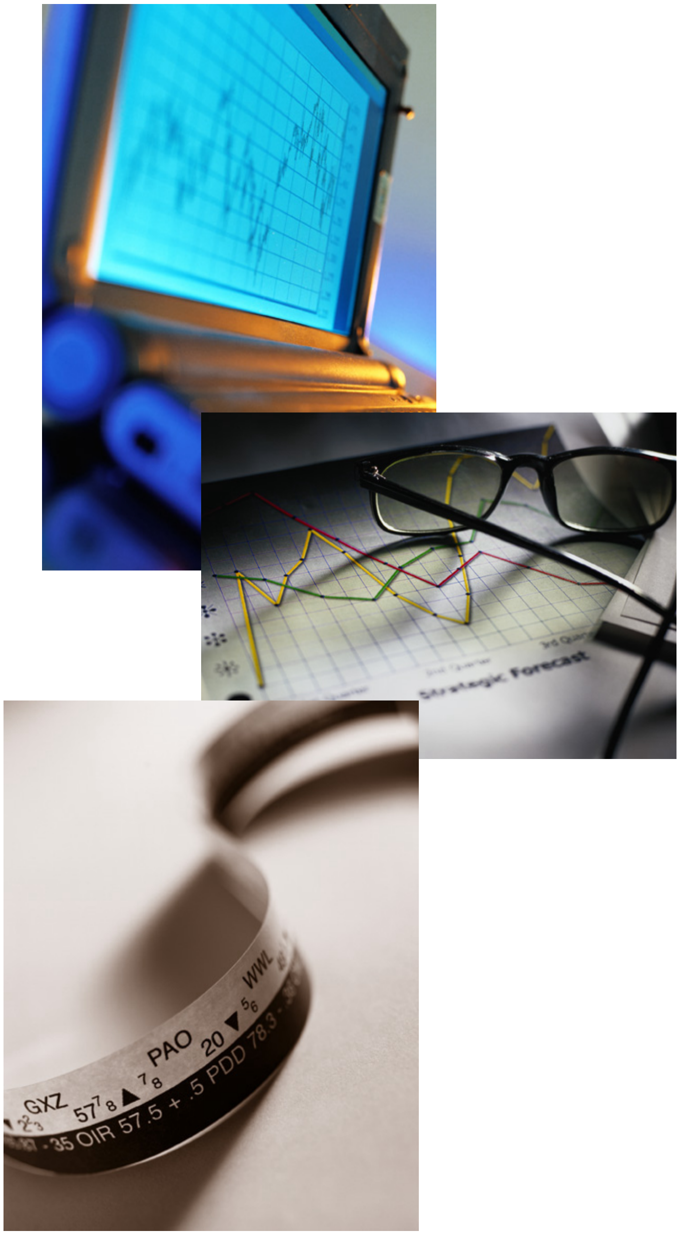

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Adaptive Near Horizon Risk Models

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Introduction

Northfield's Adaptive Near Horizon Models are a suite of models for forecasting risk over short horizons, under market conditions as they exist today. Each model retains the intuitive, proven factor structure of its long horizon counterpart. Within that structure, the model is adapted to fit instantaneously observed market information that may deviate quite dramatically from the long-term norm.

Responsiveness Depends on Horizon

Forecasting long-term behavior requires intentionally restraining news. A priori, one cannot know whether events are transient (more likely) or shifts in regime (less likely), so a sensible long-term model integrates innovations cautiously.

While a phenomenon can arise suddenly and unexpectedly, it decays gradually over weeks or months. Whether it is transient or regime shifting, the best near horizon forecast strongly resembles the latest market.

Northfield's Adaptive Near Horizon Models are based on the belief that ***current*** *market conditions contain the most relevant information about near term behavior.*

New Approach to Modeling Short-Term Behavior

Conventionally, short-term models are built following the steps for long-term models, but on high frequency, i.e. daily, data. Putting aside statistical complexities (for example, how to determine correlation between non-concurrent global markets), proceeding this way ignores everything known but not visible in return history. Why limit oneself to time-series methods when the best information isn't in time series?

The richest source of information about the near-horizon future is not the past, but is,in fact, contemporaneous sources.

Northfield's innovative approach began with our US Short-Term Equity Model, in daily production since 1998. Starting with a statistical factor analysis of daily return data, the model superimposes instantaneous information - option implied volatility levels - to capture what is both prescient and unseen in time series[1]. Leaving aside various microstructure effects, the option implied volatility represents the consensus view of option market participants about the actual volatility of the underlying security in the near future up to expiration of the option[2].

Adaptive Near Horizon Models generalize the idea of harnessing contemporaneous or forward looking information, in addition to past returns, to all risk models in all markets.

Sources of Information

The US Short-Term Equity Model incorporates volatility implied by liquid options on large cap stocks. What candidates are there for (nearly) instantaneously observable signals that exist in other markets?

---

[1] diBartolomeo, D., and S. Warrick, 2005, "Making Covariance-Based Portfolio Risk Models Sensitive to the Rate at which Markets Reflect New Information." In S. Satchell & J. Knight (Eds.), *Linear Factor Models in Finance* (pp. 249-261). Oxford, UK:Elsevier.

[2] Ederington, L. and W. Guan, 2002, "Is Implied Volatility an Informationally Efficient and Effective Predictor of Future Volatility?," *Journal of Risk*, Spring v4(3), 29-46.

1. **High/low/open/close volatility estimators** infer return volatility from intraday price extremes. Requiring only one or a few days of price history, estimates respond rapidly to changing market conditions. (See Parkinson, Garman-Klass, Rogers-Satchell, Yang-Zhang.[3])

   Differences between the volatility of an index and the average of its constituents' volatilities reveal underlying correlations.

2. **Cross-sectional volatility**, the cross-sectional standard deviation of security returns, jointly measures volatility and correlation and requires only 1 period of data[4].

3. **Implied volatility** captures the forward looking beliefs of option traders. Indices analogous to the VIX are published for many markets:

VIX	USA S&P 500
VXN	USA NASDAQ 100
RVX	USA Russell 2000
MVX	Canada TSX 60
VSTOXX	DJ EURO STOXX 50
VBEL	Belgium BEL 20
VCAC	France CAC 40
VDAX	Germany DAX
VAEX	Netherlands AEX
VSMI	Switzerland SMI
India VIX	India CNX 50
OVX	CBOE Oil
GVZ	CBOE Gold
EVZ	CBOE Euro Currency

[3] Parkinson, M., 1980, "The Extreme Value Method for Estimating the Variance of the Rate of Return," *Journal of Business*, v53(1), 61-65.
Garman, M. B., and M. J. Klass, 1980, "On the Estimation of Security Price Volatilities From Historical Data," *Journal of Business*, v53(1), 67-78.
Rogers, L. C. G., and S. E. Satchell, 1991, "Estimating Variance from High, Low, and Closing Prices," *Annals of Applied Probability*, v1(4), 504-512.
Rogers, L. C. G., and S. E. Satchell, and Y. Yoon, 1994, "Estimating the Volatility of Stock Prices: A Comparison of Methods that use High and Low Prices," *Applied Financial Economics*, v4, 241-247.
Yang, D., and Q. Zhang, 2000, "Drift-Independent Volatility Estimation Based on High, Low, Open and Close Prices," *Journal of Business*, v73(3), 477-491.

[4] See, for a good summary: Satchell, S. E. and S. Hwang, 2001, "Properties of Cross-sectional Volatility", Financial Econometric Research Centre Working Paper WP00-4, City University Business School.

The Process

Each adaptive model starts from the respective long horizon (one year) model. The model is first "trained" to fit current market information, then adjusted for statistical differences between monthly and daily or bi-weekly (10 trading days) returns.

For example, the April 15th Northfield Adaptive Horizon bi-weekly global model is the March 31st Northfield monthly global model

- adapted to April 15th market conditions and
- scaled for the statistical differences between 10 day and monthly returns.

Fitting Market Conditions

The basic idea[5]:

1. To be able to fit something, a model needs flexibility. Rather than copying the risk parameters from the long-term model unchanged, the adaptive model leaves certain parameters as free variables. For example, in the global model, the variance of the 'Continental Europe' factor might be flexible instead of taking the value from the long-term version.

2. The free parameters are fit to make the model's predictions consistent with market conditions. If, for example, the risk of the FTSE 100 were to double, the free parameters would be adjusted so that the model's predicted risk of the FTSE 100 doubles.

3. As one would expect, given many free parameters and various sources of conditioning data, the process requires mathematical machinery. The framework is based on Bayesian statistics; the parameter fitting relies on nonlinear optimization.

Short-Term vs. Long-Term Returns

Two differences between daily and monthly returns merit attention:

1. Serial correlation: daily returns exhibit reversals that wash out over longer periods. Ignoring those and simply scaling a month's forecast to one day without incorporating an adjustment for serial correlation yields a forecast that underpredicts risk (by about 50% in many markets).

2. Short term returns contain more extreme events. Naively calculating VaR assuming a normal distribution guarantees trouble[6].

The first is handled during model construction – variances in the adaptive near horizon models are appropriately inflated to the horizon. The second is a caution to users of any risk model.

---

[5] Technical details are available in a downloadable presentation on our website: "Short Term Risk from Long Term Models", http://www.northinfo.com/documents/325.pdf

[6] Examples of more robust (but unlikely to be achieved) bounds are the Cantelli inequality and Chebyshev style bounds. The Chebyshev bound is $P\{|x - \mu| > n\sigma\} < 1/n^2$. Following similar reasoning, one can derive tighter bounds using L^p norms: $P\{|x - \mu| > n\, E[|x - \mu|^p]^{1/p}\} < 1/n^p$, or kurtosis: $P\{|x - \mu| > n\sigma\} < \kappa/n^4$. Even these extreme bounds aren't bullet-proof because future variance and other moments are only forecasts in the nonstationary process of stock returns.

Coverage

Every Northfield long-term model is available in Adapted Near Horizon form. For a detailed description and methodology of Northfield's long horizon models please contact our offices.

Global Models
Global Equity (Global Equities)
Everything Everywhere (All Asset Classes)

Regional Equity Models
Asia Pacific excluding Japan
Pacific Rim
Europe
U.S. & Canada

Single-Country Equity Models
Australia
China
Japan
Swiss
U.K.
Canada

U.S. Market Specific Models
US Fundamental
US Macroeconomic
US Single Country
US REIT

Summary

The Northfield Adaptive Near Horizon Models are designed to predict near term risk in every market covered. Developed through in-house research, their innovative approach to incorporating market information gives managers and traders the most current measure of portfolio risk. Our staff is committed to making certain that clients are fully satisfied with these tools. Please call our offices or your dedicated sales representative for further information.

Appendices

For each index, we use both the volatility of the index and the average volatility of its constituents.

Training Data

Global models

<u>Global Equity</u>
US broad market index
DJIA
NASDAQ 100
S&P 500

(VXD) CBOE DJIA implied volatility
(VXN) CBOE NASDAQ implied volatility
(VIX) CBOE S&P 500 implied volatility

Argentina MERVAL
Brazil BOVESPA

Eastern Europe broad market index
Western Europe broad market index
Austria ATX
Germany DAX
France CAC 40
Italy BCI 30
Norway OSE
Spain IBEX 35
UK FTSE 100

India SENSEX
Indonesia JSX
Japan Nikkei 225
Korea KOSPI composite
Malaysia KLSE composite
Taiwan TAIEX

New Zealand NZX 50

<u>Everything Everywhere</u>
All indices used in Global Equity
CBOE US treasury interest rate options

Regional Equity Models

<u>European Union</u>
Western Europe broad market index
Austria ATX
Germany DAX
Italy BCI 30
Norway OSE
France CAC 40

Spain IBEX 35
UK FTSE 100

Pacific Rim
Indonesia JSX
Korea KOSPI composite
Malaysia KLSE composite
New Zealand NZX 50
Taiwan TAIEX

Single-Country Equity Models

Canada
Canada broad market index

Swiss
Switzerland SMI

United Kingdom
UK broad market index
UK FTSE 100

China
China broad market index

Japan
Japan broad market index
Japan Nikkei 225

Australia
Australia broad market index

U.S. Market Specific Models

US Fundamental
US broad market index
10 sector indices
DJIA
NASDAQ 100
Russell 2000
S&P 500

(VXD) CBOE DJIA implied volatility
(VXN) CBOE NASDAQ implied volatility
(RVX) CBOE R2000 implied volatility
(VIX) CBOE S&P 500 implied volatility

US Macroeconomic
US broad market index

DJIA
NASDAQ 100
S&P 500

(VXD) CBOE DJIA implied volatility
(VXN) CBOE NASDAQ implied volatility
(VIX) CBOE S&P 500 implied volatility

<u>US Single Country</u>
US broad market index
DJIA
NASDAQ 100
S&P 500

(VXD) CBOE DJIA implied volatility
(VXN) CBOE NASDAQ implied volatility
(VIX) CBOE S&P 500 implied volatility

Shortening the Timescale in the Presence of Serial Correlation

Negative serial correlation in daily returns makes returns over days or weeks considerably more volatile than one would expect given monthly numbers.

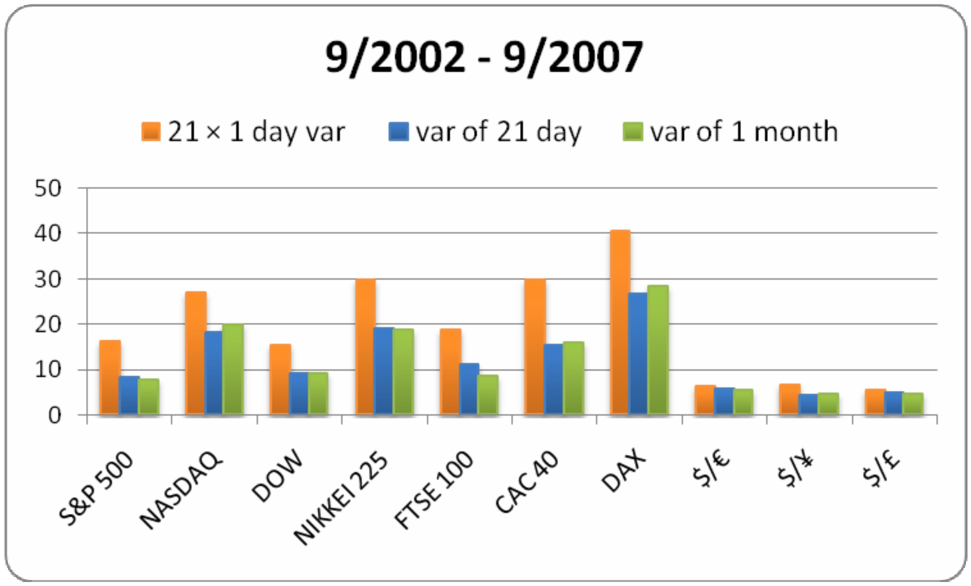
A typical month has 21 trading days.

Let $r_t =$ return on day t
$r_{21\ day,\ t} = \sum_{t=1..21} r_t$ (a month's return excluding compounding)

$$var(r_{21\ day,\ t}) = \sum_{t=1..21} var(r_t) + 2 \sum_{s < t \leq 21} cov(r_s,r_t)$$

If $\{r_t\}$ stationary and no covariance between days,
$$var(r_{21\ day}) = 21 \times var(r_{1\ day})$$

In real investment returns, however, 21 × the 1 day return variance far exceeds the variance of summed 21 day returns and the variance of monthly returns:



Northfield takes the following steps to properly project variances to short time scales:

1) Express cumulative return from time 0 → time T in terms of 1 day returns.
   $r_{0\to T} = f(r_1, \ldots, r_T) = \prod_{t=1..T}(1 + r_t) - 1$

2) Approximate linearly around the mean return, μ
   $r_{0\to T} \approx f|_{r_1\ldots r_T=\mu} + \nabla f|_{r_1\ldots r_T=\mu}(r - \mu)$
   $= \text{constant} + (1 + \mu)^{T-1}\sum_{t=1..T}(r_t - \mu)$

3) Yields cumulative return variance as 1 day return covariances
   $\mathrm{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2}\sum_{1\le j,k\le T}\mathrm{cov}(r_j, r_k)$

4) Model 1 day returns as an AR(1)
   $r_{t+1} = c + \rho r_t + \varepsilon_{t+1}$ $\qquad$ $\mathrm{var}(r_t) = \sigma^2$ for every t
   $\mathrm{cov}(r_j, r_k) = \sigma^2\rho^{|j-k|}$ $\qquad$ n day apart correlation = ρ^n

5) Substitute AR(1) covariances into formula from step #3
   $\mathrm{var}(r_{0\to T}) \approx (1 + \mu)^{2T-2}\sum_{1\le j,k\le T}\sigma^2\rho^{|j-k|}$
   $= (1 + \mu)^{2T-2}\sigma^2[T(1 + \rho)/(1 - \rho) - 2\rho(1 - \rho^T)/(1 - \rho)^2]$

6) A forecast can then be projected to any time scale by solving out σ^2 from approximations for $\mathrm{var}(r_{0\to T_1})$, $\mathrm{var}(r_{0\to T_2})$
   e.g. 1 month to 2 weeks, T_1=21, T_2=10
   1 month to 1 day, T_1=21, T_2=1

   $T_1/T_2\,\mathrm{var}(r_{0\to T_2}) \approx \mathrm{var}(r_{0\to T_1})(1 + \mu)^{2(T_2 - T_1)}$
   $\times[(1 - \rho^2) - 2\rho(1 - \rho^{T_2})/T_2] / [(1 - \rho^2) - 2\rho(1 - \rho^{T_1})/T_1]$

Bayesian Framework for Incorporating Market Information

Underlying the procedures for fitting a model to current market conditions lies a rigorous Bayesian framework.

1) Free parameters, θ, are added to make the model flexible.

   Let $\theta_1 \ldots \theta_5$ scale the std dev of the 5 region factors
   θ_6 scale the std dev of the value/growth factor
   θ_7 scale the std dev of the remaining factors
   θ_8 scale stock specific std dev

2) To each piece of market information, e.g. the option implied variance of the S&P 500, is a quantity that can be forecast by the risk model. In this case, the obvious choice is the variance of the S&P 500.

   Let $\hat{g}$ = recent level of the implied var of the S&P 500
   $\hat{g}_{avg}$ = historic level of the implied var of the S&P 500
   $\hat{s} = \hat{g} / \hat{g}_{avg}$

   g = forecast var of the S&P 500 under the original model
   g_θ = forecast var of the S&P 500 under the θ adjusted model
   $s_\theta = g_\theta / g$

3) The parameters θ are fit to make the relative increase in the forecast, s_θ, match the observed relative increase in the market, $\hat{s}$.

4) The fit criteria comes from a Bayesian model of the market information.

   $p(\theta)$ = prior distribution on parameters θ
   $p(\hat{s})$ = unconditional probability of observing statistics $\hat{s}$

   $p(\hat{s}|\theta)$ = probability of observations $\hat{s}$ given parameters θ
   $p(\theta|\hat{s})$ = probability that parameters are θ given observations $\hat{s}$

   We seek the most likely configuration of free parameters given the data, the maximum a posteriori (MAP) estimate of θ given $\hat{s}$:

$$\begin{aligned}\theta_{MAP} &= \operatorname{argmax}_\theta\ p(\theta|\hat{s}) = p(\hat{s}|\theta) \times p(\theta) / p(\hat{s}) \\ &= \operatorname{argmax}_\theta\ \log p(\theta|\hat{s}) = \log p(\hat{s}|\theta) + \log p(\theta) - \log p(\hat{s}) \\ &= \operatorname{argmax}_\theta\ \log p(\hat{s}|\theta) + \log p(\theta)\end{aligned}$$

   Assume observations $\hat{s}$ are noisy observations of predictions s_θ:

$$\hat{s} = s_\theta + \varepsilon$$
$$p(\hat{s}|\theta) = p(\hat{s}|s_\theta) = p(\varepsilon = \hat{s} - s_\theta)$$

   The distribution of noise determines the loss function:

   $\varepsilon_k \sim \text{Gaussian}[0, \sigma_k^2] \quad \rightarrow \quad \log p(\hat{s}|\theta) = \sum_k [\hat{s}_k - s_\theta^k]^2 / 2\sigma_k^2 + \text{const}$
   $\varepsilon_k \sim \text{Laplace}[0, b_k] \quad \rightarrow \quad \log p(\hat{s}|\theta) = \sum_k |\hat{s}_k - s_\theta^k| / b_k + \text{const}$

5) Nonlinear optimization finds the optimal θ's.

Relationship between Variance & Cross-Sectional Variance

Cross-sectional variance is affected by both variance and correlation. It increases as variance levels rise yet decreases as securities become more correlated. Combined with a measure of overall variance, e.g. an implied volatility index, cross-sectional variance captures changes in correlation.

Let
w_s = weight of security s in reference portfolio, $\sum_s w_s = 1$
r_s = return of security s
r = return of reference portfolio, $r = \sum_s w_s r_s$
μ_s = expected return of security s
μ = expected return of reference portfolio, $r = \sum_s w_s \mu_s$
σ_s^2 = variance of security s
σ^2 = variance of reference portfolio
ρ = average correlation between securities

$$
\begin{aligned}
\sigma_{xs}^2 &= \text{cross-sectional variance of reference portfolio} \\
&= \textstyle\sum_s w_s [r_s - r]^2 \\
&= \textstyle\sum_s w_s [(r_s - \mu_s) + (\mu_s - \mu) + (\mu - r)]^2 \\
&= \textstyle\sum_s w_s (r_s - \mu_s)^2 + \sum_s w_s (\mu_s - \mu)^2 + (\mu - r)^2 \\
&\quad + 2 \textstyle\sum_s w_s [(r_s - \mu_s)(\mu_s - \mu) + (r_s - \mu_s)(\mu - r) + (\mu_s - \mu)(\mu - r)] \\
&= \textstyle\sum_s w_s (r_s - \mu_s)^2 + \sum_s w_s (\mu_s - \mu)^2 - (r - \mu)^2 + 2 \sum_s w_s (r_s - \mu_s)(\mu_s - \mu)
\end{aligned}
$$

Taking expectations,

$$
\begin{aligned}
E[\sigma_{xs}^2] &= \textstyle\sum_s w_s \sigma_s^2 + \sum_s w_s (\mu_s - \mu)^2 - \sigma^2 \qquad \text{the 2}^{\text{nd}} \text{ term is negligible} \\
&\approx \textstyle\sum_s w_s \sigma_s^2 - \sigma^2 \\
&= \text{the average security variance} - \text{the variance of the index}
\end{aligned}
$$

Recall

$$
\begin{aligned}
\sigma^2 &= \textstyle\sum_s (w_s\sigma_s)^2 + \sum_s\sum_{t\neq s} (w_s w_t \rho\sigma_s\sigma_t) \\
&= (1 - \rho) \textstyle\sum_s (w_s\sigma_s)^2 + \rho \sum_s\sum_t (w_s w_t \sigma_s\sigma_t) \\
&= \textstyle\sum_s (w_s\sigma_s)^2 + \rho [(\sum_s w_s \sigma_s)^2 - \sum_s (w_s\sigma_s)^2]
\end{aligned}
$$

Why Northfield is right for you

Powerful, Integrated, Consistent & Comparable Risk Models

Northfield's family of risk models has been helping clients construct and analyze portfolios in many countries across the world for over 15 years. The risk models are based on sound theoretical and academic foundations. They are clear, intuitive, informative and comparable. Diverse portfolios can be analysed, using appropriate metrics, relative to standard and or customised benchmarks. Sources of systematic and security specific risk are identified quickly, clearly and easily.

Open Models: Open Systems No Black Boxes!

Northfield maintains a philosophy of openness and partnership with our clients. Northfield offers and supports "glass boxes" – there is nothing hidden. Should you want to know the full detail of how a model is put together, we will tell you, clearly. Northfield is not in the "black box" business.

Global, Regional, Country & Asset Coverage

The coverage of assets in the Northfield family of risk models is huge. From the Everything Everywhere ("EE") global fixed income and equity risk model, to the Global, Single Country / Regional, and specialist equity risk models, coverage includes over 57,000 equities and about 400,000 fixed income instruments. Additional EE data coverage includes 1,100,000 U.S. muni bonds, 1,000,000 mortgage backed securities and agency pass-throughs, and 100,000 U.S. collateralized mortgage obligations and asset backed securities. Should your portfolios contain assets not included in the system (private equity holdings, very new IPO's etc. etc.) we give you the tools and understanding to add them yourself.

Sophisticated, Flexible, Robust, Open Analytical Systems

"Just like it says on the box" - Northfield systems are flexible, robust and open. Inputs can be managed and changed to reflect your views. Output can be saved as text files and used in any manner of your choosing. Available on the PC, Unix, Linux and multiple partner platforms, Northfield's analytical tools are widely respected for their reliability and functionality.

Partners

Northfield has partnered with selective business information services companies to enhance clients' ability to access Northfield analytics via multiple platforms. Northfield partners include FactSet, ClariFi, Quantitative Services Group, SoftPak, Thomson Reuters and others.

Innovation

Northfield constantly strives to add more useful features and functions for your use. Examples of recent innovation include: The ability to manage long-short hedge funds appropriately as a single entity, accurately and conveniently managing composite assets as part of a portfolio, the ability to manage non-linear transaction costs during the optimization process.

Excellent Training, Support and Solutions

Northfield staff attentively assist customers with excellent training and support, based on many years experience.

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