Fat Tails, Tall Tales, Puppy Dog Tails

Dan diBartolomeo

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Goals for this Talk

- Survey and navigate the enormous literature in this area
- Review the debate on assumed distributions for stock returns
- Consider the implications of the various possible conclusions on asset pricing, portfolio construction and risk management

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Return Distributions

- While traditional portfolio theory assumes that returns for equity securities and market are normally distributed, there is a vast amount of empirical evidence that the frequency of large magnitude events *seems much greater than is predicted by the normal distribution with observed sample variance parameters*
- Three broad schools of thought:
  - Equity returns have stable distributions of infinite variance.
  - Equity returns have specific, identifiable distributions that have significant kurtosis (fat tails) relative to the normal distribution (e.g. a gamma distribution)
  - Distributions of equity returns are normal at each instant of time, but look fat tailed due to time series fluctuations in the variance

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Stable Pareto Distributions

- Mandelbrot (1963) argues that extreme events are far too frequent in financial data series for the normal distribution to hold. He argues for a stable Paretian model, which has the uncomfortable property of infinite variance
- Mandelbrot (1969) provides a compromise, allowing for "locally Gaussian processes"
- Fama (1965) provides empirical tests of Mandelbrot's idea on daily US stock returns. Finds fat tails, but also volatility clustering
- Lau, Lau and Wingender (1990) reject the stable distribution hypothesis
- Rachev (2000, 2003) deeply explores the mathematics of stable distributions

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A Bit on Stable Distributions

- General stable distributions have four parameters
  - Location (replaces mean)
  - Scale (replaces standard deviation)
  - Skew
  - Tail Fatness
- Some moments are infinite
- Except for some special cases (e.g. normal) there are no analytical expressions for the likelihood functions
- Estimation of the parameters is very fragile. Many, many different combinations of the four parameters can fit data equally well
- These distributions do have time scaling (you should be able to scale from daily observations to monthly observations, etc.)

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Specific Fat Tailed Distribution

- Gulko (1999) argues that an efficient market corresponds to a state where the informational entropy of the system is maximized
- Finds the risk-neutral probabilities that maximize entropy
- The entropy maximizing risk neutral probabilities are equivalent to returns having the Gamma distribution
- Gamma has fat tails but only two parameters and finite moments
- Has finite lower bound which fits nicely with the lower bound on returns (i.e. -100%)
- Derives an option pricing model of which Black-Scholes is a special case

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Time Varying Volatility

- The alternative to stable fat-tailed distributions is that returns are normally distributed at each moment in time, but with time varying volatility, giving the illusion of fat tails when a long period is examined
- Rosenberg (1974?)
  - Most kurtosis in financial time series can be explained by *predictable* time series variation in the volatility of a normal distribution
- Engle and Bollerslev: ARCH/GARCH models
  - Models that presume that volatility events occur in clusters
  - Huge literature. I stopped counting when I hit 250 papers in referred journals as of 2003
- LeBaron (2006)
  - Extensive empirical analysis of stock returns
  - Finds strong support for time varying volatility, but very weak evidence of actual kurtosis

The Remarkable Rosenberg Paper

- Unpublished paper by Barr Rosenberg (1974?), under US National Science Foundation Grant 3306
- Builds detailed model of time-varying volatility in which long run kurtosis arises from two sources
  - The kurtosis of a population is an accumulation of the kurtosis across each sample sub-period
  - Time varying volatility and serial correlation can induce the appearance of kurtosis when the distribution at any one moment in time is normal
  - Predicts more kurtosis for high frequency data
- An empirical test on 100 years of monthly US stock index returns shows an R-squared of .86
- Very reminiscent of subsequent ARCH/GARCH models

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ARCH/GARCH

- Engle (1982) for ARCH, Bollerslev (1986) for GARCH
- Conditional heteroscedasticity models are standard operating procedure in most financial market applications with high frequency data
- They assume that volatility occurs in clusters, hence changes in volatility are predictable
- Andersen, Bollerslev, Diebold and Labys (2000)
  - Exchange rate returns are Gaussian
- Andersen, Bollerslev, Diebold and Ebens (2001)
  - The distribution of stock return variance is right skewed for arithmetic returns, normal for log return
  - Stock returns must be Gaussian because the distribution of returns/volatility is unit normal

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Recent Empirical Research

- Lebaron, Samanta and Cecchetti (2006)
- Exhaustive Monte-Carlo bootstrap tests of various fat tailed distributions to daily Dow Jones Index data using robust estimators
- Propose a novel adjustment for time scaling volatilities to account for kurtosis, in order to use daily data to forecast monthly volatility
- Conclusion: “No compelling evidence that 4^{th} moments exist”
  - If variance is unstable, then its difficult to estimate
  - High frequency data is less useful
  - Use robust estimators of volatility
  - Estimation error of expected returns dominates variance in forming optimal portfolios

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More Work on Fat Tails

- Japan Stock Returns
  - Aggarwal, Rao and Hiraki (1989)
  - Watanabe (2000)
- France Stock Returns
  - Navatte, Christophe Villa (2000)
- Option implied kurtosis
  - Corrado and Su (1996, 1997a, 1997b)
  - Brown and Robinson (2002)
- Sides of the debate
  - Lee and Wu (1985)
  - Tucker (1992)
  - Ghose and Kroner (1995)
  - Mittnik, Paolella and Rachev (2000)
  - Rockinger and Jondeau (2002)

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The Time Scale Issue

- Almost all empirical work shows that fat tails are more prevalent with high frequency (i.e. daily rather than monthly) return observations
- Lack of fat tails in low frequency data is problem for proponents of stable distributions,
  - the tail properties should time scale
  - maybe we just don't have enough observations when we use lower frequency data for apparent kurtosis to be statistically significant
- Or the observed differences in higher moments could be a mathematical artifact of the way returns are being calculated
  - Lau and Wingender (1989) call this the "intervaling effect"

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The Curious Compromise of Finanalytica

- The basic concepts of stable fat tailed distributions and time-varying volatility models are clearly mutually exclusive as explanations for the observed empirical data
- From the Finanalytica website:
  - "uses proprietary generalized multivariate stable (GMstable) distributions as the central foundation of its risk management and portfolio optimization solutions"
  - "Clustering of volatility effects are well known to anyone who has traded securities during periods of changing market volatility. Finanalytica uses advanced volatility clustering models such as stable GARCH..."
- Svetlozar Rachev and Doug Martin are really smart guys so I'm putting this down to pragmatism rather than schizophrenia

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Kurtosis versus Skew

- So far we've talked largely about 4^{th} moments
- We haven't done much in terms of economic arguments about why fat tails exist, and at least appear to be more prevalent with higher frequency data
- Many of the same arguments apply to skew (one fat tail),
  - consistent prevalence of negative skew in financial data series
- Harvey and Siddique (1999) find that skew can be predicted using an autoregressive scheme similar to GARCH

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Cross-Sectional Dispersion

- When we think about “fat tails” we are usually thinking about time series observations of returns
- For active managers, the cross-section of returns may be even more important, as it defines the opportunity set
- DeSilva, Sapra and Thorley (2001)
  - if asset specific risk varies across stocks, the cross-section should be expected to have a unimodal, fat-tailed distribution
- Almgren and Chriss (2004)
  - provides a substitute for “alpha scaling” that sorts stocks by attractiveness criteria, then maps the sorted values into a fat-tailed multivariate distribution using copula methods

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What's the Problem with Daily Returns Anyway?

- Financial markets are driven by the arrival of information in the form of "news" (truly unanticipated) and the form of "announcements" that are anticipated with respect to time but not with respect to content.

- The time intervals it takes markets to absorb and adjust to new information ranges from minutes to days. Generally much smaller than a month, but up to and often larger than a day. That's why US markets were closed for a week at September 11th.

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Investor Response to Information

- Several papers have examined the relative market response to “news” and “announcements”
  - Ederington and Lee (1996)
  - Kwag Shrieves and Wansley(2000)
  - Abraham and Taylor (1993)
- Jones, Lamont and Lumsdaine (1998) show a remarkable result for the US bond market
  - Total returns for long bonds and Treasury bills are not different if announcement days are removed from the data set
- Brown, Harlow and Tinic (1988) provide a framework for asymmetrical response to “good” and “bad” news
  - Good news increases projected cash flows, bad news decreases
  - All new information is a “surprise”, decreasing investor confidence and increasing discount rates
  - Upward price movements are muted, while downward movements are accentuated

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Implications for Asset Pricing

- If investors price skew and/or kurtosis, there are implications for asset pricing
- Harvey (1989) finds relationship between asset prices and time varying covariances
- Kraus and Litzenberger (1976) and Harvey and Siddique (2000) find that investors are averse to negative skew
  - diBartolomeo (2003) argues that the value/growth relationship in equity returns can be modeled as option payoffs, implying skew in distribution
  - If the value/growth relationship has skew and investors price skew, then an efficient market will show a value premium
- Dittmar (2002) find that non-linear asset pricing models for stocks work if a kurtosis preference is included
- Barro (2005) finds that the large equity risk premium observed in most markets is justified under a “rare disaster” scenario

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Portfolio Construction and Risk Management

- Kritzman and Rich (1998) define risk management function when non-survival is possible
- Satchell (2004)
  - Describes the the diversification of skew and kurtosis
  - Illustrates that plausible utility functions will favor positive skew and dislike kurtosis
- Wilcox (2000) shows that the importance of higher moments is an increasing function of investor gearing

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Optimization with Higher Moments

- Chamberlin, Cheung and Kwan(1990) derive portfolio optimality for multi-factor models under stable paretian assumptions
- Lai (1991) derives portfolio selection based on skewness
- Davis (1995) derives optimal portfolios under the Gamma distribution assumption
- Hlawitscka and Stern (1995) show the simulated performance of mean variance portfolios is nearly indistinguishable from the utility maximizing portfolio
- Cremers, Kritzman and Paige (2003)
  - Use extensive simulations to measure the loss of utility associated with ignoring higher moments in portfolio construction
  - They find that the loss of utility is negligible except for the special cases of concentrated portfolios or “kinked” utility functions (i.e. when there is risk of non-survival).

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Conclusions

- The fat tailed nature of high frequency returns is well established
- The nature of the process is usually described as being a fat tailed stable distribution or a normal distribution with time varying volatility
- The process that creates fat tailed distributions probably has to do with rate at which markets can absorb new information
- The existence of fat tails and skew has important implications for asset pricing
- Fat tails probably have relatively lesser importance for portfolio formation, unless there are special conditions such as gearing that imply non-standard utility functions

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