Optimal Trading Strategy With Optimal Horizon

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Trading – An Integral Part of Investment Process

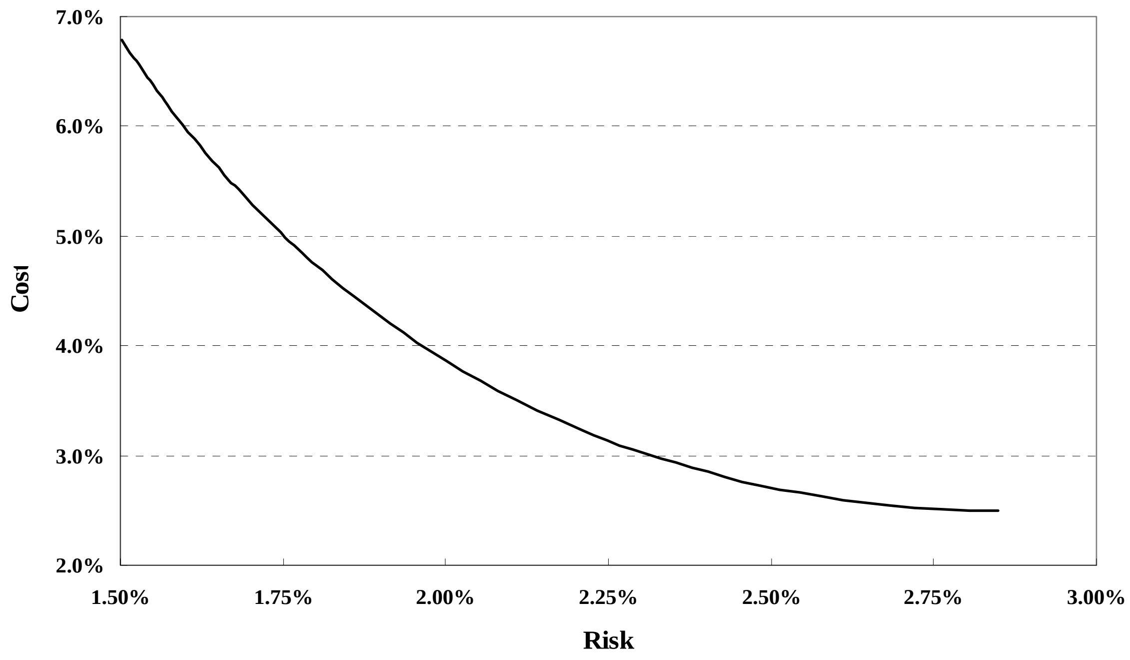
- **Alpha forecasting**
- **Portfolio construction**
- **Trading – portfolio implementation**
- **Performance attribution**

Conflicting Objectives in Trading

➢Immediacy and costs

- Alpha capture
- Risk reduction
- Labor costs
- Opportunity costs

- Market impact



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Optimal Trading Strategies

- **Optimal trading path (sequence) with minimum costs for a given level of risk**

$$h^*(t), t \in [0, T], \quad T \text{ is the trading horizon.}$$

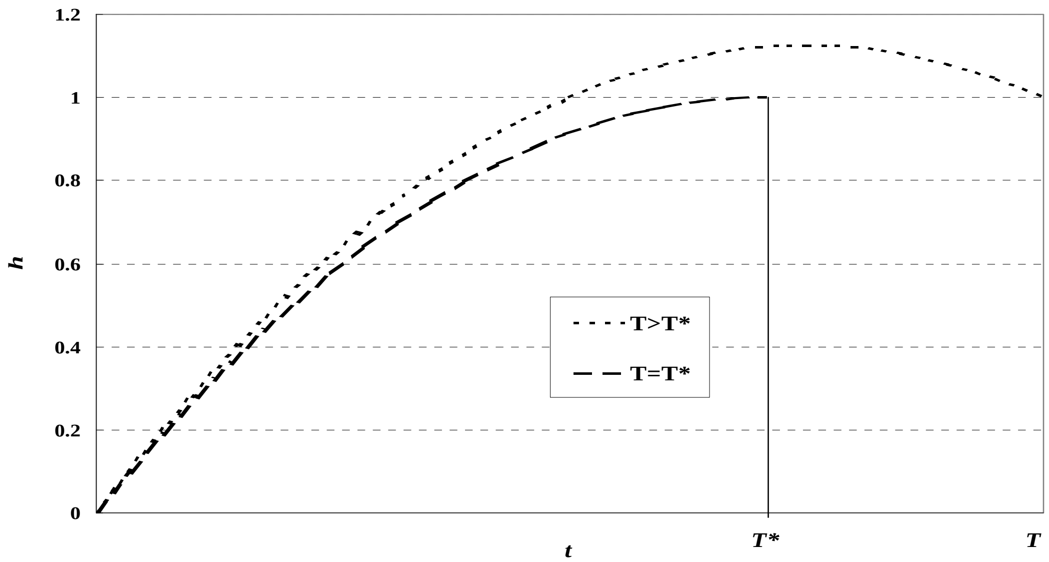
- **Previous researches (Grinold & Kahn 1999, Almgren & Chriss 2000) used a fixed horizon *T***

- **Extension to optimal trading strategy with optimal horizon (Qian 2008 JOIM, Qian, Hua, Sorensen 2007)**

$$h^*(t), t \in [0, T^*].$$

Optimal Horizon

- **Horizon is not known in advance**
  - Single stocks versus baskets
- **It is optimal along two dimensions**
- **Flip-floping in optimal trading with fixed horizon**



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Mathematical Model - Inputs

- **Trade weight** Δw **and trade path** $h(t)\Delta w,\ h(0)=0$ and $h(T)=1$
- **Trade shortfall** $h(t)\Delta w-\Delta w=\Delta w\left[h(t)-1\right]$

- **Return shortfall** $f\,\Delta w\left[h(t)-1\right]dt$
- **Shortfall variance** $\sigma^2(\Delta w)^2\left[h(t)-1\right]^2 dt$
- **Fixed cost** $c\left|\Delta w\right|T,\ c>0$
- **Market impact** $\psi(\Delta w)^2\left[\dot{h}(t)\right]^2 dt\ ,\psi>0$

Mathematical Model – Objective Function

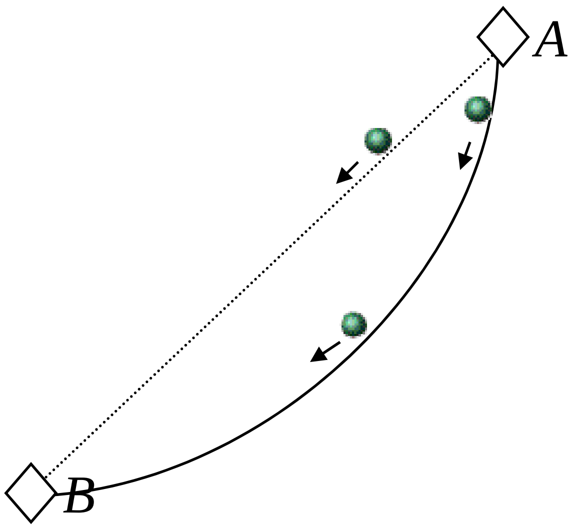
- **Find path and horizon** $h^*(t), t \in [0, T^*]$. **that maximize**

$$J = \int_0^T f\,\Delta w\left[h(t)-1\right]dt - \frac{1}{2}\lambda\int_0^T \sigma^2(\Delta w)^2\left[h(t)-1\right]^2 dt - c\left|\Delta w\right|\int_0^T dt - \psi\int_0^T (\Delta w)^2\left[\dot{h}(t)\right]^2 dt$$

- **Similar to MV optimization that maximizes expected return for a given level of risk**

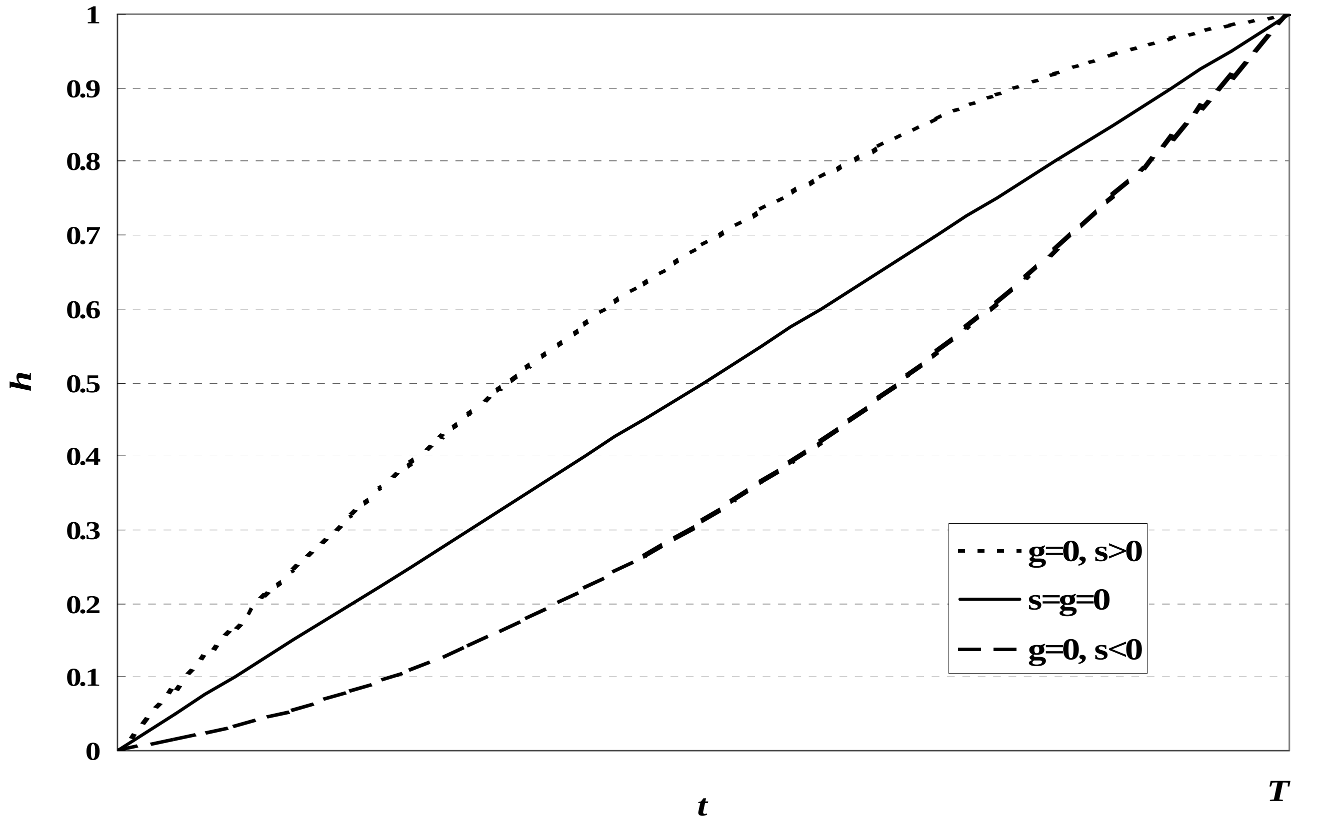
Mathematical Model – Solutions

- **Method of calculus of variation**
  - Find optimal function instead of optimal parameter
- **Ordinary differential equation for** $h(t)$
- **Boundary condition for** $h(t)$



Solution – No Risk Aversion

- **Three different expected returns (*s*)**



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Solution – No Risk Aversion

- **Optimal horizon**

$$T^* = \frac{2\sqrt{\psi}}{\sqrt{c} + \sqrt{c + f}}$$

- **Horizon should be longer if**
  - Market impact is high
  - Fixed cost is low
  - Return is low (if it agrees with the trade)

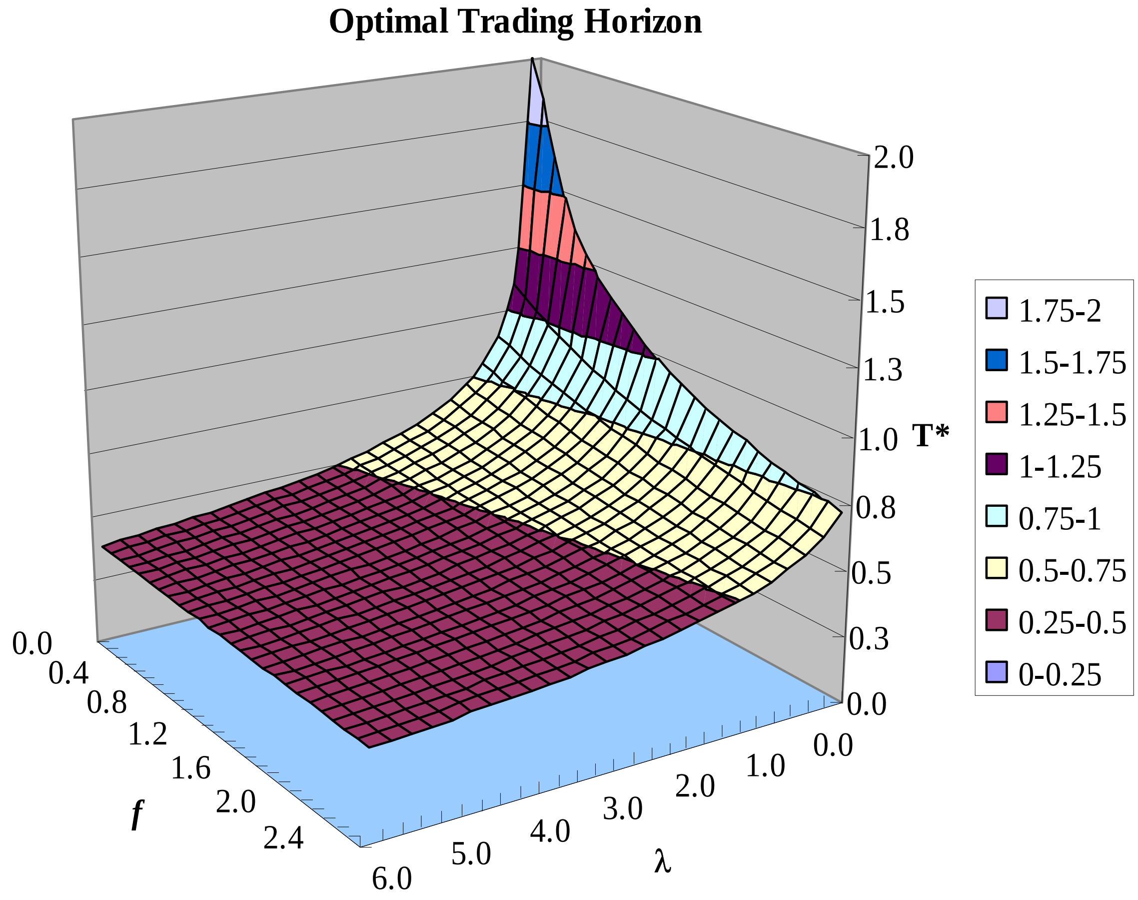
Numerical Examples

- **Base parameter assumption. Optimal horizon = 0.52 day**

f	1% / day
σ	4% / day
λ	2 day / %
c	0.1% / day
ψ	0.5 % day
$s = f/2\psi$	1 / day^2
$g = \sqrt{\lambda\sigma^2/2\psi}$	5.7 / day
$p = \sqrt{c/\psi}$	0.45 / day

Numerical Examples

➢ Changing parameters – case I



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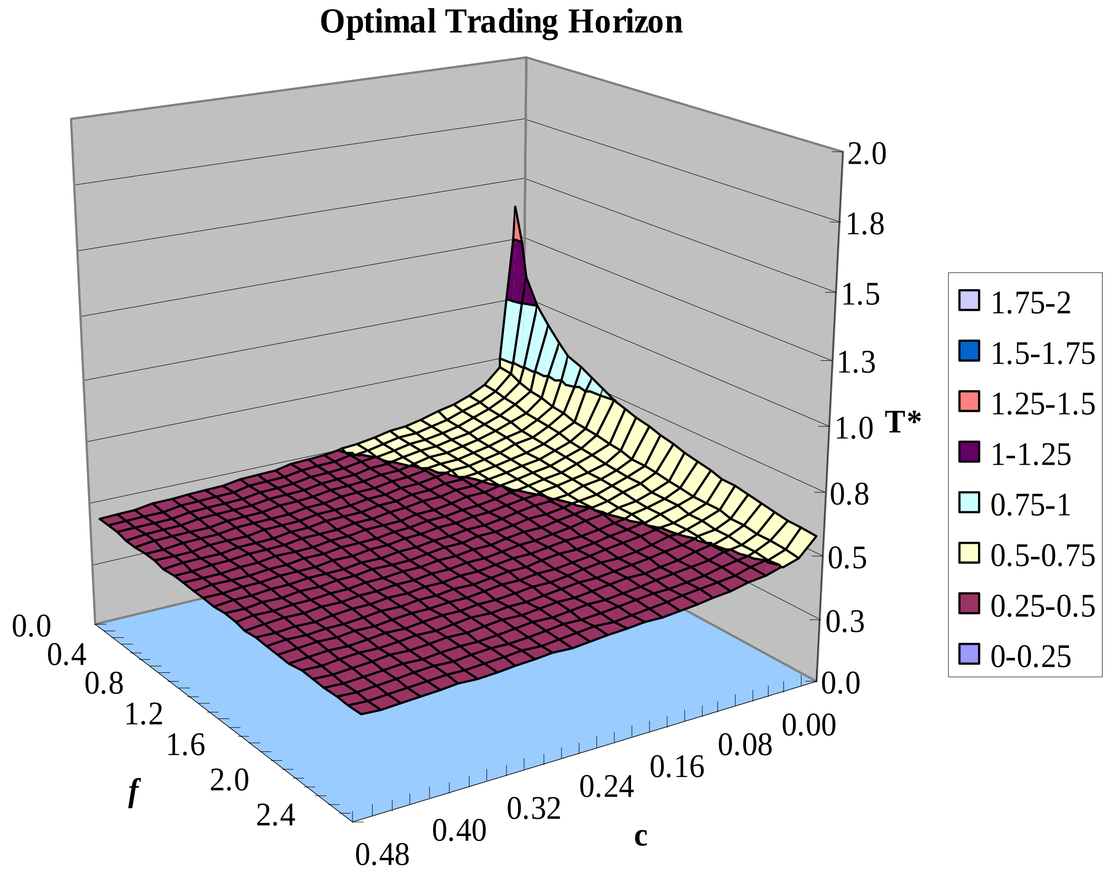
Numerical Examples

➢Changing parameters – case II



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Summary

- **There is often an optimal trading horizon with optimal trading strategy**
- **Our analytic solution shows the optimal horizon depends on**
  - Expected return
  - Stock volatility
  - Fixed cost
  - Market impact
- **The solution can be extended to portfolios of stocks**
- **Practical applications hinge on modeling of multiple processes**

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