Short Term Risk from Long Term Models

Northfield Information Services

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Motivation for Forecasting Short Term Risk

- Our Everything, Everywhere model is spreading among both large pension funds and multi-strategy hedge funds & "fund of funds"
  – It offers more detailed analysis at the individual issue level than most popular multi-asset class risk systems

- By construction, hedge funds experience/suffer the near term
  – Leverage
  – High turnover
  – Fickle investors
  – Risk is monetary loss (VaR, CVaR), not volatility (variance drain)

- UCITS3 regulations require many European mutual funds to provide VaR estimates over short horizons

Material taken from diBartolomeo, "Short Term Risk from Long Term Models Part 1", Oct 2007, Key Largo, FL

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One Approach to Short Term Modeling

- The usual answer
  - Change structure to factors relevant in the short term
  - Increase the frequency of observations (e.g. monthly → daily)
  - Shorten the sample period (e.g. 5 yrs → 3 mo)
  - Hope GARCH type volatility adjustment catches shocks

- … ignores serious problems at the individual security level
  - Negative serial correlation due to short term reversal effects
  - Positive serial correlation in illiquid instruments
  - Less Gaussian return distributions as time intervals shrink
  - Unobservable correlation when trading hours don't overlap

Material taken from diBartolomeo, "Short Term Risk from Long Term Models Part 1", Oct 2007, Key Largo, FL

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Differences Between Long and Short Horizon Risk

- Negative serial correlation
  - Daily overreactions & reversals, which cancel out over time, become significant e.g. under leverage
- Contagion / panic
  - Liquidity demands can drive up short-term correlations
- Transient behavior
  - A long term model intentionally integrates new phenomena slowly: Is the future like the past or are we in and concerned about a present shift?
- More extreme events in the short-term
  - 3 std deviations contains less probability mass. 99% VaR is farther away from the mean

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Our Short Term Model for US Equities, in Production Daily Since 1998

- Incorporate current, forward looking market information - option implied volatility
  - Calculate the implied volatility for each security in the S&P 500
  - The forecast for each of these securities is implied vol × its 30 day avg ratio of time-series to implied vol
  - Start with a blind factor model (on de-trended returns)
  - Project forecasts above onto the model's factor variances
    - Spreads pervasive information to those stocks without options
  - What remains is company specific (e.g. Bill Gates run over by a bus) and is applied to stock specific risk
  - See Chapter 12, Linear Factor Models in Finance, Satchell and Knight, editors

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Generalizing the Idea

- Take any risk model. e.g. one of our models estimated monthly
- Add flexibility points and fit to information about current conditions
- Adjust for statistical differences between short and long term returns
- Many benefits of this approach
  - Avoids statistical complexities of high frequency data
  - Keeps with familiar factor structure
  - Common factor structure for long and short horizons permits interpolating any horizon in between
  - Works with any factor model

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Statistical Differences: An Extreme Example of Serial Correlation

- Big drops in Aug 2007

8/3	SP500 ↓ 2.7%, R2000 ↓ 3.6%
8/6-8	SP500 ↑ 4.5%, R2000 ↑ 5.3%
8/9	SP500 ↓ 3.0%, R2000 ↓ 1.4%
8/10	SP500 ↑ 0.0%, R2000 ↑ 0.5%
8/13-15	SP500 ↓ 3.2%, R2000 ↓ 4.7%
8/16-17	SP500 ↑ 2.8%, R2000 ↑ 4.5%,
8/27-28	SP500 ↓ 3.2%, R2000 ↓ 3.9%
8/29	SP500 ↑ 2.2%, R2000 ↑ 2.5%

- Over the month, SP500 up 1.3%, R2000 up 2.2%

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Observing Typical Serial Correlation

- Say 21 trading days in a month

  r_t = return on day t

  $r_{month} = \prod_{t=1..21} (1+r_t) - 1$

  $r_{21\ day\ sum} = \sum_{t=1..21} r_t$ (a month w/o compounding)

- $var(r_{21\ day\ sum}) = \sum_{t=1..21} var(r_t) + 2 \sum_{s<t} cov(r_s, r_t)$

- If stationary and no covariance between days, $var(r_{21\ day\ sum}) = 21 \times var(r_{1\ day})$

  annualized: $250/21 \times var(r_{21\ day\ sum}) = 250 \times var(r_{1\ day})$

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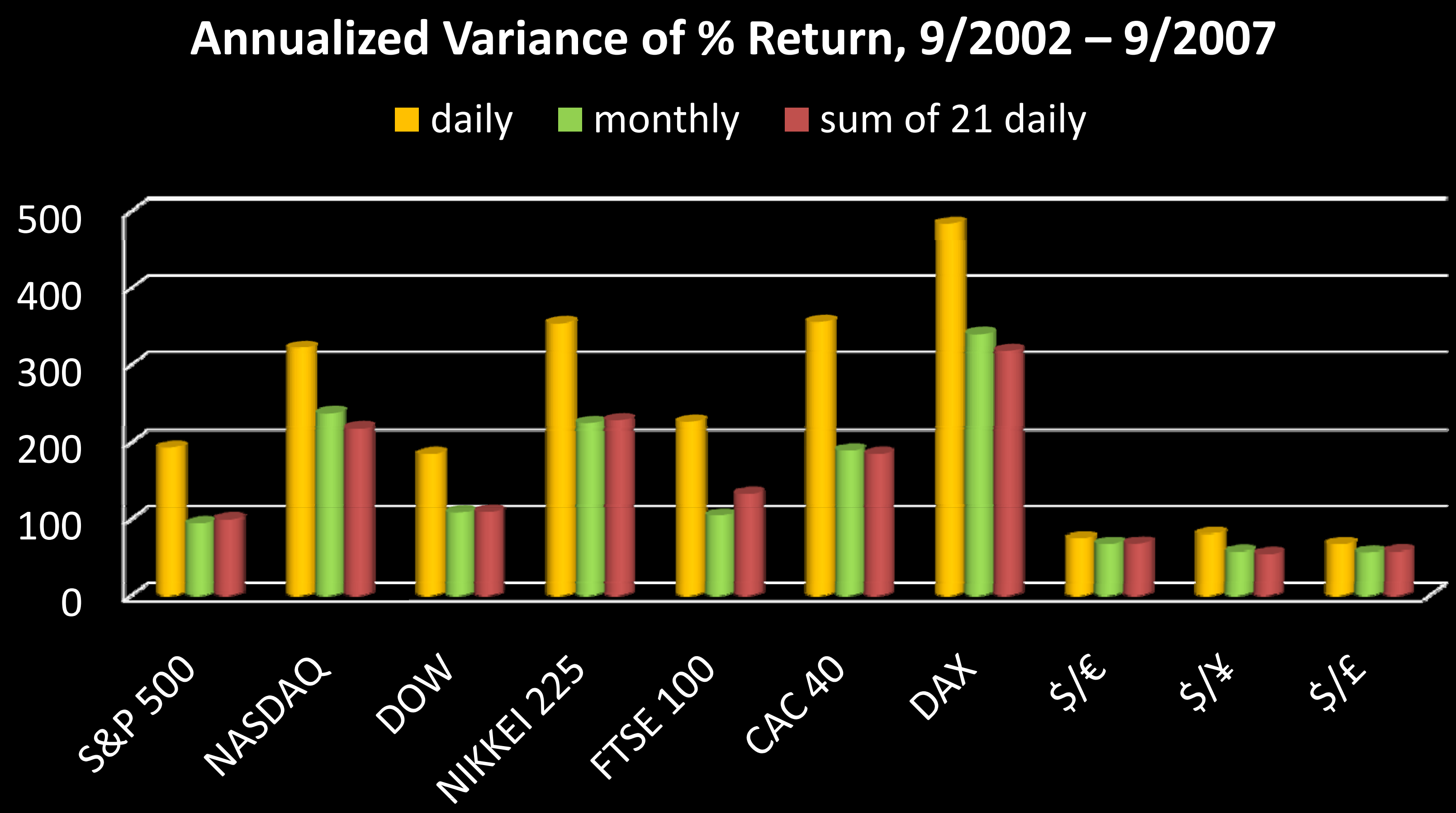
(Annualized) 1 day vol > 1 month vol: Negative Serial Correlation



Adjusting for Serial Correlation

- Goal: the variance inflation introduced by negative serial correlation

- Express cumulative return from time 0 → time T in terms of 1 day returns:

  $r_{0\to T} = f(r_1, \ldots, r_T) = \prod_{t=1..T} (1 + r_t) - 1$

- Approximate linearly around mean return, μ

  $r_{0\to T} \approx f|_{r_1 \ldots r_T = \mu} + \nabla f|_{r_1 \ldots r_T = \mu} (\mathbf{r} - \mu)$
  $= \text{constant} + (1 + \mu)^{T-1} \sum_{t=1..T} (r_t - \mu)$

- Yields cumulative return variance as 1 day return covariances

  $\text{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2} \sum_{1 \le j,k \le T} \text{cov}(r_j, r_k)$

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Adjusting for Serial Correlation (2)

- Suppose 1 day returns follow an AR(1)
  $r_{t+1} = c + \rho r_t + \varepsilon_{t+1}$ same $\mathrm{var}(r_t)$ every day, σ^2
  $\mathrm{cov}(r_j, r_k) = \sigma^2 \rho^{|j-k|}$ n day apart correlation = ρ^n

- Recall previous expression for variance of cumulative return as sum of covariances of 1 day returns
  $\mathrm{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2} \sum_{1 \le j,k \le T} \mathrm{cov}(r_j, r_k)$

- Substitute AR(1) covariances
  Result is in terms of 1 day variance and 1 day apart correlation
  $\mathrm{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2} \sum_{1 \le j,k \le T} \sigma^2 \rho^{|j-k|}$
  $= (1 + \mu)^{2T-2} \sigma^2 [T(1 + \rho)/(1 - \rho) - 2\rho(1 - \rho^T)/(1 - \rho)^2]$

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Adjusting for Serial Correlation (3)

- Now, can project a forecast from one horizon to another by solving out σ^2 from approximations for $var(r_{0 \to T_1})$, $var(r_{0 \to T_2})$
  e.g. 1 month to 1 week, $T_1=21$, $T_2=5$
  1 month to 1 day, $T_1=21$, $T_2=1$

- $var(r_{0 \to T}) \approx (1 + \mu)^{2T-2}\, \sigma^2\, [T(1 + \rho)/(1 - \rho) - 2\rho(1 - \rho^T)/(1 - \rho)^2]$

$\rightarrow \sigma^2 \approx var(r_{0 \to T}) / \{(1 + \mu)^{2T-2}\, [T(1 + \rho)/(1 - \rho) - 2\rho(1 - \rho^T)/(1 - \rho)^2]\}$

- $T_1/T_2\, var(r_{0 \to T2}) \approx var(r_{0 \to T_1})\, (1 + \mu)^{2(T2 - T1)}$
  $\times [(1 - \rho^2) - 2\rho(1 - \rho^{T2})/T_2] / [(1 - \rho^2) - 2\rho(1 - \rho^{T1})/T_1]$

What value for ρ?

9/02 – 9/07	Variance Ratio: Annualized Daily/Monthly	Measured Serial Correlation of 1 Day Returns	Serial Correlation Implied by Variance Ratio
S&P 500	2.1	-0.11	-0.36
NASDAQ	1.4	-0.04	-0.17
DOW	1.7	-0.10	-0.27
NIKKEI 225	1.6	0.00	-0.24
FTSE 100	2.2	-0.13	-0.38
CAC 40	1.9	-0.04	-0.32
DAX	1.4	-0.08	-0.19
$/€	1.1	-0.01	-0.05
$/¥	1.4	-0.02	-0.18
$/£	1.2	0.02	-0.09

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Current Market Information

- "Instantaneous" volatility and covariance measures
  - cross sectional
  - option implied
  - high/low/open/close (Parkinson, Garman-Klass, Beckers, Rogers-Satchell, Yang-Zhang)
    - std dev of return is proportional to log(high) – log(low)
  - return spreads

- ~~Liquidity – volume, short interest, bid/ask spreads~~

- Method works only with information that can be predicted by the covariance model

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Bending to Fit Information

- Make model flexible by adding free parameters θ

- Example 1: In Northfield's global model, let

  $\theta_1 \dots \theta_5$ scale std dev of the 5 region factors
  θ_6 scale std dev of the value/growth factor
  θ_7 scale std dev of all the remaining factors
  θ_8 scale stock specific std dev for U.S. & E.U. stocks
  θ_9 scale stock specific std dev for all other stocks
  $\theta_1 \dots \theta_9 \geq 0$

- Example 2: Correlation tightening/loosening

  correlation matrix = $\theta_1 \times (11^T) + \theta_2 \times \mathbf{I} + (1 - \theta_1 - \theta_2) \times$ original
  $\theta_1, \theta_2 \geq 0, \quad \theta_1 + \theta_2 \leq 1$

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Bending to Fit Information (2)

- For each observed statistic (piece of market information)
  e.g. option implied variance of S&P 500
  - $\hat{g}$ = recent value
  - $\hat{g}_{avg}$ = average value over estimation period
- Find a related statistic that can be predicted by the model
  e.g. variance of S&P 500 portfolio
  - g_θ = predicted value under model adjusted by θ
  - g = predicted value under model
- Choose θ to make the increase in the forecast, (g_θ/g), match the increase in market information, $(\hat{g}/\hat{g}_{avg})$

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Examples of Information

- $\hat{g}$ = implied variance from options on SP500 index
  g = variance of SP500
  (a measure of volatility that includes correlation)

- $\hat{g}$ = average range variance estimates of EAFE constituents
  g = average variance of EAFE constituents
  (a measure of volatility separate from correlation)

- $\hat{g}$ = cross-sectional variance of Nikkei 225
  g = xc var (ignoring difference in means) of Nikkei 225

- $\hat{g}$ = median squared spread between 2 baskets of securities
  g = variance of long/short portfolio of same securities

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Making the Intuition Rigorous

- Bayesian Framework

  $p(\theta) =$ prior distribution on parameters θ

  $p(\hat{g}) =$ unconditional probability of observing statistics $\hat{g}$

  $p(\hat{g}|\theta) =$ probability of observations $\hat{g}$ given parameters θ

  $p(\theta|\hat{g}) =$ probability that parameters are θ given observations $\hat{g}$

- Maximum a posteriori (MAP) estimate of θ given $\hat{g}$

  $\theta_{MAP} = \text{argmax}_\theta \; p(\theta|\hat{g}) = p(\hat{g}|\theta) \times p(\theta) / p(\hat{g})$

  $\theta_{MAP} = \text{argmax}_\theta \; \log p(\theta|\hat{g}) = \log p(\hat{g}|\theta) + \log p(\theta) - \log p(\hat{g})$

  $\theta_{MAP} = \text{argmax}_\theta \; \log p(\hat{g}|\theta) + \log p(\theta)$

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Making the Intuition Rigorous (2)

- Recall $\theta_{MAP} = \text{argmax}_\theta \; \log p(\hat{g}|\theta) + \log p(\theta)$

- Assume observations $\hat{g}$ are noisy observations of predictions g_θ:

$$\hat{g} = g_\theta + \varepsilon$$

$$p(\hat{g}|\theta) = p(\hat{g}|g_\theta) = p(\varepsilon = \hat{g} - g_\theta)$$

- The distribution of noise determines the fit criterion:

$\varepsilon_k \sim \text{Gaussian}[0, \sigma_k^2] \quad \rightarrow \quad \log p(\hat{g}|\theta) = \sum_k [\hat{g}_k - g_\theta^k]^2 / 2\sigma_k^2 + \text{const}$

$\varepsilon_k \sim \text{Laplace}[0, b_k] \quad \rightarrow \quad \log p(\hat{g}|\theta) = \sum_k |\hat{g}_k - g_\theta^k| / b_k + \text{const}$

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Summarizing the Framework

- Add parameters to model and tune so increases in predictions match recently observed increases
- Scale long horizon variance #'s to account for the reversing swings that occur day to day
- Be aware that short term returns follow a different distributional shape. Extreme events are more frequent

Example: Aug 31, 2007

	Median Value Relative to Trailing 5 Yrs		
	Jul '07	Aug '07	Last 3 Days of Jul '07
VIX	1.10	1.72	1.62
1 Day Intraday Log(High/Low):			
S&P 500	1.05	2.06	2.11
Avg of S&P 500 stocks	1.03	1.70	1.64
Nikkei 225 (¥)	0.56	1.31	1.28
IShare S&P Eur 350	1.03	1.73	1.59
1 Week Daily Close Log(High/Low):			
€ (in $)	0.71	0.94	
¥ (in $)	1.04	1.63	

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Tune NIS Global Model to Current Conditions

- Factors:

  Sector, Region, Economic Variables, Pricing Spreads, Blind Factors, Currencies

- Add parameters to make model flexible

  θ_1 × std dev of E.U. & Scandinavian currencies

  θ_2 × std dev of all other currencies

  θ_3 × std dev of sector factors

  θ_4 × std dev of stock specific risk

  θ_5 × std dev of all remaining factors

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The Tuned Model

- Adjusted volatility predictions

Predicted	Data	Observed Relative Level	In Tuned Model
S&P 500	Hi/Lo	2.06	1.93
Avg of S&P 500 stocks	VIX	1.72	1.72
Nikkei 225 (¥)	Hi/Lo	1.31	1.32
S&P Eur 350 ($) IShare	Hi/Lo	1.73	1.86
€ (in $)	Hi/Lo	0.94	0.93
¥ (in $)	Hi/Lo	1.63	1.63

- Changes to the model's factors

Factor	Dilation Applied to Std Dev
E.U. & Scandinavian currencies	0.93
All other currencies	1.63
Sector factors	2.14
Stock specific risk	1.78
All remaining factors	0.65

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Applying the Serial Correlation Adjustment

- Recall $T_1/T_2\ \mathrm{var}(r_{0\to T2}) \approx \mathrm{var}(r_{0\to T_1})\ (1+\mu)^{2(T2-T1)}$
  $\times\ [(1-\rho^2) - 2\rho(1-\rho^{T2})/T_2]\ /\ [(1-\rho^2) - 2\rho(1-\rho^{T1})/T_1]$
- Suppose $\rho = -.30$, $\mu = 0$
  - Note: can apply a different serial correlations to each factor and, by security, to stock specific risk
- Monthly → weekly std dev adjustment ($T_1 = 21$, $T_2 = 5$) = 1.05
- Monthly → daily std dev adjustment ($T_1 = 21$, $T_2 = 1$) = 1.34

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Summary

- 2 Major pieces to this framework
  - Training to fit current market information
  - Adjusting for distributional differences
- The principles are general and work with any risk model (and many other types of modeling problems. The approach was motivated by the style of models used in computer vision.)
- In construction at Northfield, to make a short horizon version of our EE model

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