The P/B-ROE Model Revisited

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Agenda

- Characterizing a good equity model: Its virtues and uses
- Static vs. dynamic models
- The P/B-ROE model: Closed form & approximate solutions
- Cross-sectional explanation using the P/B-ROE model
- Cross-sectional prediction using the P/B-ROE model
- Time-series explanation using the P/B-ROE model
- Time-series prediction using the P/B-ROE model

What Characterizes a Good Model?

- Economic realism in its intellectual underpinnings
  - Must be grounded in a realistic view of the firm
  - Must allow the incorporation of economic constraints
    - e.g. Earnings cannot grow faster than revenues in perpetuity
- Parsimony and computability
  - Should require relatively few inputs
  - Inputs should be readily available or easily estimated from data
- Widespread applicability
  - Model prices should explain prevailing prices without significant bias
  - Model residuals should predict future returns
  - Should be applicable in cross-section and time-series

Who Might Use a Good Model?

- Corporate officers
  - If the model can guide them on how best to increase firm value
- Fundamental analysts
  - If the model can help them better evaluate a firm and its management
- Investment bankers and buyers and sellers of companies
  - If the model can generate unbiased valuations
- Investors
  - If the model's residuals are predictive of future returns

Models in Widespread Use Today

- Dividend Discount Model (J.B. Williams, 1938): $P_0 = \sum_{i=1}^{\infty} \frac{E[FCF_i]}{(1+k)^i}$
  - Intellectual root of almost all models in use today
- Gordon Growth model (1962): $P = \frac{\text{Free Cash Flow}_1}{k - g}$
  - Free cash flows grow at a constant rate in perpetuity
- Edward-Bell-Ohlson Equation (1961): $P = B_0 + \sum_{i=1}^{\infty} \frac{E[(r_i - k) \times B_{i-1}]}{(1+k)^i}$
  - Apply clean surplus relationship to DDM and rearrange terms
- Various multi-stage versions of the DDM
  - 3 stages model growth, steady state and decline

Static vs. Dynamic Models

- A static model evaluates price at a point in time
  - Estimate inputs at fixed points in time, discount back to get today's price
  - Examples: DDM, EBO

- A dynamic model evolves some function of price over time
  - Some evolve price, others evolve a valuation ratio
  - Trajectory must be consistent with the model: a hint of continuous time
  - Examples: Options (Black-Scholes), pricing a zero-coupon bond
    - Bond price trajectory must be consistent with the yield curve

- Both static and dynamic models can have the same intellectual roots
  - Both ultimately give us a fix on today's price
  - Choice of one over the other is empirical – which works better in practice

A Brief History of Dynamic Models

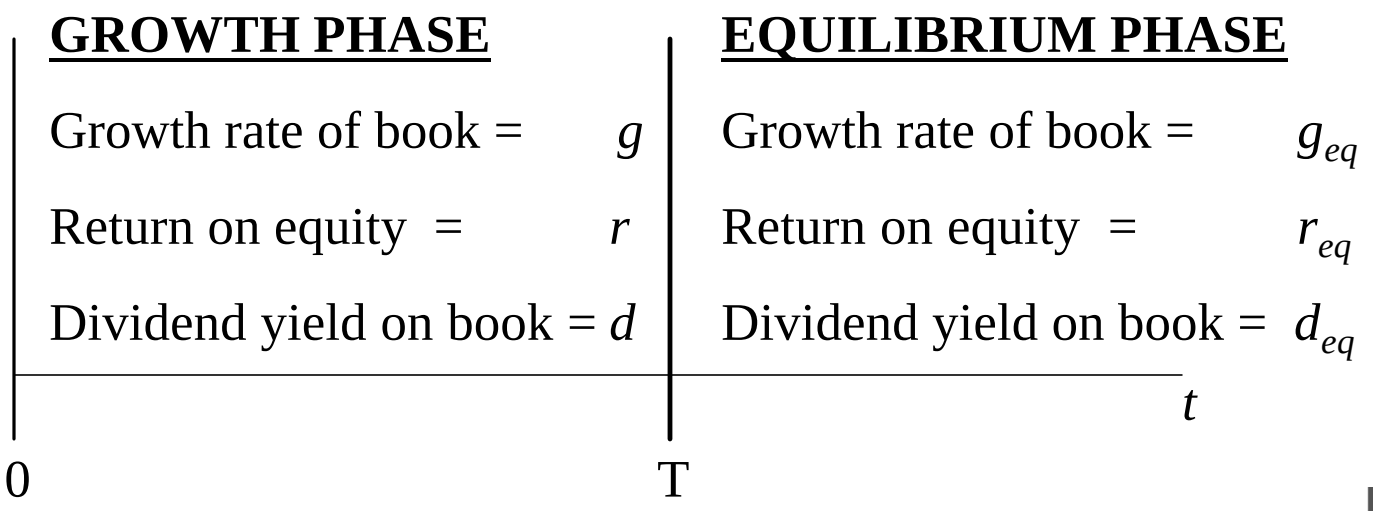
- Jarrod Wilcox (FAJ 1984): *P*/*B*-*ROE* model.
  - Two stage growth model ,with first phase ending at time *T*.
  - Determine the trajectory of *P*/*B* subject to the constraint $P/B_T=1$
  - Obtain today's *P*/*B* from trajectory & terminal condition: $\ln(P/B)=(r-k)T$

- Tony Estep (FAJ 1985, JPM 2003): *T* (or Total Return) model
  - Follows *P*/*B*-*ROE* logic, but arbitrarily sets time horizon to 20 years
  - Derives and tests a holding period return: $T=g+\left(\frac{r-g}{P/B}\right)+\frac{\Delta P/B}{P/B}(1+g)$

- Marty Leibowitz (FAJ 2000): P/E Forwards And Their Orbits
  - P/E must evolve along certain paths (orbits) determined by *k*
  - Has implications for current P/E
  - Theoretical, no tests of explanatory or predictive power

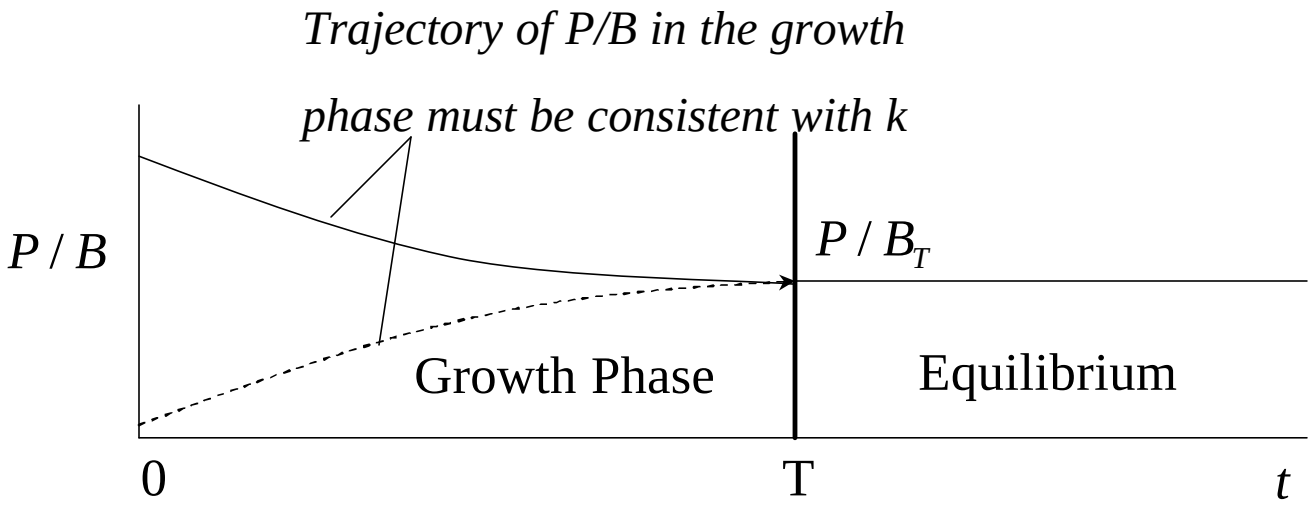
Our Two-Stage Dynamic Model

- Firm has two stages – growth phase ($t<T$) and equilibrium phase ($t>T$)
- Distinct growth rates, *ROE*s, and dividend yields in these two phases
- Capital structure is time-invariant – firm is self financing
- Exogenously determined expected return is time-invariant



Economic Intuition and More Notation

- We evolve P / B_t = Price-to-Book ratio at time t



- D_t = Cumulative dividend process at time t
- r = Instantaneous ROE = growth + dividend yield on book = $g+d$
- k = Required shareholder return, assumed constant for all $t>0$.

Exact Solution - I

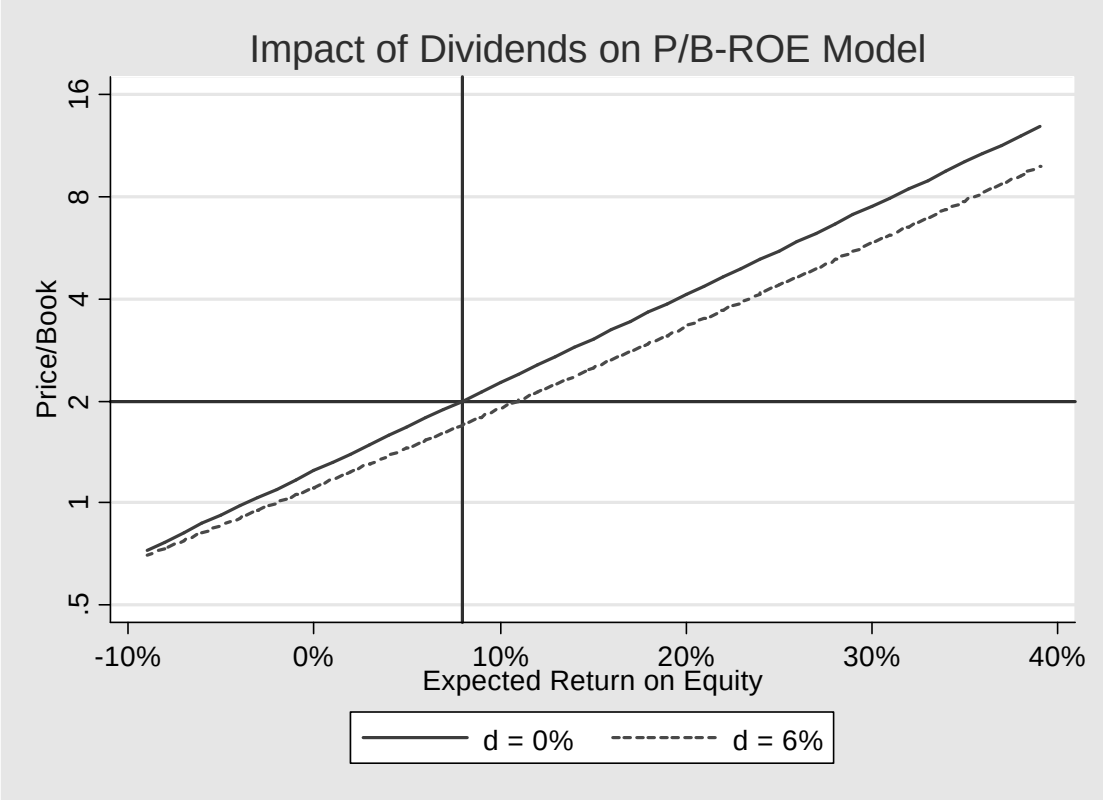
- Total Return = Price Return + Dividend yield
- If all parameters are time invariant:
- $$k = \text{Total Return} = \frac{\frac{\partial P_t}{\partial t} + \frac{\partial D_t}{\partial t}}{P_t} = \frac{1}{P_t}\frac{\partial P_t}{\partial t} + \frac{1}{P_t}\frac{\partial D_t}{\partial t} \quad (1)$$
- In addition, we always have $P_t = B_t \times P / B_t$ (2)
- Differentiate (2) w.r.t. time and divide by price to get
- $$\frac{1}{P_t}\frac{\partial P_t}{\partial t} = \frac{1}{P / B_t}\frac{\partial P / B_t}{\partial t} + \frac{1}{B_t}\frac{\partial B_t}{\partial t} \quad (3)$$

Exact Solution - II

- Substitute and rearrange to get $P / B_t \times (k - g) = \frac{\partial P / B_t}{\partial t} + d$
- Solve this differential equation to give
- $P / B_0 = P / B_T \times e^{(g-k)T} + \frac{d}{k-g} * \left(1 - e^{(g-k)T}\right)$
- $d / k\text{-}g = P/B^* = P/B$ if the initial conditions prevail in perpetuity.
- $$\ln\left(\frac{P / B_0 - P / B^*}{\frac{r_{eq} - k}{k - g_{eq}} - \frac{r - k}{k - g}}\right) = (g - k)T$$

Approximation: All Profits Are Reinvested In Growth Phase

- Then $d=0$, $r = g$, and $P / B_0 = P / B_T \times e^{(r-k)T}$



Approximate Solution: The *P/B-ROE* Model

- Take natural log on both sides to get

$$\ln(P/B_0) = \ln(P/B_T) + (r-k)T = [\ln(P/B_T) - kT] + rT$$

- In Jarrod's 1984 paper, $P/B_T = 1$, but this is unrealistic today

- The *P/B-ROE* model can be estimated from data via OLS regression
  - Can proxy *r* with *ROE*, as profitability tends to be stable and mean-reverting.
  - Can use analysts' estimates to further enhance our estimate of *r*.
  - Hard to extract information from constant, so focus on estimating *T*

- Run cross-sectional (U.S. stocks) and time-series (S&P 500) regressions

- Determine fit of regression (cross-sectional & time-series explanation)

- Use residuals to forecast returns (cross-sectional and time-series prediction)

Cross Sectional Explanation

- How much should CEO's expect stock price to increase for each 1% in additional *ROE*?
- Sample: ValueLine Datafile 1988-2002, companies with fiscal year-ends in December, positive book value, and *ROE* between -10% and +40%. Over 20,000 observations.
- Run panel regression of ln(*P/B*) against *ROE*
- Slope of regression line depends on past volatility of *ROE*.
- We interpret this slope as a measure of the investment horizon *T*.

Long-term Panel Results

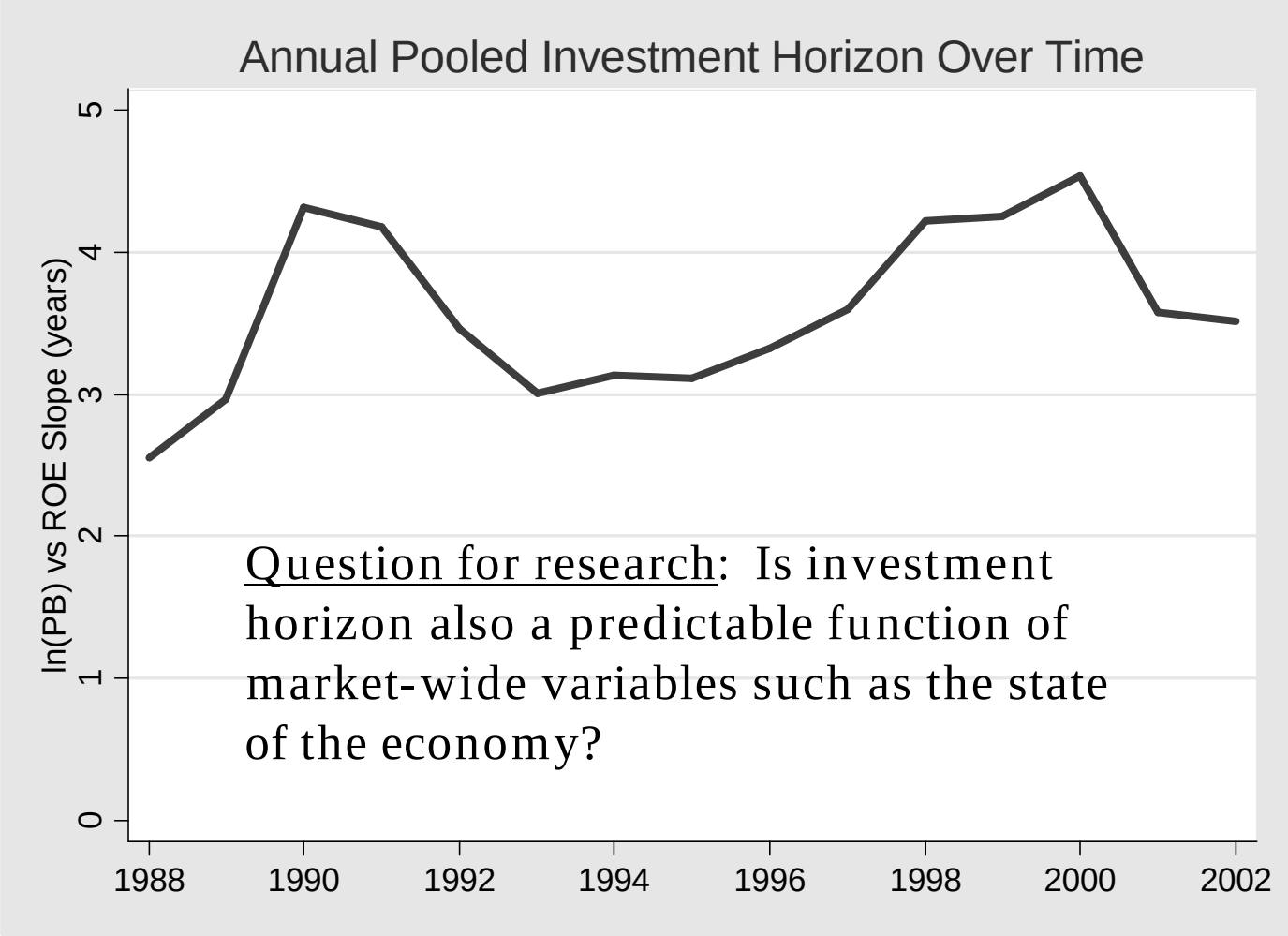
QUARTILES:	*1* *Highest 5-year ROE Volatility*	*2*	*3*	*4* *Lowest 5-year ROE Volatility*
***T* (years)**	2.37	3.87	5.00	6.47

- The pooled slope within each year of the 1988-2002 period is 3.66 years. For very stable companies it rises to about 9 years.
- A stable *ROE* allows projecting recent values further into the future.
- Independent of risk premium, *ROE* stability can either help or hurt through its impact on investment horizon T.

Example Drawn From 1988-2002 Averages

- Consider a stock with *ROE* = 15%, and in the 4th quartile of *ROE* stability (5-year standard deviation of *ROE* < 2.5%).
- Regression slope (our estimate of *T*): 6.47 years
- Question: Other things equal, how much higher would its price be if its *ROE* were 20%?
- Answer: Its stock price would have been 38% higher, not counting any increase in book value *B*.

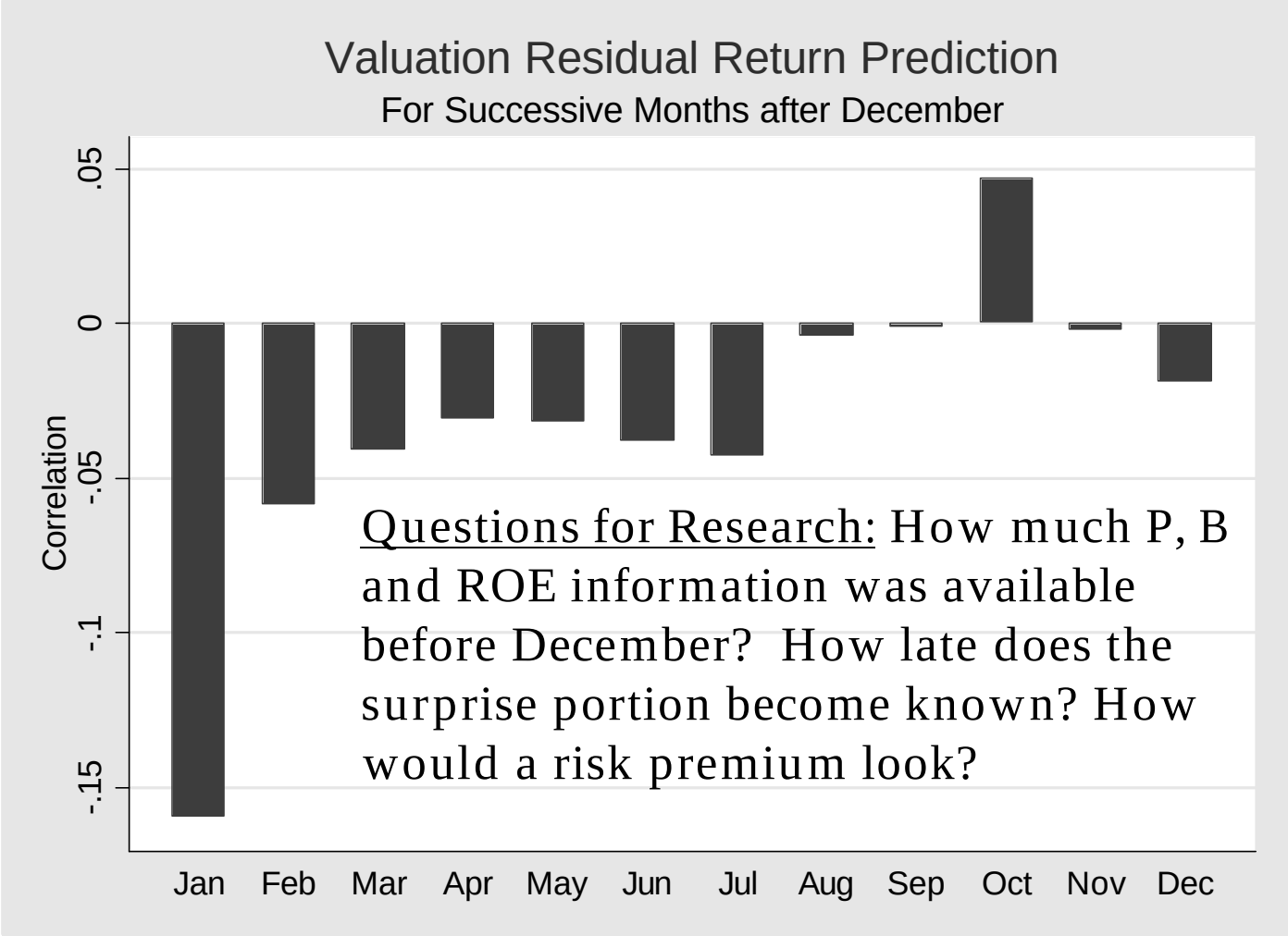
How Predictable is the Investment Horizon?



Interpretation of Explanatory Models

- Across the full sample, R^2 = 26%. It approaches 50% for more stable companies. R^2 biased upward by random *B*, and downward by pooling across company types and time.
- Statistical models involving valuation ratios should be translated into standard errors in log price to judge their merits.
- Apparent degrees of freedom are inflated because of clustering of observations by industry. However, they are still very large.
- Though useful in practice, interpreting slope as *T* may also incorporate an errors-in-variables bias from using *ROE* as a proxy for *r*.
- In an efficient market, even a very good explanatory model for prices may not forecast returns.

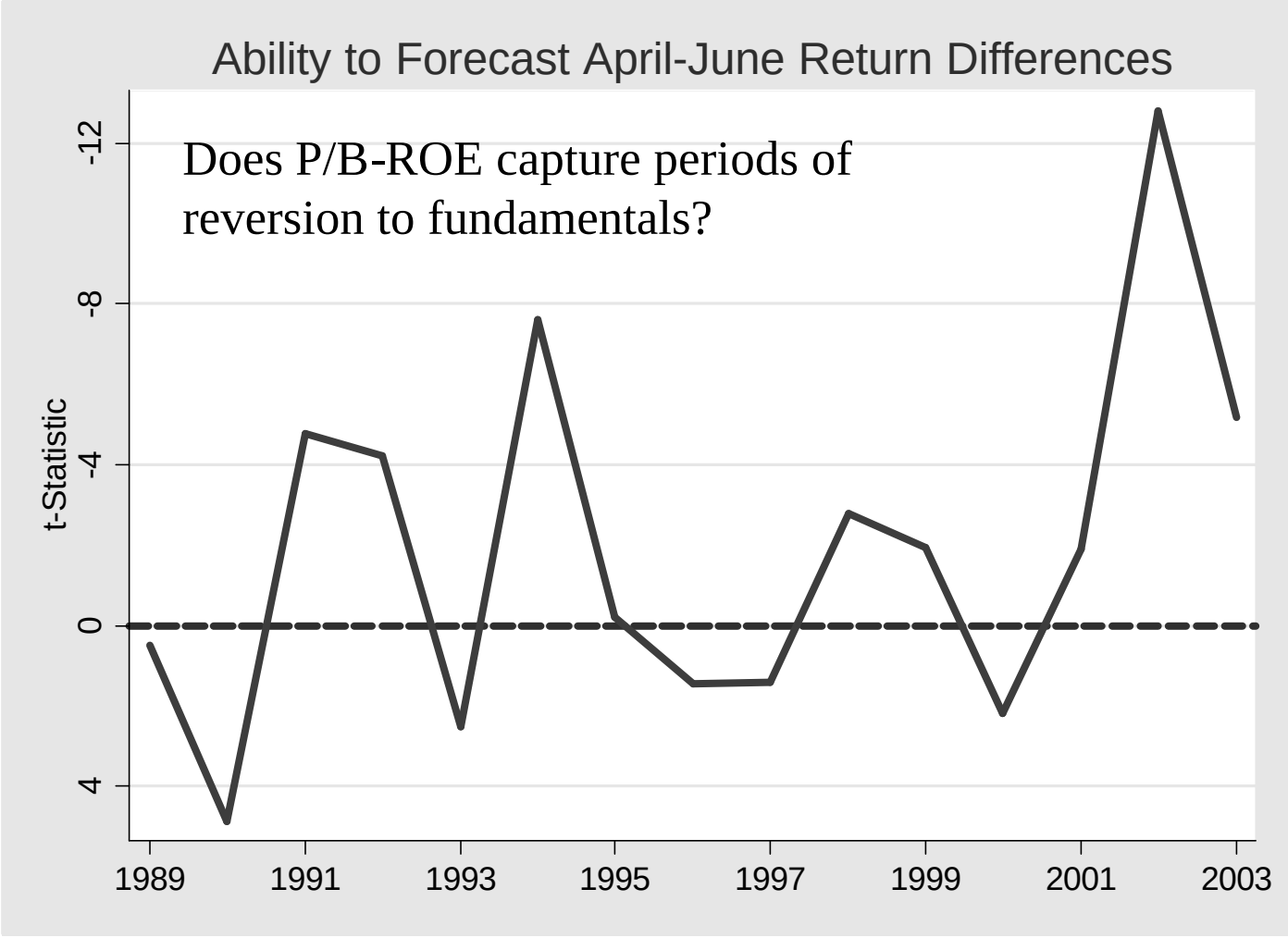
Cross Section Prediction



Regression Coefficients
1988-2002 December Residuals vs. Future Returns.

Period	*OLS Coefficient*	*t-statistic*	*Adjusted R^2*
January – March	-.055	-21.2	2.22%
April – June	-.022	-8.7	0.38%
July – September	-.011	-4.5	0.10%

Hypothetical Cross-sectional Return Forecast Success



Cross-Sectional Summary

- *P/B-ROE* gives both the company and the market a helpful tool to calibrate the impact of financial plans on shareholder value.
- Model residuals have predictive power, and are likely to be a useful addition to the investor's toolbox, even before disaggregating by time and industry.
- *P/B-ROE* allows a value approach for growth stocks, and is less biased against high quality growth than are traditional ratios like *P/E*, *P/B*, *P/S*, and *P/CF*.

Why Improve Explanatory Models for the S&P 500?

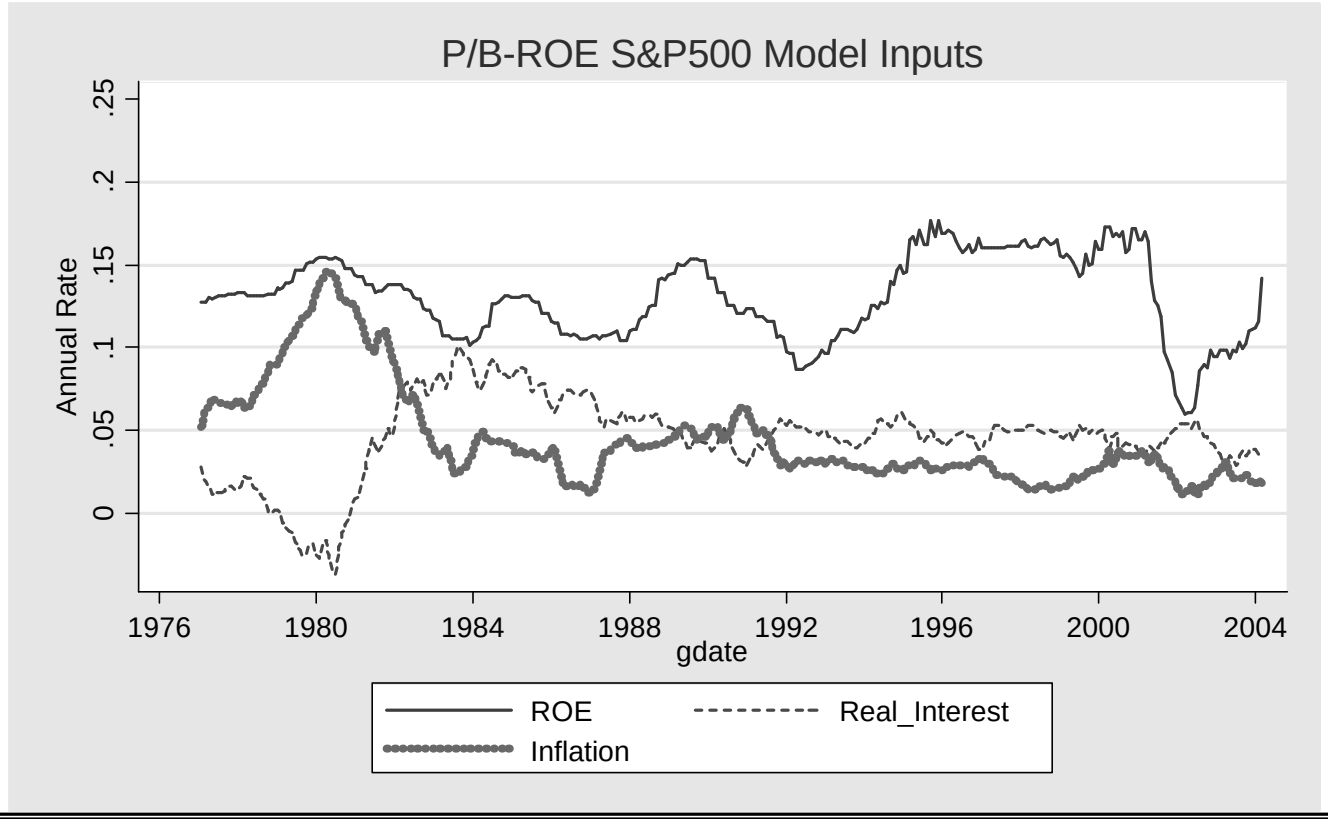
- To increase market stability by showing relevance of fundamentals and identifying bubbles.
- To better show forecasters the impact of changes in fundamentals.
- If the market departs from forecastable fundamentals, to help forecast returns using valuation residuals.

Relevant Structure

- If $\ln(P/B) = \ln(P/B_T) + T * (r - k)$, comparisons of E/P to interest rates (so-called Fed Model) are badly mis-specified.
    - See also "Fight The Fed Model" by Cliff Asness (JPM, 2003)
- Changing monetary inflation complicates this picture further
- Higher rates of inflation both:
    - Raise nominal *k*
    - Lower replacement cost profitability and thus *r* from reported ROE.
- We therefore model ln(P/B) as a linear function of ROE, inflation, and real interest rates.

Model Inputs (updated)

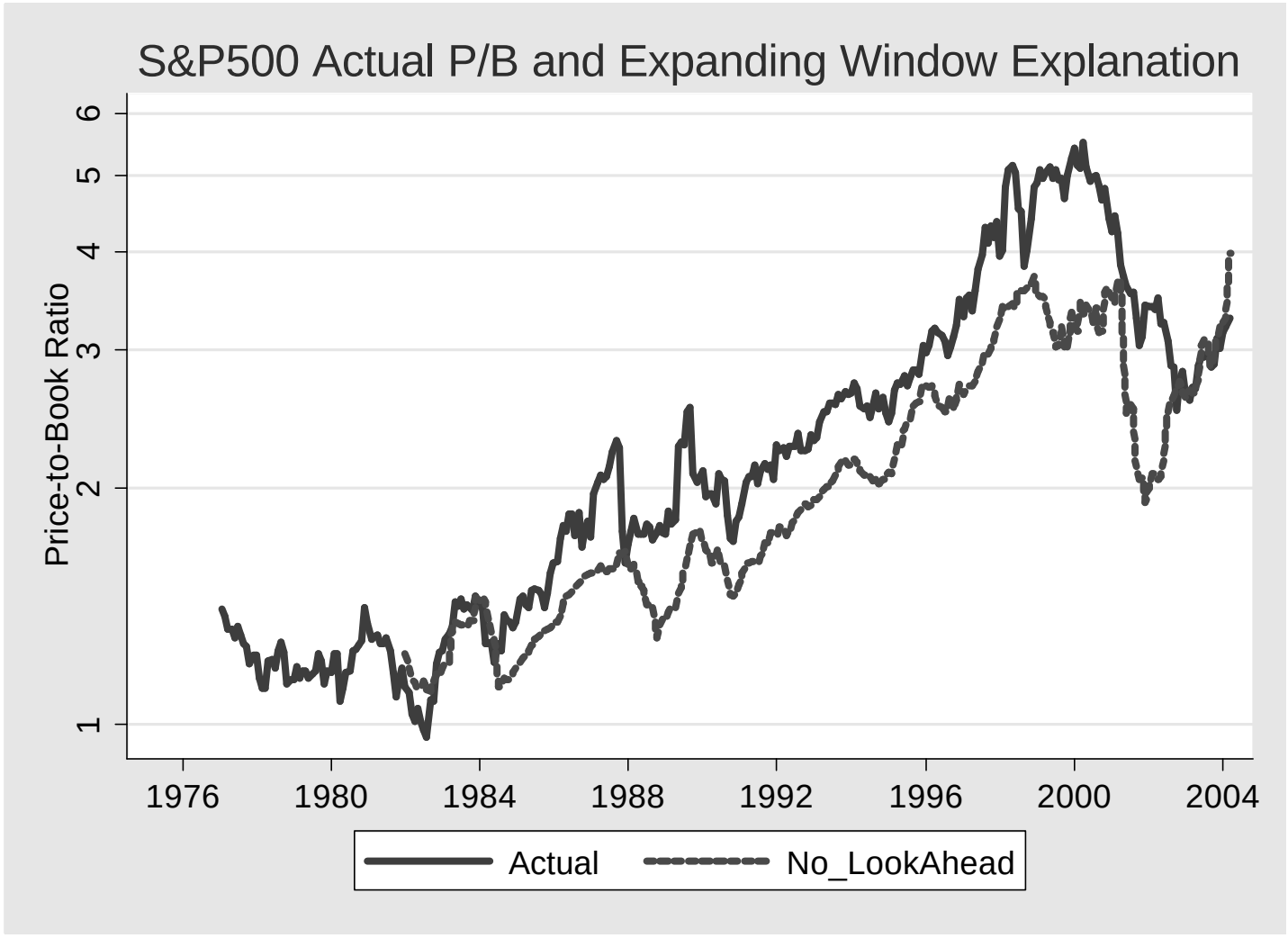
- ROE: S&P 500's S E/ S P
- Inflation: 12 Month CPI % Change
- Real Interest: Moody's AAA Yield – 12 Month CPI % Chg.



Full Sample S&P500 Index Model

- Because of omitted variables, the model errors are highly autocorrelated. R-squared of the fit is highly inflated.
- $\ln(P/B)_{S\&P500} = 1.1 + 6.3 * ROE - 15.9 * Inflation - 8.0 * Real\ Interest$
- When appraising the model's hypothetical use as an prediction tool, it is important to avoid look-ahead bias.
  - Use expanding window regression after 5-year warm-up period.

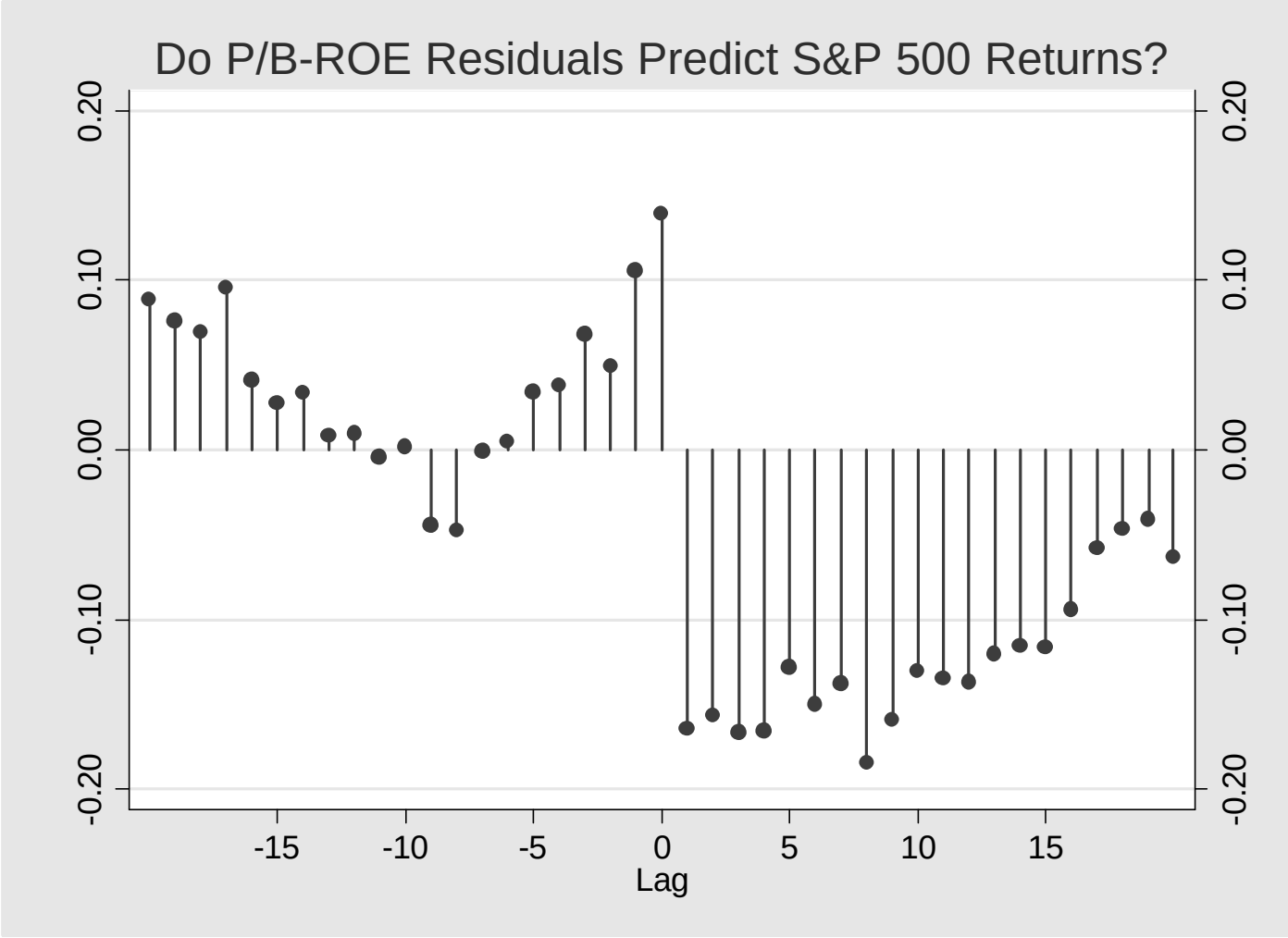
What Does P/B-ROE Tell US? (updated)



Using A Regression Model With Unstable Missing Variables

- We know that the regression model is not fully satisfied
  - The process is not stable
- Residuals are highly autocorrelated due to missing variables
  - Changes in risk preference?
  - Changing ROE cross-sectional dispersion?
  - Changing taxation?
- Consequently, we do not assume that correlation automatically translates into a successful investment decision,
- But...

Correlation: *P/B-ROE* Residuals vs. 1 month S&P 500 Returns



Predicting S&P 500 Returns with *P/B-ROE* Residuals

	Coefficient	*Std. Error*	*t*	*P>|t|*	*95% Confidence Interval*	
1 month return	-0.0418	0.017	-2.41	0.016	-0.0759	-0.0077
3 month return	-0.1199	0.050	-2.39	0.018	-0.2188	-0.0210
6 month return	-0.2243	0.093	-2.41	0.017	-0.4077	-0.0409

- All t-statistics are corrected for correlation (Newey-West)
- Predictions are both economically and statistically significant

P/B-ROE Time-Series Confirms Cross-section

- Implied investment horizon *T* against ROE for the S&P500 is similar to that found in cross-section for stocks in the most stable quartile.

- When supplemented by allowance for time-varying inflation and interest rates, *P/B-ROE*:
  - Identifies key fundamentals controlling valuation, useful for planning
  - Is structurally different from *E/P* versus interest rate comparisons
    - See "Fight The Fed Model", Cliff Asness (JPM 2003)
  - Provides useful short-term coincident explanation

- In addition, its residuals also show potential for use as an ingredient in tactical asset allocation (TAA)

Summary

- *P/B-ROE* is both simple and effective for a wide range of problems
- Some investment managers have used *P/B-ROE* for many years as an additional valuation factor...
- But it not used as widely as it could be:
  - By CEO's, CFO's and analysts
  - And for identifying undervalued growth stocks
  - And to better identify bubbles
  - And as an ingredient in tactical asset allocation
- And generally to enhance the importance of fundamentals as opposed to momentum in investing and pricing.