Comprehensive Risk and Performance Attribution

Northfield Information Systems
2007 Research Conference
Ocean Reef Club
Key Largo, Florida

Barry Feldman, Ph.D., CFA
Prism Analytics
barry@prismanalytics.com

October 22, 2007

Highlights

- A theory of attribution
- Returns-based risk decomposition
- Portfolio-based performance attribution

1. Theory

A new approach to attribution (Feldman, 2007)

- Start with a model of utility maximizing behavior
- Consider a random order model of the relative importance of *the factors* contributing to utility
- Look for a distribution over orders to describe the probability of being ordered by relative importance
- Assume
  - Random order consistency
  - Functional separability
  - Exclusion
- Proportional marginal attribution (PMA) is
  - The unique resulting expectation of factor contributions
  - A powerful and theoretically consistent attribution method

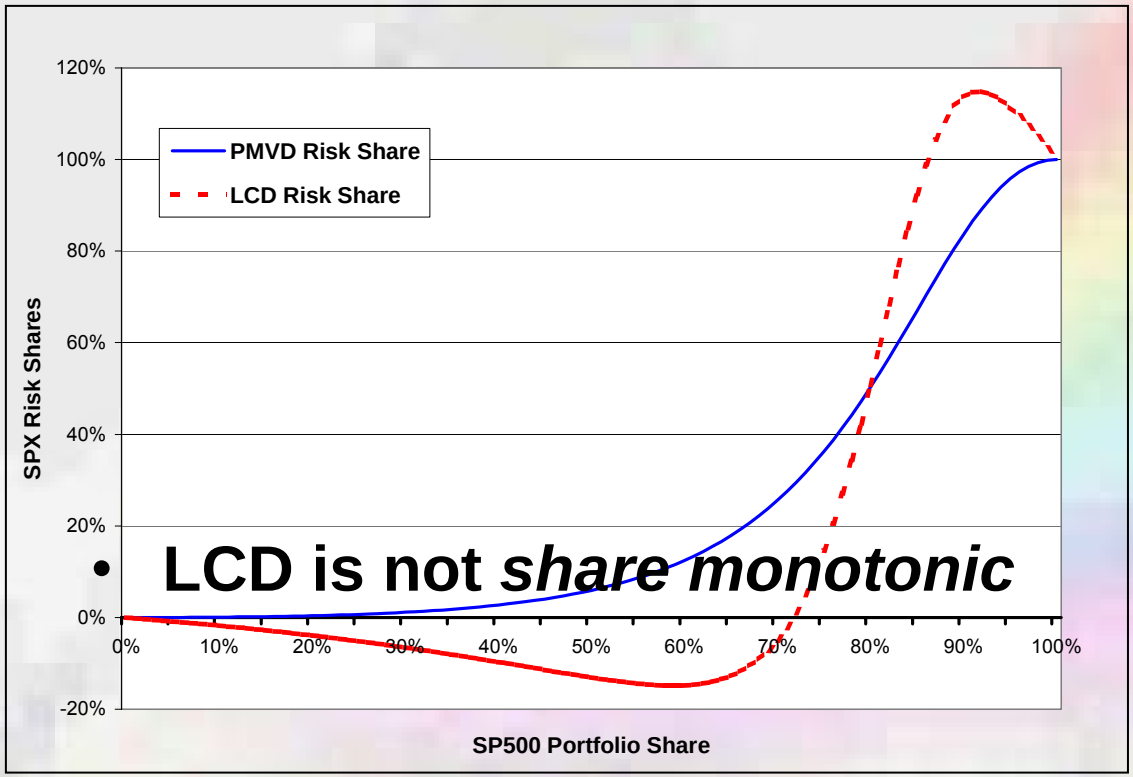
2. Returns-based risk attribution

Returns-based risk attribution / decomposition

- Let the utility function of an analyst be the R^2 of a factor model and use PMA
  - This PMVD
- Let the utility function of an analyst be based on the |R| of a quantile regression and use PMA
  - This is PMQD
- *Linear covariance decomposition* or *LCD* is the standard method of variance decomposition
- The key advantage of PMA over LCD and statistical significance measures is that PMA is *designed* to take factor correlation into account

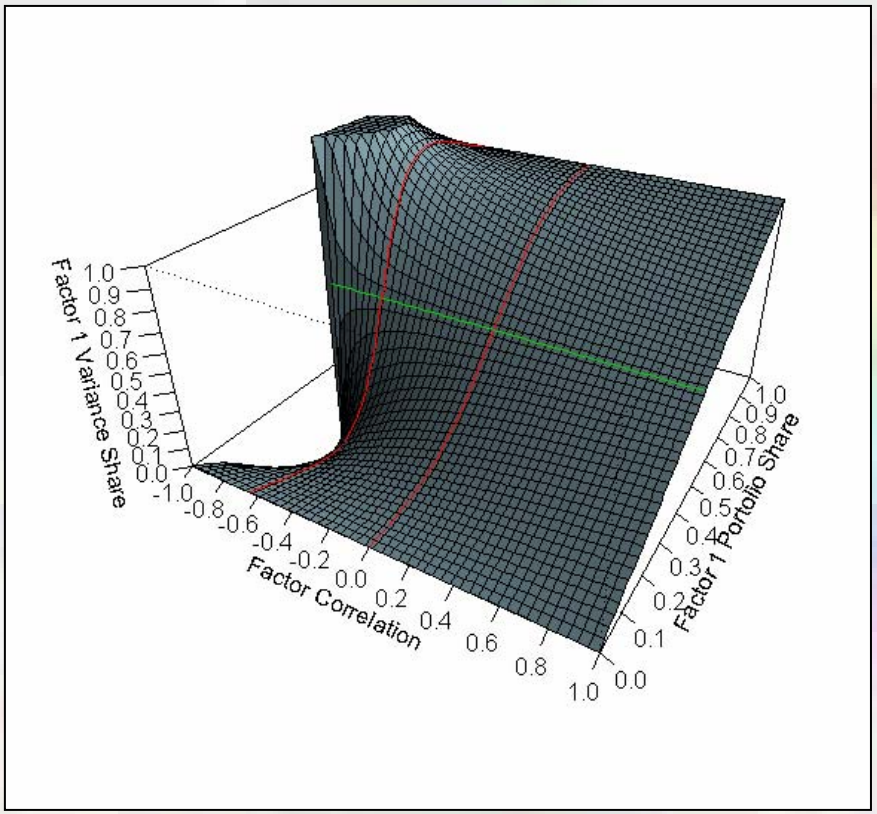
3. Variance decomposition: The two-factor case

Example: Portfolios of SPDRs and VIX futures



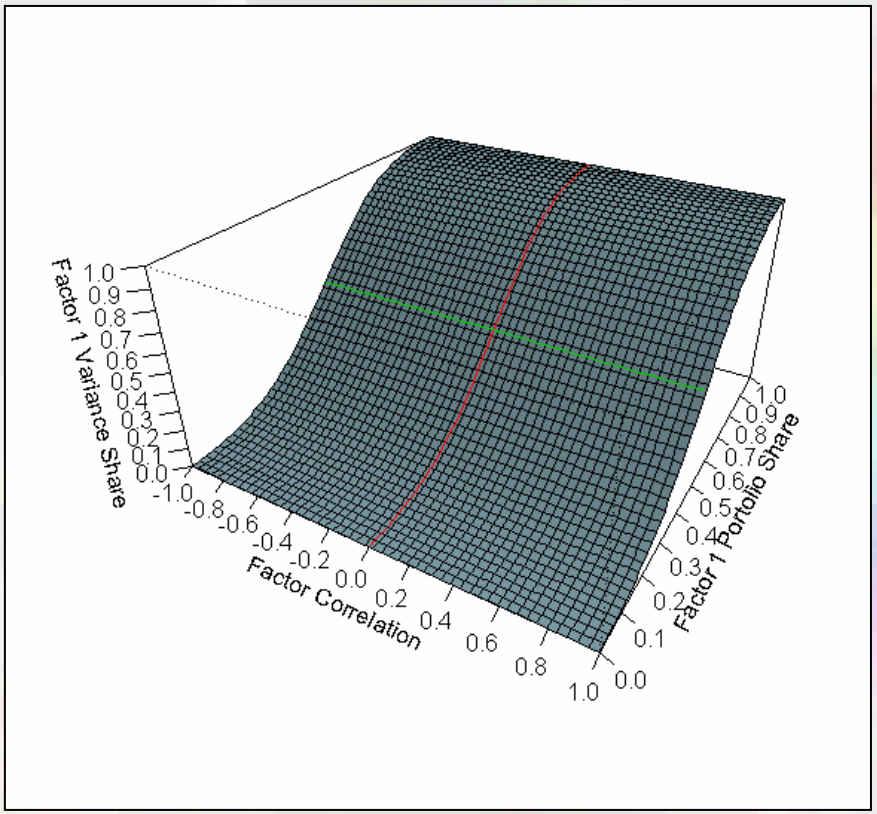
- PMVD SPDR attribution in blue, LCD SPDR attribution in red
- Horizontal axis shows SPDR portfolio percentage
- LCD gives negative attributions and those over 100%
- Correlation between SPDRs and VIX: -0.64

Standard variance decomposition (LCD) in the general two-factor case



- Attributions diverge to plus and minus infinity as factor correlation approach -1.0

PMVD in the general two-factor case

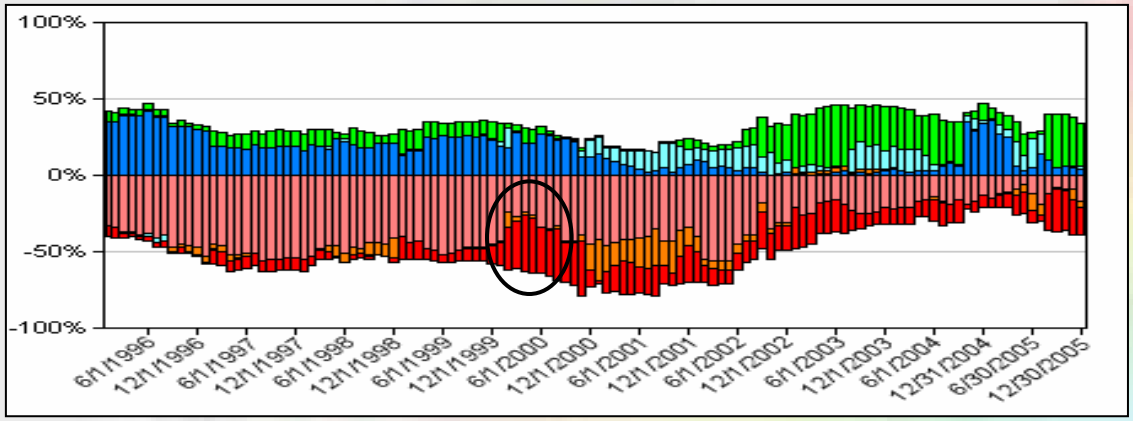


- PMVD attributions are independent of correlation in the two-factor case

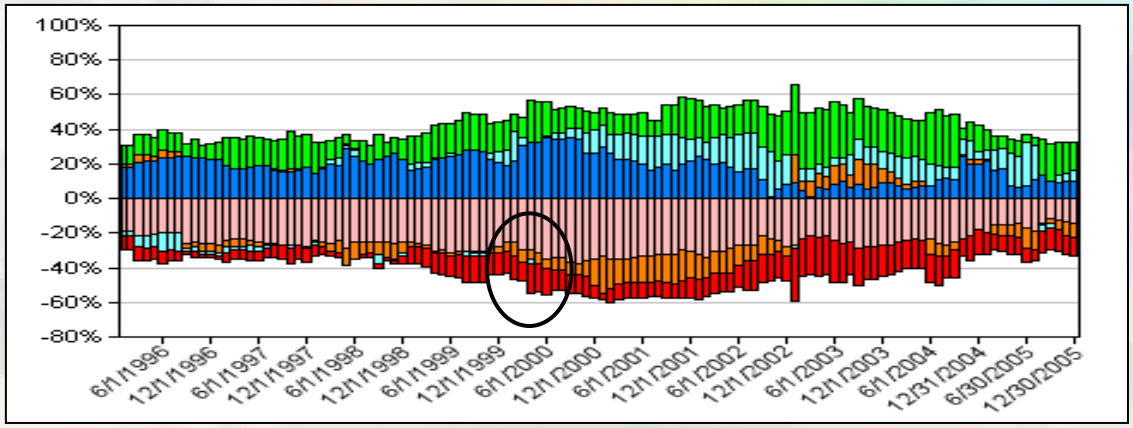
4. Negative factor relationships

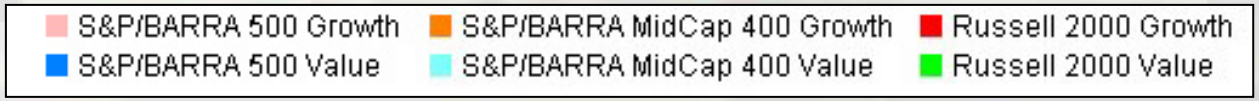
Portable alpha example: A portfolio long the RAFI 1000 and short the Russell 1000

PMVD



Factor Model



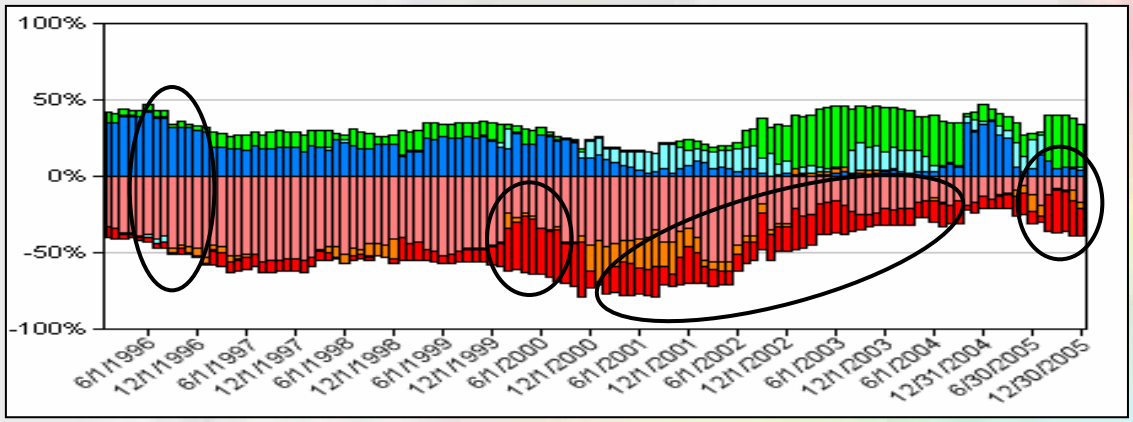


- PMVD exposures consistent with the factor model

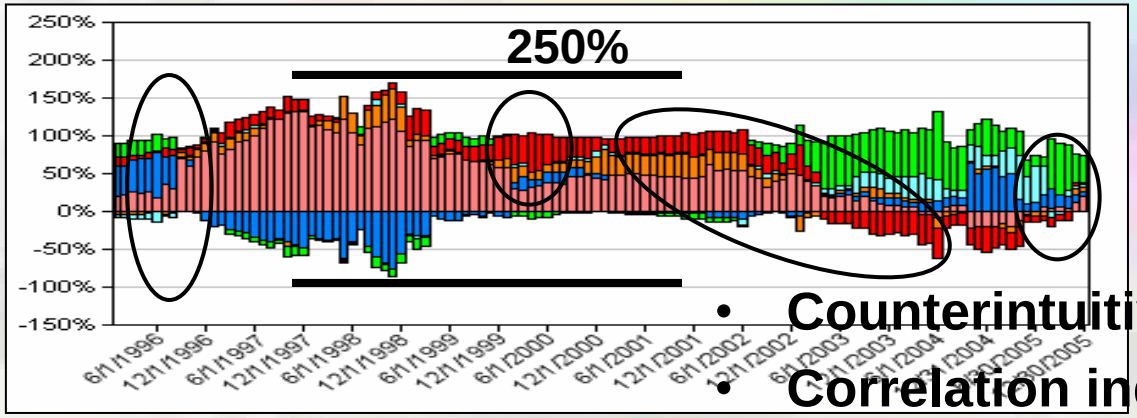
RAFI 1000 performance data courtesy of Research Affiliates.

Portable alpha example: PMVD and LCD compared

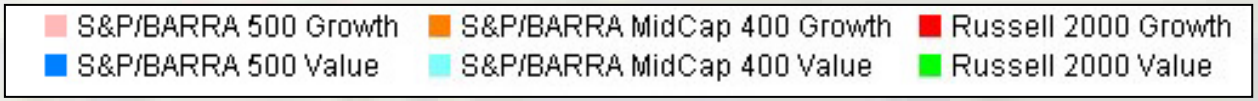
PMVD



LCD



- **Counterintuitive exposures**
- **Correlation induced bias**
- **Colinearity inflation**



RAFI 1000 performance data courtesy of Research Affiliates.

5. PMVD put to a statistical test

Laudus Rosengberg Long/Short Fund
Is the LCG PMVD attribution reasonable?

Basic factor model

Factor	Beta	T-Stat	p-value	PMVD
LCG	-18%	-1.23	0.221	-9.2%
LCV	-20%	-0.94	0.351	-6.4%
MCG	36%	2.50	0.015	5.8%
MCV	-20%	-0.88	0.382	-0.9%
SCG	-52%	-3.76	0.000	-26.7%
SCV	36%	1.78	0.079	3.6%
INTL	9%	0.67	0.503	0.7%
EMERG	2%	0.20	0.839	0.1%
HighYld	-8%	-0.47	0.643	-0.1%
LT	16%	1.15	0.255	1.1%
TBILL	-252%	-1.18	0.242	-1.0%

***R^2*: 55.52%**

Combined LC factor

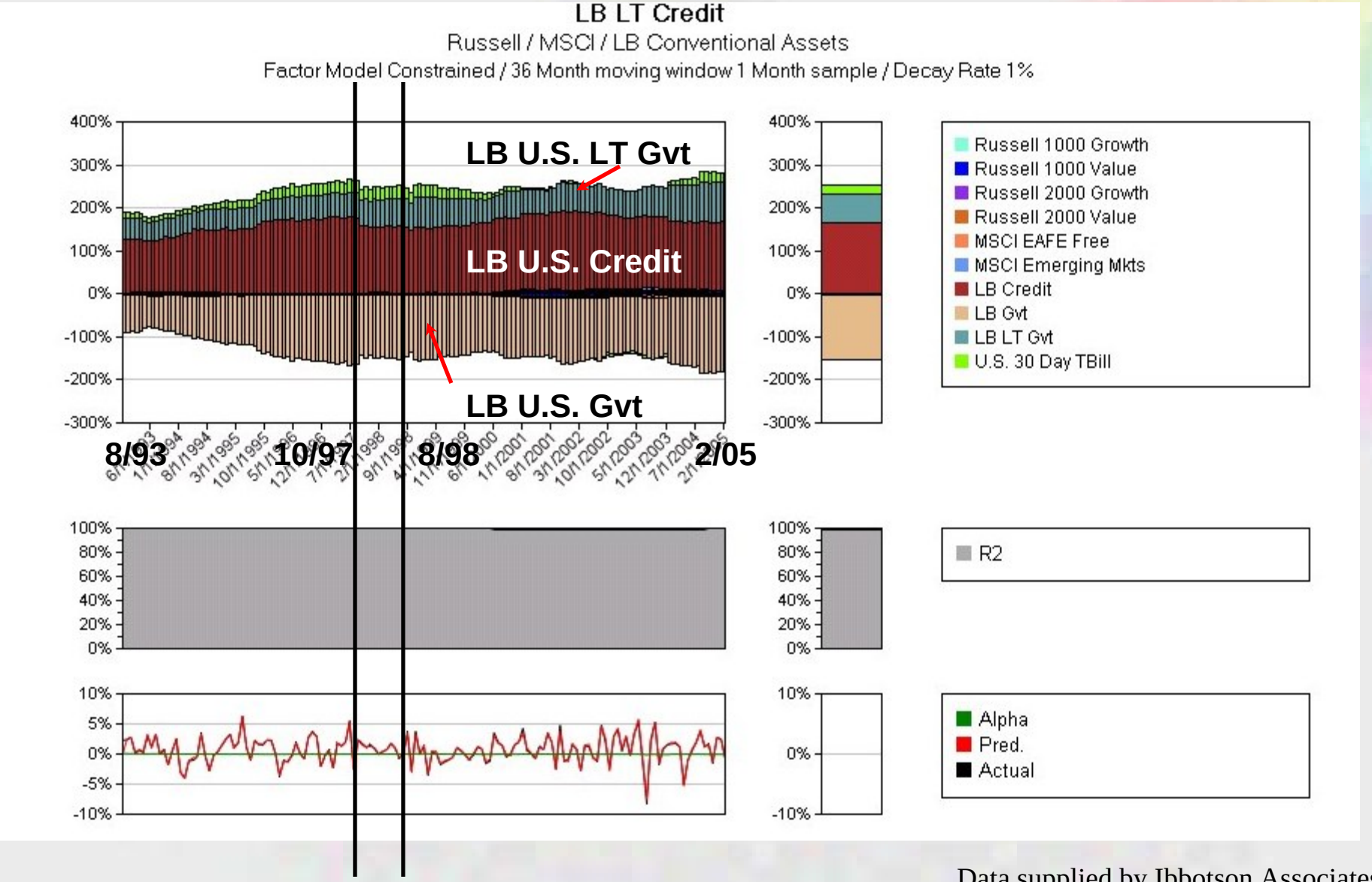
Factor	Beta	T-Stat	p-value	PMVD
LC	-35%	-2.11	0.038	-16.6%
MCG	36%	2.59	0.011	6.2%
MCV	-22%	-1.12	0.265	-1.2%
SCG	-52%	-3.83	0.000	-25.1%
SCV	37%	1.87	0.066	2.8%
INTL	9%	0.64	0.522	0.8%
EMERG	2%	0.22	0.829	0.1%
HighYld	-9%	-0.52	0.608	-0.2%
LT	16%	1.24	0.219	1.2%
TBILL	-254%	-1.21	0.232	-1.0%

***R^2*: 55.19%**

- *F*-test constraint *p*-value = **.**47
- Correlations degrade LCG stat. sig. levels

6. Extreme positive colinearity

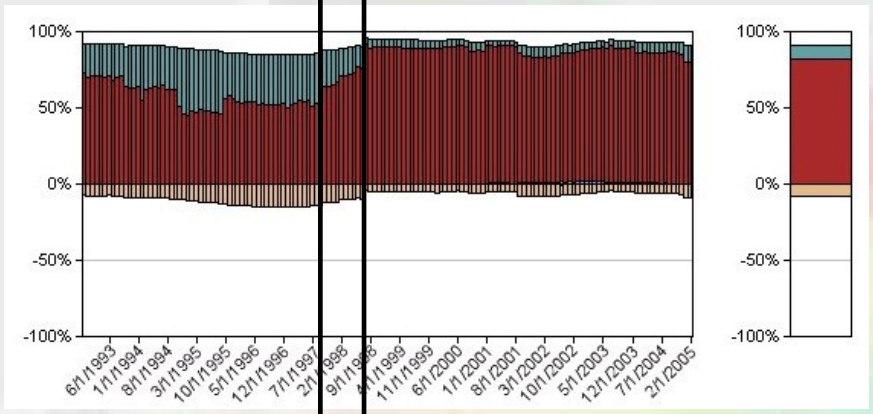
Example from Feldman (2006): LB US Long Term Credit



Data supplied by Ibbotson Associates.

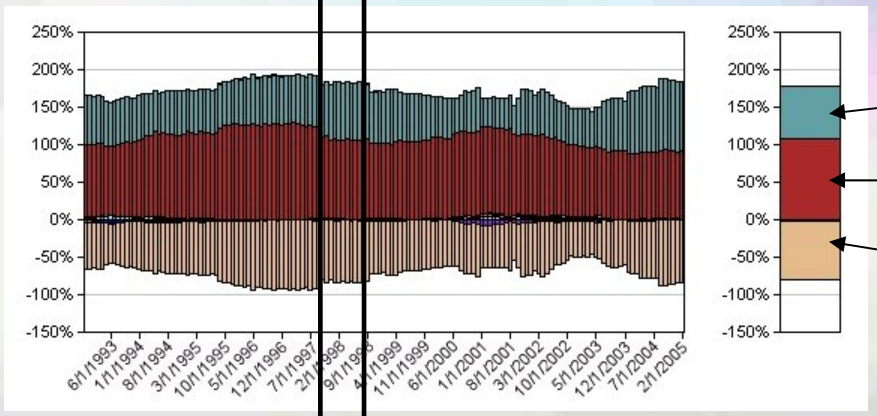
PMVD shows evidence of a 1997-1998 structural break

PMVD



8/93 **10/97** **8/98** **2/05**

LCD



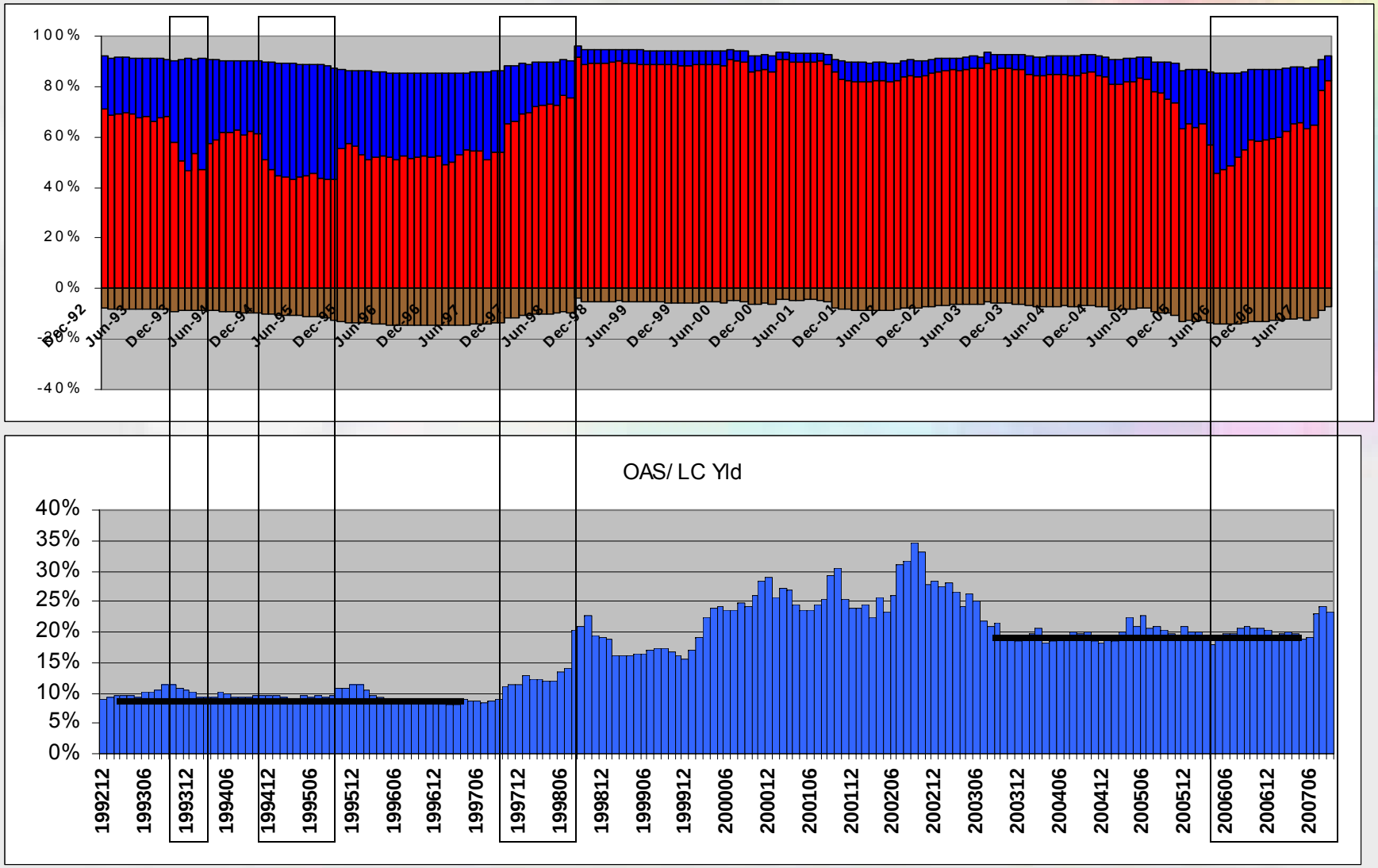
LB U.S. LT Gvt

LB U.S. Credit

LB U.S. Gvt

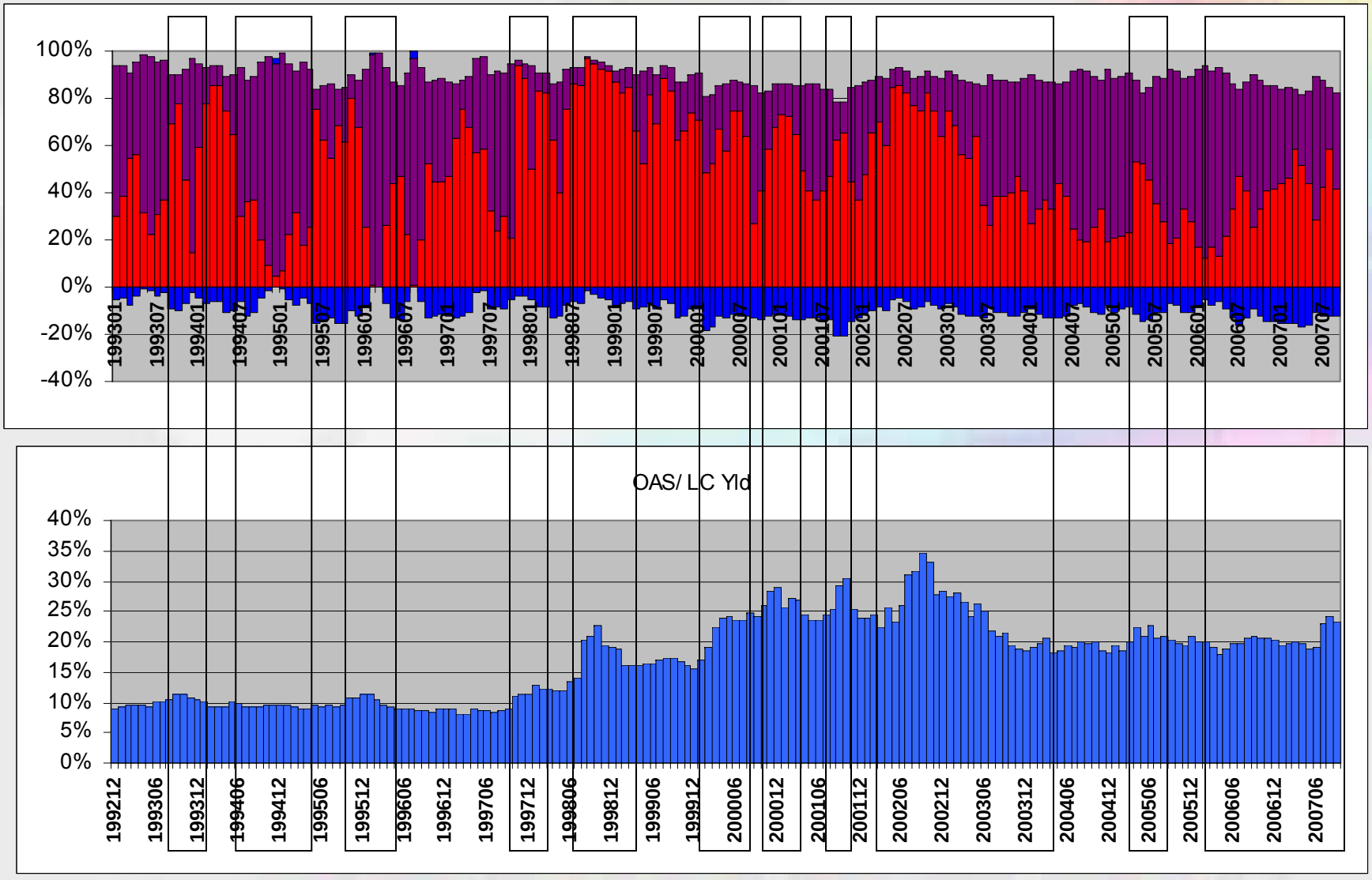
Data supplied by Ibbotson Associates.

PMVD: December 1992 to August 2007



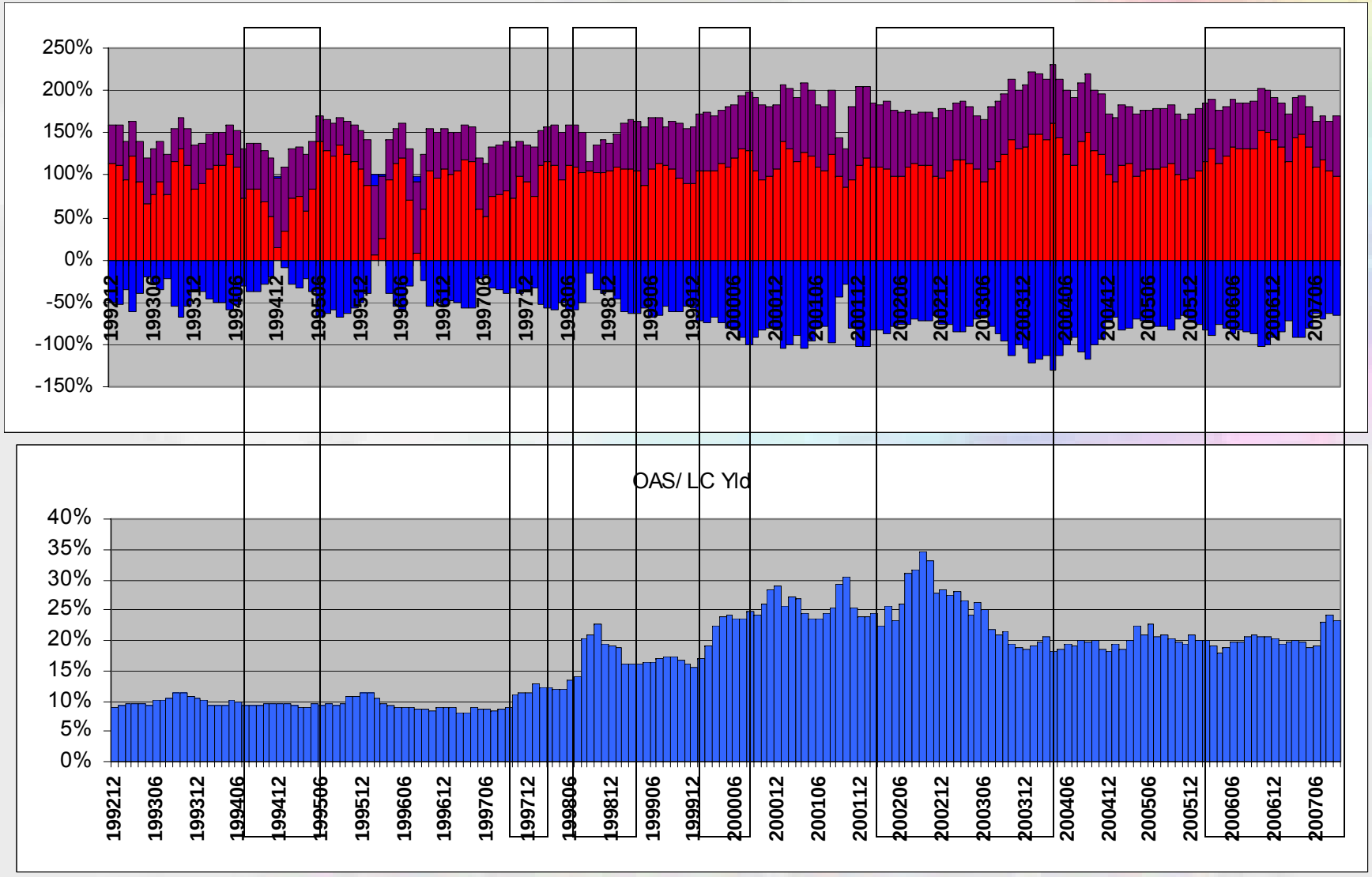
Data courtesy of Morningstar.

PMVD analysis based on daily data



Data courtesy of Morningstar.

LCD with daily data



Data courtesy of Morningstar.

Long-term credit example conclusions

- PMVD appears to capture information in credit markets, sometimes before it is reflected in spreads
- Many historical events clearly identified
- Other factors also reflected in PMVD results
- Noise likely also present (Gromping, 2006)
- LCD appears to capture little information

7. Proportional marginal quantile risk attribution

Quantile regression

- Koenker and Bassett (1978) and Bassett and Chen (2001)
- Exact analogy to least squares
- Estimate conditional quantile (e.g. median) instead of conditional mean
- Minimize the sum of absolute quantile deviations
- Attribution approach is exactly analogous to OLS
- Operational difference with OLS: choice of quantile

PMQD example: Analysis of CTA performance

- Based on Premia Capital Management daily P/L for 2005

Sector	PMVD	Quantile		
		1%	50%	99%
Agriculture	20%	66%	16%	10%
Energy	13%	31%	10%	58%
Metals	19%	1%	26%	7%
Livestock	37%	2%	34%	5%
Precious	1%	0%	8%	19%
UST5TR	10%	1%	7%	0%
R1 or R2	1.9%	24.5%	1.5%	3.2%

- PMVD and median quantile PMQD components similar
- Agriculture and energy dominate 1% quantile downside risk
- Energy dominates 99% quantile upside potential
- Metals and livestock show median quantile risk but little tail risk

P/L data courtesy of Premia Capital Management.

8. Portfolio performance attribution

Attribution of portfolio performance

- Use a utility function to quantify the benefit of historical or forecast investment performance of an optimized portfolio of assets
  - Determine the utility share of each asset
  - Divide the utility share of an asset by its portfolio weight
- This is proportional marginal performance attribution (PMPA)

Standard portfolio optimization problem

MANAGER	Historical Return	Return Adjustment	Adjusted Return	Std. Dev.	Annualized Sharpe Ratio
DFA U.S. Large Company Institutional	0.90%	0.40%	1.30%	3.34%	1.19
Robeco WPG 130/30 Large Cap Core Intl	0.75%	0.60%	1.35%	3.93%	1.06
Bridgeway Ultra-Small Company	1.96%	-0.20%	1.76%	5.44%	1.02
ProFunds Real Estate UltraSector Svc	1.67%	-0.30%	1.37%	6.78%	0.63
AllianceBernstein Intl Growth C	1.70%	-0.10%	1.60%	3.93%	1.28
Oppenheimer Emerging Growth A	1.28%	0.30%	1.58%	6.23%	0.80
Matthews China	2.15%	-0.50%	1.65%	5.02%	1.04
Putnam High Yield M	0.87%	-0.10%	0.77%	1.43%	1.51
Vanguard Long-Term Bond Index	0.55%	0.25%	0.80%	2.57%	0.88
Mellon Short Term U.S. Govt Secs M	0.20%	0.20%	0.40%	0.39%	2.24

- 10 Managers
- Historical returns, standard deviations and correlations based on 60 months of date to June 2007
- Return adjustments constitute one relatively optimistic assessment of the coming year
- MVO: Quadratic utility with $\lambda=10$

Data courtesy of MPI

Performance attribution example: Assets and performance characteristics

MANAGER	MVO Portfolio Allocation	PMA	PMPA	Annualized Sharpe Ratio
DFA U.S. Large Company Institutional	5.2%	6.3%	1.20	1.19
Robeco WPG 130/30 Large Cap Core Intl	11.8%	13.1%	1.11	1.06
Bridgeway Ultra-Small Company	8.5%	7.2%	0.85	1.02
ProFunds Real Estate UltraSector Svc	1.9%	0.3%	0.17	0.63
AllianceBernstein Intl Growth C	15.7%	16.3%	1.03	1.28
Oppenheimer Emerging Growth A	0.0%	0.0%	--	0.80
Matthews China	21.7%	25.0%	1.15	1.04
Putnam High Yield M	0.0%	0.0%	--	1.51
Vanguard Long-Term Bond Index	32.6%	31.3%	0.96	0.88
Mellon Short Term U.S. Govt Secs M	2.7%	0.6%	0.21	2.24

- PMPA: Proportional marginal portfolio attribution
- PMPA takes manager correlations to the portfolio into account
- PMPA results surprisingly similar to to Sharpe ratio for some managers

9. Limitations

PMA Limitations

- Computing time
  - Increases sharply with number of factors
  - Solution: approximation methods
- Attribution precision
  - Examined in Feldman (2005) and Gromping (2007)
  - PMA is estimated consistently
  - Bootstrapped confidence intervals

10. Your turn

11. Wrap up

Get more information from your data

- PMA methods have a strong theoretical foundation
- PMA is share monotonic
- PMA methods work
- LCD and statistical significance measures have serious limitations as risk attribution methods
  - LCD and statistical significance measures are vulnerable to correlation-driven risk attribution distortions
  - LCD risk exposures can be completely suppressed

References

- Bassett, Gilbert and Hsiu-Lang Chen (2001): "Quantal style: return-based attribution using regression quantiles," *Empirical Economics*, 26:293:305.
- Feldman, Barry (2005): *Relative Importance and Value*, www.prismanalytics.com/docs/RelativeImportance.pdf
- ____ (2006): "Using proportional marginal variance decomposition to understand hedge fund performance drivers," in *Portfolio Analysis: Advanced Topics in Performance Measurement, Risk and Attribution*, Timothy Ryan, ed., Risk Books db.riskwaters.com/public/showPage.html?page=book_page&tempPageName=315943
- ____ (2007): *A Theory of Attribution*, working paper, www.ssrn.com/abstracts=988860
- Grömping, Ulrike (2007): "Estimators of relative importance in linear regression based on variance decomposition," *American Statistician*, 61(2) 139-147, www.tfh-berlin.de/~groemp/downloads/amstat_preprint.pdf
- Koenker, Roger and Gilbert Bassett (1978): "Regression quantiles," *Econometrica*, 46:33-50.