Multiple Managers, Multiple Accounts and Trading "Fairness"

Dan diBartolomeo

Naples, FL

January 2008

Northfield
INFORMATION SERVICES, INC.

A Lot to Talk About Today

- We will re-introduce Centralized Portfolio Management
  - a non-traditional multiple manager scheme introduced by Northfield in 1992 to ***enhance returns***
  - particularly valuable to taxable investors
  - rapidly gaining assets under management for the first firm to implement it, Vanguard Australia
- We will extend concepts from CPM to:
  - provide a resolution to the “trading fairness” issue associated with the market impact of block trades across multiple portfolios
  - provide an optimal “global” solution to a set of separate, but related accounts, such as those held by various members of a family
- Each of these topics is probably enough for a separate hour talk so we’re going to move quickly

Northfield
INFORMATION SERVICES, INC.

The Best Wine We Ever Served

- Centenary Solera Madera, 1845 was served at our 1992 client conference at Bretton Woods
  - This was the last time I spoke about Centralized Portfolio Management at a Northfield event
  - Brokers thought it would reduce trading (and their incomes)
  - Asset managers thought it would cause reductions in management fees (and hence their incomes)
  - Plan sponsors thought I was calling them stupid for not having done it already
  - Everyone agreed it was probably analytically correct but were very unhappy with me. As usual, I sought help from the bartender. The Madera was hastily added to the dinner menu.
- Everyone, including me, agreed that no one would ever do it in the real world. *Everyone was wrong.*

Northfield
INFORMATION SERVICES, INC.

The Basic Motivation for Using Multiple Managers

- *The purpose of using multiple active managers is to add return*, not to provide diversification.
  - If an investor wishes to simply reduce the risk of under-performing a particular benchmark index, it may be accomplished efficiently by owning an index fund. Such passive investment also reduces fees and trading costs
  - It must therefore be true that the value added by active management arises from the manager's efforts to create forecasts that are meaningfully predictive of future index-relative security returns (that is, alpha)
  - There may be some logic in having multiple active managers in an effort to prudently protect the fund against embezzlement or other illegal actions on the part of investment agents, but this protection could be met equally well with multiple passive managers

Northfield
INFORMATION SERVICES, INC.

The Multi-Manager Problem

- In the typical multiple manager portfolio, a single manager has influence over each dollar of the total fund.
  - This methodology rests on two beliefs: that each manager is expected to produce superior performance on average and that the certainty of any particular manager producing superior performance is low.
  - Hence, there is a need to diversify with multiple managers so as to decrease the uncertainty associated with each manager's forecasts of future alphas.
  - By allowing multiple managers to operate within the same universe of securities with similar constraints, the aggregate position of the total fund with respect to any particular security is reflective of the consensus alpha forecast of the managers. In this fashion, the fund benefits from the improvement in the aggregate quality of the return forecasts.

Northfield
INFORMATION SERVICES, INC.

Multi-Manager Correlation

- Unfortunately, there exists the possibility that manager alphas will be correlated.
  - For example, if a fund had ten managers, but all ten simultaneously adopted a similar strategy (e.g., mid-cap, low P/E), little improvement in the quality of the alpha forecasts would result.
  - Even worse, the overall fund would be subject to large residual risk. The presumed diversification benefit of multiple managers would vanish if each manager held a similar portfolio
  - To prevent such situations, it is common practice for plan investors to hire multiple active managers, but with distinct mandates. For example, the investor would hire separate managers for large-capitalization and small capitalization stocks (or growth/value, etc.).
  - Once we have separated the manager mandates into mutually exclusive habitats, we now have only the predictive skills of a single manager applying to each security. *The improvement in aggregate forecast quality has been lost.*

Northfield
INFORMATION SERVICES, INC.

The Crossing Cost Problem

- One way to preserve the benefits of multiple managers without using distinct habitats would be to allow all managers the same mandate, but select managers that will be uncorrelated in their future alpha forecasts.
  - This unfortunately leads to the problem of internal crossing.
- Let's assume that the price of stock XYZ has just run up in reaction to the announcement of better than expected earnings. Value manager "A" believes that stock is now overpriced and sells it. Growth manager "B" believes that the stock has good future prospects and buys it.
  - The investor has just incurred two transaction costs, *possibly a capital gain tax, a possible wash sales violation,* and continues to pay two active management fees while nothing has happened to the total portfolio.
  - When we expand this example to industries and sectors, multiple active managers will inevitably spend most of their time in a meaningless but expensive tug-of-war.

Northfield
INFORMATION SERVICES, INC.

Capital Allocation and Risk Budgeting for Multi-manager Portfolios

- Allocating capital across multiple active/passive managers is a complex issue
  - Managers can be of varying levels of aggressiveness
  - Active managers generally are judged on *pre-tax* benchmark relative performance, while investors receive after tax returns
  - Investors have varying degrees of concern about both benchmark relative and absolute risk
  - The correlation of active returns across managers can be very unstable as market conditions change
- Shifting money between managers to rebalance capital allocations is expensive
  - *May involve capital gain taxes as well as transaction costs*
  - *Managers are hesitant to accept "legacy" positions from other managers that could reduce their performance record*

Northfield
INFORMATION SERVICES, INC.

Simple Multi-Disciplinary Accounts Distort Manager Intent

Assume we have a two manager portfolio with 50% of capital allocated to each.

- Manager A thinks XON is a great stock and overweights it 3%. He has market weighting on the airline sector. Manager B has a big 5% overweight in airlines (which he considers undervalued) and so overweights XON 3% as a hedge against oil prices rising.
- If we just add the positions together, we will have a 3% overweight in XON and a 2.5% overweight in airlines.
- However, Manager B wanted to overweight XON as a hedge against his big airline bet. Since the airline sector is now just overweighted by 2.5% in the aggregate portfolio, the correct overweight for XON in aggregate is 2.25% not 3%
- Even with some kind of tax overlay process in place, its inefficient because each manager makes risk/reward decisions myopically without knowledge of what the overall portfolio looks like.

Northfield
INFORMATION SERVICES, INC.

The Rosenberg Solution

- Rosenberg (1977) introduced the “centralized” multiple-adviser fund as an alternative
  - Outside managers are hired to advise the investor
  - They inform the investor of their market, security return and possibly risk forecasts on a continuous basis
  - The investor (or an agent) computes a statistical consensus of the forecasts and manages a single large portfolio in a fashion consistent with the consensus forecasts of the hired advisers.
  - A proof is presented that a multiple manager fund can at best equal the efficiency of the centralized multiple-adviser fund. Many of the aforementioned drawbacks to the current approach are resolved in the multiple-adviser scheme.
  - The problem with this scheme is that it requires managers to deliver explicit numerical forecasts to the sponsor both for market returns and for the relative returns of each security. Many successful investment firms are very qualitative in their approach and use a great deal of subjective judgment that is not easily converted to a certainty adjusted return forecast.

"Side Funds" Can Patch Multiple Manager Portfolios

- Tierney (1980) introduced the "completeness fund"
  - Deals with the possibility that a multiple manager fund had segmented mandates that left gaps relative to the overall benchmark investor
  - A completeness fund is simply a passive fund structured to fill in the gaps left by the aggregate of the manager mandates relative to the overall benchmark.
- Ferguson (1978) describes the "inventory fund" to address unnecessary crossing costs.
  - An inventory fund is a passively managed account that could (but is not obligated) to act as the other side of trades initiated by a given investor's active managers.
  - Consider our earlier example of an internal cross: Managers "A" and "B" transact with the "inventory account". The trades are now only book entries, with no fees and no tax implications
  - Wagner and Zipkin (1978) estimate the potential cost savings from the use of inventory funds.

Northfield
INFORMATION SERVICES, INC.

Key Insight

- Almost all asset managers use risk models from an outside vendor for portfolio analysis and construction
  - Suggests that managers think the risk models work pretty well
  - Managers see their “value added” in superior return forecasting
- If everyone roughly agrees on the covariance among securities, then we can infer manager “alpha” forecasts from the portfolio they choose to hold.
  - Active managers must think their portfolio is optimal or they would hold a different portfolio
  - Expected returns must offset marginal risks at optimal weights
  - Sharpe (1974), Fisher (1975)
- If we know the aggressiveness level of a manager’s strategy, we can obtain the alphas directly.
  - If not we can estimate risk tolerance from observed risk values
  - Alternatively, we can use the implied rank values and then map into the expected cross-section of returns

Northfield
INFORMATION SERVICES, INC.

The Easy Thing to Do with Implied Alphas

- Build customized “separate accounts” to meet the heterogeneous needs of taxable clients from a model portfolio such as a successful mutual fund
  - Easy to create portfolio versions at different aggressiveness levels, portfolio sizes, SRI constraints or tax considerations
  - Model portfolios can be run with any strategy, fundamental or quantitative
  - Relative performance across versions makes clear to investors the tradeoffs they are experiencing between “best ideas” investment performance and the influence of customizing to their preferences and constraints
  - Rationally migrate portfolios from legacy positions toward the model portfolio *over time to minimize capital gain realization*

Northfield
INFORMATION SERVICES, INC.

Centralized Portfolio Management

- Combine “implied alpha” methods with Rosenberg’s “centralized adviser” structure
- Each external manager runs a paper portfolio, as is done with managers that provide model portfolios for “wrap” programs, reporting trades to the investor’s agent
- Use statistical methods to combine the implied alphas across all managers, and apply to running one portfolio
  - The consensus alpha is used on the entire value of the portfolio, so you *get the return benefit of “two heads are better than one”. We get the sum of the manager’s knowledge, not the average.*
  - See Johnson (1972), Rudd and Clasing (1982)
  - Risk control is internally consistent as the alphas are implied using the same model used for portfolio construction of the central account

Northfield
INFORMATION SERVICES, INC.

Other Benefits of CPM

- Almost all the “crossing costs” disappear
- Taxes are a lot easier to manager. No more wash sales
- Rebalancing costs disappear as shifting managers adjust changing weights in the formation of consensus alphas
- Manager aggressiveness and turnover levels no longer impact the aggressiveness of the whole
  - They are just part of the weighting of the alpha consensus
- Investors have direct control over commission spending
- You can give negative weight in the consensus to consistently bad managers
- Managers that are capacity constrained can participate
- No more myopic risk controls at the individual manager level filter through to the central portfolio

Northfield
INFORMATION SERVICES, INC.

The “Long Only” Problem

- If the external active managers are “long” only, the implied alphas will be biased
  - Will exclude the large negative values that would be implied by short positions
  - Positive skew in the distribution of implied alpha values
  - The distribution of the implied alphas should be symmetric about its mean and the alpha of the market portfolio should be around zero
- For positions that hit the zero weight bound, adjust the implied alpha
  - Subtract some constant X times the asset specific risk of that stock, making the implied alphas more negative
  - Select X such that the market portfolio alpha is close to zero and the distribution of implied alpha has insignificant skew

Northfield
INFORMATION SERVICES, INC.

Other Possible Pitfalls

- The implied alphas can be biased through estimation errors in the risk model
  - You're no worse off than in conventional management that is using the same risk model (i.e. just about everybody)
- External managers could concentrate on illiquid positions to boost returns on their paper portfolio
  - Use an agreed upon market impact model to estimate "trading costs" of the external paper portfolios and judge performance after costs
- Managers could "front run" the central portfolio if they know their weight in the consensus is high

Northfield
INFORMATION SERVICES, INC.

Somebody Did It

- Centralized Portfolio Management has been successfully implemented by Vanguard Australia
  - Even large pension funds in Australia are subject to some capital gain taxes, so taxes are a big deal
  - The Australian market is small so hot managers hit capacity constraints quickly
  - Assets under management are growing quickly with a couple of major Australian state funds participating
  - August 7, 2007 press release quotes Scott Lawrence of Vanguard: "Results to date are a 60 basis point improvement in returns with a 50 to 70 percent reduction in turnover"

Northfield
INFORMATION SERVICES, INC.

Ensuring Trading Fairness

- If a manager has multiple portfolios under their control there is an SEC mandated responsibility to trade “fairly” for all accounts
- If you assume that trading costs are non-linear in trade size (i.e. market impact exists) you have a problem whenever you try to optimally manage more than one heterogeneous account
- Given different objectives and tax circumstances, you may be buying a security in some accounts and selling in others. You should take advantage of the potential internal cross to avoid market impact

Northfield
INFORMATION SERVICES, INC.

The Fairness Problem

- Imagine you have 100 heterogeneous portfolios to rebalance
  - When you optimize account #1 you don't know anything about expected trade sizes, so we assume zero market impact
  - As you optimize accounts # 2, 3, 4, ... you can aggregate the expected trades across all accounts and adjust your transaction cost assumptions accounting for subsequent optimizations
  - By the time you get to account #100, the trading cost assumptions are markedly different than you started with so the "optimal" trades you got for account #1 *are very sub-optimal given the revised cost estimates*
  - The later accounts in the optimization queue benefit at the expense of the early accounts, violating fairness requirements

Northfield
INFORMATION SERVICES, INC.

Current Approaches

- Many “wrap” SMA managers simply require every account to be identical, grossly suboptimal for heterogeneous, taxable investors
- The “Ostrich” method
  - A bad but popular alternative is to ignore aggregate market impact, treat each account myopically and hope your traders can manage to keep costs down
- Set maximum aggregate trade sizes that you believe you can transact with low market impact
  - Now account #1 is in the best position, and the accounts early in the queue unfairly benefit
  - Account #1 gets first choice of available supplies of securities, while account #100 can only transact what is “left over” after the rest of the accounts have done their trades
  - This is a real mess if you have large accounts at the end of the list. If you put the larger accounts first in queue, your actions may be interpreted as being explicitly unfair to the small accounts

Northfield
INFORMATION SERVICES, INC.

Lets Consider Basic Cases

- Assume you have N portfolios all of equal size, objectives, holdings and tax circumstances
  - You'll want to do the same thing in every account so all you have to do is optimize one account and adjust your market impact assumptions for trades to reflect aggregate share amounts being N times as big
- Now lets assume you have N accounts of unequal size but with the same objectives, holdings and tax circumstances
  - Similar to above but we need to adjust our market impact assumptions in the optimization to reflect the ratio of the total value of all accounts (T) to the value of the portfolio being optimized (X_i)
  - Your expected trades are T/X_i times larger so cost assumptions need to be adjusted

The Not So Basic Real World Case

- Now lets think again about our 100 heterogeneous portfolios
  - Our ratio of the total trade in stock j (T_j) to the trade in stock j in account i (X_{ij}) is going to be specific to each individual security, not just to each specific account, because we're making different trades with different stocks in different accounts
  - The problem is circular. The total T is the sum of the individual trades (the X values), but the optimal trades forming vector X are conditional on expected costs that are a function of T
  - We're back to the classic "chicken and egg" problem

Northfield
INFORMATION SERVICES, INC.

Suggested Methodology: Smart Trial and Error

- Obtain a preliminary estimate of the trades in each account before you actually optimize (the X_{ij} values)
- Sum the X_{ij} values to get estimates of the total trade T_j for each stock
- Calibrate your transaction cost assumptions based on the estimated T_j values
- Optimize all accounts using the transaction cost assumptions and observe the recommended trades
  - If the recommended trades are close to the preliminary estimated Xij values, you're done
  - If you're not close enough, go back to the second step, revise the X_{ij} values and repeat the remaining steps

Northfield
INFORMATION SERVICES, INC.

A Preliminary Estimate of Required Trades

- For each portfolio i, obtain the vector of implied alphas via the methods previously described
- Compare the vector of implied alphas for account i to the vector of your expected alphas
  - If the implied alpha is higher than the expected alpha we're going to want to sell this stock in this account. If the implied alpha is lower than the expected alpha we're going to want to be buying this stock in this account
- Within each account, the *active weights will bear a specific algebraic relationship to the alpha*
  - From this relation we can roughly approximate the expected trades if there are no position size constraints or impediments (big tax costs of a sale of low cost basis shares)
  - If there are constraints, we can use the lesser of the estimated trade and largest allowable trade given the constraints as our "rough estimate"

Northfield
INFORMATION SERVICES, INC.

The Relation Between Alpha and Active Weight

- If we assume that all stocks are of equal volatility and equally correlated with one another, then unconstrained active weights are linearly related to alpha
  - Clarke, deSilva, and Thorley (2002)
  - Changes in optimal weight are proportional to incremental alpha
- If we have a risk model we can do better
  - Think of a two asset optimization where one asset is portfolio i, and the other asset is stock j with an initial zero weight
  - Use the risk model to get estimates of the volatility of each asset and the correlation between them. See diBartolomeo (1998)
  - Since the optimization has only two assets, the unconstrained optimum has a closed form algebraic solution
  - If stock j is a constituent of portfolio i, net the new and old positions to determine the required trade

The Multiple Related Account Problem

- Multiple accounts held by various members of the same family represent a very special challenge to wealth management organizations.
  - A single family could have accounts that are held in the name of the father, mother, various children, plus trust funds
  - Some of these accounts will be taxable, with a variety of tax circumstances and legacy holdings. Other accounts within the set are likely to be tax-deferred retirement accounts or possibly offshore structures.
  - Even a single individual with a tax deferred account must make the interconnected choice of *asset allocation* and *asset location*.
  - In addition, each of the accounts may have a different level of risk tolerance.

Northfield
INFORMATION SERVICES, INC.

The MRA Structure

- Within a set of multiple related accounts (MRA), there are separate pools of money in various sub-accounts
  - Funds cannot be moved from one to another
  - In some cases, the client has multiple managers each actually managing a "sleeve" or distinct sub-account.
  - In other cases, there are sub-accounts for legal reasons (e.g. husband and wife, 401K, trust fund, etc.)
  - There is no central actual account. Each sub-account may have its own benchmark, tax circumstances and risk tolerance.
- Our goal in this case is to do a "global" optimization of the client's MRA portfolio across all sub-accounts:
  - Caveat: What is optimal for a family group as a whole is not likely to be optimal for any one member of the group. This can lead to some rather nasty issues in divorces and across generations

Northfield
INFORMATION SERVICES, INC.

MRA Optimization Goals

- Our goals in this case are to do a "global" joint optimization of the client's MRA portfolio across all sub-accounts:
  - tax efficient across the entire MRA portfolio
  - capture as much of the investment performance as possible as would otherwise have occurred in each sub-account
  - keep each of the sub-accounts meeting any position size constraints that were originally imposed on that sub-account
  - No money can move between sub-accounts
- This would be easy to optimize as one big portfolio if the tax rates and risk tolerance were uniform across the sub-accounts, but they aren't.

Northfield
INFORMATION SERVICES, INC.

Unifying the Tax Rates

- Our basic approach is to transform the problem we have (multiple heterogeneous sub-accounts) to a mathematically equivalent problem where all the sub-accounts have uniform properties of tax rate and risk tolerance
  - Assume we have an individual with both taxable and tax-deferred accounts. To simulate tax deferred positions, we can *pretend* that the cost basis of any position in the tax-deferred account is always equal to the current market price. This removes any influence of capital gain taxes from the optimization on those specific securities
  - For different tax rates across different taxable entities, adjust the cost basis of the positions so that the value of the potential taxes on positions are proper. If I choose a "global" capital gain tax rate of 20%, and one sub-account is taxed at 10%, we adjust the position cost bases to make gains and losses half as large for that sub-account.

Northfield
INFORMATION SERVICES, INC.

Unifying Risk Tolerance

- Create separate versions of each security, including cash for each account such as IBM_Husband, IBM_Wife, IBM_IRA using the Northfield composite asset function
  - We need to use the composite asset function rather than add records to the database in order to account for the correlation being one, not zero, between the “asset specific” risks of different versions of the same security
  - Multiple lots of the “version securities” are permitted
- To account for different risk tolerances across sub-accounts we adjust the composition of the composites
  - Assume #1 is twice as risk tolerant (in variance terms) as account #2. We just treat a 100 share IBM position in account #1 as composite asset (IBM_1) with an underlying portfolio of 100 shares of IBM.
  - For account #2 that is only half as risk tolerant, we create the mathematical equivalent by keeping the same risk tolerance coefficient, but representing the underlying portfolio of the composite asset as ($100 * 2^{.5}$) shares of IBM, with an offsetting negative position in cash.

Northfield
INFORMATION SERVICES, INC.

Handling Constraints

- Once we've got the security "versions" created by sub-account, we can add a variety of constraints
  - Position size constraints within a sub-account, or across all sub-accounts of the MRA
  - Sector or factor constraints at the sub-account or MRA level
  - Constraints that prevent money from moving between sub-accounts
  - Wash sale limitations for multiple accounts held by the same taxable entity using an approach similar to our "trading fairness" methods
- Most typical optimization constraints (i.e. limit turnover) still work on the "global" solution
- Mechanics of the entire process including use of implied alphas was described in diBartolomeo (2005)

Northfield
INFORMATION SERVICES, INC.

Conclusions

- Efficient methodologies now exist for dealing with the heterogeneity across high net worth investors
- Centralized Portfolio Management can be very usefully applied to HNW portfolios with multiple managers in order to improve returns
- The implied alpha concept on which CPM is based also provides solutions to the “trading fairness” problem
- Very complex cases, such as finding the globally optimal solution across “Multiple Related Accounts” problem can be handled with a combination of “implied alpha” and composite asset methods

Northfield
INFORMATION SERVICES, INC.

References

- Berkowitz, Stephen, Dennis Logue and Eugene Noser. “The Cost of Transactions on the NYSE,” *Journal of Finance*, 1988
- Best, Michael and Robert Grauer. “On the Sensitivity of Mean-Variance Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results,” *Review of Financial Studies*, 1991.
- Broadie, Mark. “Computing Efficient Frontiers Using Estimated Parameters”, *Annals of Operations Research*, 1993.
- Chandra, Amit. “Some Nuances of Return Modeling: The Long and Short of It”, *Proceedings of the Northfield Investment Research Conference*, Jekyll Island, Georgia, 1997.
- Clasing, Henry and Andrew Rudd. *Modern Portfolio Theory.* Homewood, Illinois: Dow-Jones Irwin, 1982
- Chopra, V.K. and W.T.Ziemba, “The Effect of Errors In Means Variances and Covariances on Optimal Portfolio Choice”, *Journal of Portfolio Management*, 1993

Northfield
INFORMATION SERVICES, INC.

References

- Ellis, Charlie, “Small Slam”, *Financial Analyst Journal*, January 1997
- Ferguson, Robert. “Do Market Inventory Funds Make Sense?” *Financial Analysts Journal*, May 1978.
- Fisher, Lawrence. “Using Modern Portfolio Theory to Maintain an Efficiently Diversified Portfolio”, *Financial Analysts Journal*, 1975.
- Grinold, Richard and Ronald Kahn. *Active Portfolio Management*, New York: McGraw-Hill, 1995.
- Horan, Michael and David Johnsen. “Soft Dollars: Theory and Evidence”. Working Paper SUNY Buffalo, 1997.
- Johnson, J. *Econometric Methods*, New York: McGraw-Hill, 1972
- Kalberg, J.T and W.T. Ziemba, “Mis-Specification of Portfolio Selection Problems”, in *Risk and Capital*, editors G. Bamberg and A. Spreman, Lecture Notes in Economics and Mathematical Sciences 227 (Springer, New York, 1984).

Northfield
INFORMATION SERVICES, INC.

References

- Kritzman, Mark and Don Rich, "Risk Containment for Investors with Multivariate Utility Functions", *Journal of Derivatives*, 1997.
- Keim, Donald and Ananth Madhavan, "The Cost of Institutional Equity Trades", *Financial Analysts Journal*, July 1998
- Michaud, Richard. *Efficient Asset Management*, Boston: Harvard Business School Press, 1998.
- Plexus Group, "Commentary 54: The Official Icebergs of Transaction Costs," www.plexusgroup.com, 1998
- Rosenberg, Barr. "Institutional Investment with Multiple Portfolio Managers", *Proceedings of the Seminar on the Analysis of Security Prices*, University of Chicago, 1977, pp. 55-160.
- Sharpe, William. "Imputing Expected Portfolio Returns from Portfolio Composition", *Journal of Financial and Quantitative Analysis*, 1974.
- Tierney, David, "The Marriage of Judgement and Techology: the Second Potent Generation of Index Funds". Presentation to the Eighth Annual American Pension Conference. January 11, 1980.

Northfield
INFORMATION SERVICES, INC.

References

- Wagner, Wayne and Carol Zipkin. “Can Inventory Funds Improve Active Equity Performance? *Financial Analysts Journal*, May 1978.Clarke, Roger, Harindra de Silva and Steven Thorley. "Portfolio Constraints And The Fundamental Law Of Active Management," Financial Analyst Journal, 2002, v58(5,Sep/Oct), 48-66.
- diBartolomeo, Dan. “Optimization of Composite Assets Using Implied Covariance Matrices”, Northfield Working Paper, December 1998, http://www.northinfo.com/documents/58.pdf
- diBartolomeo, Dan. “Optimization of Multiple Related Accounts”, Northfield News, May 2005. http://www.northinfo.com/documents/133.pdf
- diBartolomeo, Dan “A Radical Proposal for the Operation of Multiple Manager Investment Funds”, Northfield Working Paper, August 1999, http://www.northinfo.com/documents/61.pdf

Northfield
INFORMATION SERVICES, INC.