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Short-Term Risk From Long-Term Models

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Setup

- Goal: A model that forecasts short-term portfolio variance
- Believe structure of long-term model
- Long horizon intentionally downplays recent information
- Short time scale returns are negatively serial correlated and have more big moves

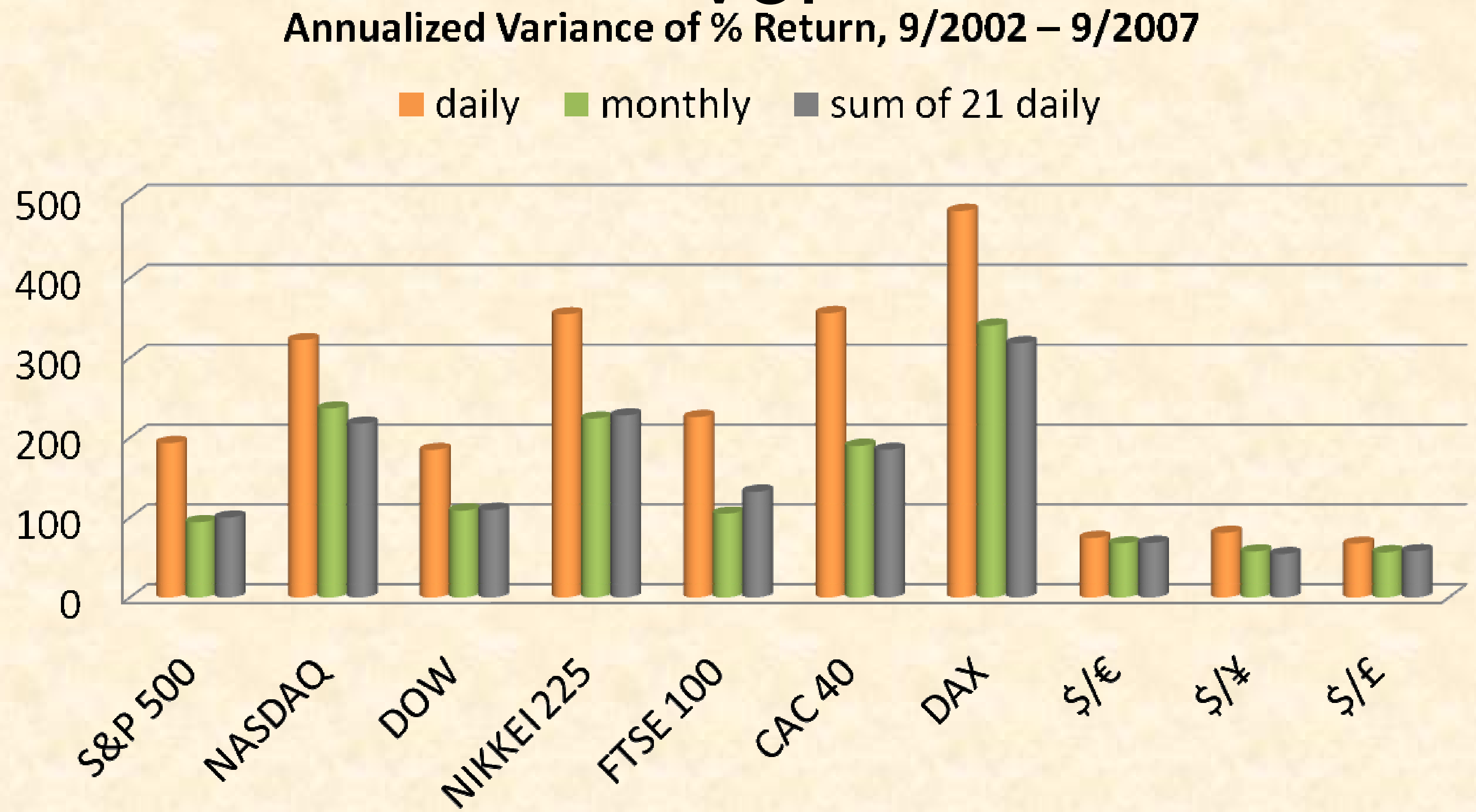
Framework That Welcomes Any Model

1 "Bend" model to current conditions

2 Scale variances to account for serial correlation in short-term returns

3 Be aware that distribution has bigger tails than normal

I: Negative Serial Correlation (time scaled) 1 day vol > 1 month vol



Adjusting for Serial Correlation

- Goal: The short time scale variance inflation introduced by negative serial correlation

- Express cumulative return from time 0 → time T in terms of 1 day returns:
  $r_{0\to T} = f\,(r_1, \ldots, r_T) = \prod_{t=1..T} (1 + r_t) - 1$

- Approximate linearly around mean return, μ
  $r_{0\to T} \approx f|_{r_1 \ldots r_T = \mu} + \nabla f|_{r_1 \ldots r_T = \mu}\, (\mathbf{r} - \boldsymbol{\mu})$
  $= \text{constant} + (1 + \mu)^{T-1} \sum_{t=1..T} (r_t - \mu)$

- Yields cumulative return variance as 1 day return covariances
  $\text{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2} \sum_{1 \le j,k \le T} \text{cov}(r_j, r_k)$

Adjusting for Serial Correlation (cont)

- Suppose 1 day returns follow an AR(1)

  $r_{t+1} = c + \rho r_t + \varepsilon_{t+1}$ same $var(r_t)$ in every period, σ^2

  $cov(r_j, r_k) = \sigma^2 \rho^{|j-k|}$ n period apart correlation $= \rho^n$

- Put cumulative variance in terms of 1 day variance and correlation

  $var(r_{0\to T}) \approx (1 + \mu)^{2T-2} \sigma^2 [T(1 + \rho)/(1 - \rho) - 2\rho(1 - \rho^T)/(1 - \rho)^2]$

- Project a forecast from one horizon to another by solving out σ^2 from approximations for $var(r_{0\to T_1})$, $var(r_{0\to T_2})$

  e.g. 1 month to 1 week, $T_1 = 21$, $T_2 = 5$ (or to 1 day, $T_2 = 1$)

  $T_1/T_2 \, var(r_{0\to T_2}) \approx var(r_{0\to T_1}) (1 + \mu)^{2(T_2 - T_1)} \times [(1 - \rho^2) - 2\rho(1 - \rho^{T_2})/T_2] / [(1 - \rho^2) - 2\rho(1 - \rho^{T_1})/T_1]$

What value for ρ?

9/02 – 9/07	Variance Ratio: Annualized Daily/Monthly	Measured Serial Correlation of 1 Day Returns	Serial Correlation Implied by Variance Ratio
S&P 500	2.1	-0.11	-0.36
NASDAQ	1.4	-0.04	-0.17
DOW	1.7	-0.10	-0.27
NIKKEI 225	1.6	0.00	-0.24
FTSE 100	2.2	-0.13	-0.38
CAC 40	1.9	-0.04	-0.32
DAX	1.4	-0.08	-0.19
$/€	1.1	-0.01	-0.05
$/¥	1.4	-0.02	-0.18
$/£	1.2	0.02	-0.09

II: Sources of Current Information

- Volatility and covariance measures
  - cross sectional
  - option implied
  - high/low/open/close (Parkinson, Garman-Klass, Beckers, Rogers-Satchell, Yang-Zhang)
  - return spreads

- ~~Liquidity – volume, short interest, bid/ask spreads~~

- *Method works only with information that can be predicted by the covariance model*

Bending to Fit Information

- Make model flexible by adding parameters $\boldsymbol{\theta}$
- Example 1: In Northfield's global model, let
  $\theta_1 \ldots \theta_5$ scale variances of the 5 region factors
  θ_6 scale variance of the value/growth factor
  θ_7 scale variance of all the remaining factors
  θ_8 scale stock specific variance for U.S. & E.U. stocks
  θ_9 scale stock specific variance for all other stocks
  $\theta_1 \ldots \theta_9 \geq 0$
- Example 2: Correlation tightening/loosening
  correlation matrix = $\theta_1 \times (\mathbf{1}\mathbf{1}^T) + \theta_2 \times \mathbf{I} + (1 - \theta_1 - \theta_2) \times$ original
  $\theta_1, \theta_2 \geq 0, \quad \theta_1 + \theta_2 \leq 1$

Bending to Fit Information - II

- For each observed statistic (piece of information)
  e.g. option implied variance of S&P 500
  $\hat{g}$ = recent value
  $\hat{g}_{avg}$ = average value over estimation period

- Find a related statistic that can be predicted by the model
  e.g. variance of S&P 500
  g_θ = predicted value under model adjusted by θ
  g = predicted value under model

➢ *Choose θ to make the ratios $(\hat{g}/\hat{g}_{avg})$ and (g_θ/g) match*

Examples of Information

- $\hat{g}$ = implied variance from options on SP500 index
  g = variance of SP500
  (a measure of volatility that includes correlation)

- $\hat{g}$ = average range variance estimates of EAFE constituents
  g = average variance of EAFE constituents
  (a measure of volatility separate from correlation)

- $\hat{g}$ = cross-sectional variance of Nikkei 225
  g = xc var (ignoring difference in means) of Nikkei 225

- $\hat{g}$ = median squared spread between 2 baskets of securities
  g = variance of long/short portfolio of same securities

Making the Intuition Rigorous

- Bayesian Framework

  $p(\boldsymbol{\theta})$ = prior distribution on parameters $\boldsymbol{\theta}$

  $p(\hat{\mathbf{g}})$ = unconditional probability of observing statistics $\hat{\mathbf{g}}$

  $p(\hat{\mathbf{g}}|\boldsymbol{\theta})$ = probability of observations $\hat{\mathbf{g}}$ given parameters $\boldsymbol{\theta}$

  $p(\boldsymbol{\theta}|\hat{\mathbf{g}})$ = probability that parameters are $\boldsymbol{\theta}$ given observations $\hat{\mathbf{g}}$

- Maximum a posteriori (MAP) estimate of $\boldsymbol{\theta}$ given $\hat{\mathbf{g}}$

  $\boldsymbol{\theta}_{MAP} = \text{argmax}_{\boldsymbol{\theta}}\ p(\boldsymbol{\theta}|\hat{\mathbf{g}}) = p(\hat{\mathbf{g}}|\boldsymbol{\theta}) \times p(\boldsymbol{\theta}) / p(\hat{\mathbf{g}})$

  $\boldsymbol{\theta}_{MAP} = \text{argmax}_{\boldsymbol{\theta}}\ \log p(\boldsymbol{\theta}|\hat{\mathbf{g}}) = \log p(\hat{\mathbf{g}}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta}) - \log p(\hat{\mathbf{g}})$

  $\boldsymbol{\theta}_{MAP} = \text{argmax}_{\boldsymbol{\theta}}\ \log p(\hat{\mathbf{g}}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$

Making the Intuition Rigorous - II

- Recall $\boldsymbol{\theta}_{MAP} = \text{argmax}_{\boldsymbol{\theta}}\ \log p(\hat{\mathbf{g}}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$

- Assume observations $\hat{\mathbf{g}}$ are noisy observations of predictions $\mathbf{g}_{\boldsymbol{\theta}}$:

$$\hat{\mathbf{g}} = \mathbf{g}_{\boldsymbol{\theta}} + \boldsymbol{\varepsilon}$$

$$p(\hat{\mathbf{g}}|\boldsymbol{\theta}) = p(\hat{\mathbf{g}}|\mathbf{g}_{\boldsymbol{\theta}}) = p(\boldsymbol{\varepsilon} = \hat{\mathbf{g}} - \mathbf{g}_{\boldsymbol{\theta}})$$

- The distribution of noise determines the fit criterion:

$\varepsilon^k \sim \text{Gaussian}[0, \sigma_k^2] \ \rightarrow\ \log p(\hat{\mathbf{g}}|\boldsymbol{\theta}) = \Sigma_k\, [\hat{g}^k - g_{\boldsymbol{\theta}}^k]^2 / 2\sigma_k^2 + \text{const}$

$\varepsilon^k \sim \text{Laplace}[0, b_k] \ \rightarrow\ \log p(\hat{\mathbf{g}}|\boldsymbol{\theta}) = \Sigma_k\, |\hat{g}^k - g_{\boldsymbol{\theta}}^k| / b_k + \text{const}$

Summarizing the Framework

- Add parameters to model and tune so that increases in predictions match recently observed increases

- Scale long horizon variance #'s to account for the reversing swings that occur day to day

- We have not fixed a distributional shape

  Be aware that short term returns have more extreme events

Example: Aug 31, 2007

	Median Value Relative to Trailing 5		
	Jul '07	Aug '07	Last 3 Days of Jul ’07
VIX	1.10	1.72	1.62
1 Day Intraday Log(High/Low):			
S&P 500	1.05	2.06	2.11
Avg of S&P 500	1.03	1.70	1.64
Nikkei 225 (¥)	0.56	1.31	1.28
IShare S&P Eur	1.03	1.73	1.59
1 Week Daily Close Log(High/Low):			
€ (in $)	0.71	0.94	
¥ (in $)	1.04	1.63	

Tune NIS Global Model to Current Conditions

- Factors:

  Sector, Region, Economic Variables, Pricing Spreads, Blind Factors, Currencies

- Add parameters to make model flexible

Parameter	Scales Standard Deviation of
θ_1	E.U. & Scandinavian currencies
θ_2	All other currencies
θ_3	Sector factors
θ_4	Stock specific risk
θ_5	All remaining factors

The Tuned Model

Factor	Std Dev Scaled By
E.U. & Scandinavian currencies	0.93
All other currencies	1.63
Sector factors	2.14
Stock specific risk	1.78
All remaining factors	0.65

Predicted	Data	Observed Relative	In Tuned Model
S&P 500	Hi/Lo	2.06	1.93
Avg of S&P 500	VIX	1.72	1.72
Nikkei 225 (¥)	Hi/Lo	1.31	1.32
S&P Eur 350 ($)	IShare	1.73	1.86
€ (in $)	Hi/Li	0.94	0.93
¥ (in $)	Hi/Lo	1.63	1.63

Serial Correlation Adjustment

- Suppose ρ = –.30, μ = 0

  Note: Can apply a different serial correlations to each factor and, by security, to stock specific risk

- Monthly → weekly std dev adjustment (T_1 = 21, T_2 = 5)

  1.05 in "annualized units"

- Monthly → daily std dev adjustment (T_1 = 21, T_2 = 1)

  1.34 in "annualized units"