearly traditional optimization with time made endogenous. Think of many stocks all called IBM that each can only be traded in one period: IBM (to trade Monday), IBM (to trade Tuesday), etc.

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An important nuance

- When incorporating market impact into portfolio optimization processes, the result assumes that the trade of a given size will have a certain degree of market impact, inclusive of breaking the trade into a series of smaller orders
- When doing trade scheduling, you are breaking the trade into smaller orders so the market impact function associated with each small (but not further divisible) order may have a different shape

Setting the Objective for Scheduling

- Consider a set of undone orders as a long short-portfolio that you are liquidating
  - You are long shares you do have and don't want
  - You are short shares you do want and don't have
- The normal mean-variance objective function

$$U = A - \sigma^2/T - (C*A)$$

- Works just fine except the sign on alpha is reversed from the norm
  - You are currently short stocks that do want. The reason you want them is that they have positive alpha
- We can't get all our trades done in one shot, so we need a multi-period representation
  - Price impact in one period changes available prices in subsequent periods.

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The Trade Schedule

- Lets assume we want to finish all our open trades over a two trading pay period
- We can break the two days into discrete time blocks, either by clock time or by “expected share of day’s volume” (e.g. each block is the length of clock time that usually trades 5% of the days volume
- Think of a spreadsheet where each order is a row and each time block is a column.
  - We want the matrix of orders such that all orders are completed the end of the schedule
  - That maximizes our objectives: capturing short term alpha, minimizing risk and market impact
- After each period is experienced, we can check that the expected orders were executed, if not, we can re-run the schedule based on the remaining shares and time periods

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Conclusions

- We now have excellent facilities to incorporate market impact estimates into optimization procedures
- Market impact estimates must be
  - Rationality bounded based on reasonable assumptions
  - Include cross market impact of concurrent trades
  - Include the expected degree of break up into small orders
- Efficacy of market impact models should be evaluated in a value weighted fashion

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