Beyond Linear Factor Models

A Theoretical and Practical Insight

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Linear Models are Good

- Linear phenomena are usually larger than non-linear ones
- Factors have an economic meaning
- No need to be a quant to understand
- Industry standard
- Easy to find good linear factor models
- Easy to compare

Linear Models are Bad

Non-linear phenomena do exist:

- **Lehman**
- **LTCM**
- **Hedge fund returns**
- **Hedge fund lock-ups after Lehman**
- **…**

What is a Linear Factor Model?

$$Y = \alpha + \sum_{i=1}^{N} \beta_i X_i + U$$

- Y is the return of the security

- X_i is the return of the i-th factor

- β_i is the exposure to the i-th factor

- U is the residual which is uncorrelated to each factor

A Wrong Formula

One may be tempted to write Taylor’s formula

$$Y = \alpha + \beta X + \gamma X^2 + \delta X^3 + \dots$$

But the residuals would be correlated!

An Ugly Formula

The correct formula is

$$Y = \sum_{n=0}^{+\infty} \frac{1}{n!} \mathsf{E}[\partial^n Y] H_n(X)$$

Where the Hermite polynomials are defined by

$$H_0(X) = 1$$
$$H_1(X) = X$$
$$H_2(X) = X^2 - 1$$

$$H_n(X) = X H_{n-1}(X) - \partial H_{n-1}(X)$$

But…

Assumptions are necessary:

- **Factors need to be normally distributed**
- **Factors need to be jointly normally distributed**
- **Factors need to be normally distributed and independent**

Ouch

Normalizing

If the distribution of the i-th factor is “known”, then replace it with

$$X_i' = N^{-1}(F_i(X_i))$$

“known” can mean:

- **the empiric distribution**
- **a particular distribution that (more or less) fits the data**
- **a distribution generated by some central moments**
- **…**

Diagonalizing

A copula approach that works nicely is possible

Provided the copula is “known”

“known” can mean:

- the empiric copula
- a particular copula that more or less fits the data
- a particular copula generated by some joint central moments
- …

Hidden Non-Linear Behaviour

Rearranging terms…

$$Y = \sum_{n=0}^{+\infty} \frac{1}{n!} \mathsf{E}[\partial^n Y] H_n(X)$$

$$= \mathsf{E}[Y] + \mathsf{E}[\partial Y] X + \frac{\mathsf{E}[\partial^2 Y]}{2}(X^2 - 1) + \frac{\mathsf{E}[\partial^3 Y]}{3!}(X^3 - 3X) + \dots$$

$$= \left(\mathsf{E}[Y] - \frac{\mathsf{E}[\partial^2 Y]}{2}\right) + \left(\mathsf{E}[\partial Y] - \frac{\mathsf{E}[\partial^3 Y]}{2}\right) X + \frac{\mathsf{E}[\partial^2 Y]}{2} X^2 + \dots$$

$$= \left(\mathsf{E}[Y] - \frac{\mathsf{E}[\partial^2 Y]}{2}\right) + \left(\mathsf{E}[\partial Y] - \frac{\mathsf{E}[\partial^3 Y]}{2} + \frac{\mathsf{E}[\partial^2 Y]}{2} X\right) X + \dots$$

…shows how we can have

$$\beta \neq \mathsf{E}[\partial Y] = \mathsf{E}[\Delta]$$

A Hedge-Fund Factor Model

Fung, Hsieh, Agarwal and others have developed over the past decade a LFM for hedge funds returns

It includes

An equity market factor

An equity size spread factor

A bond factor

A credit spread factor

Three factors composed of look-back-straddles (on commodities, FX, bonds)

Implied vol

Implied skew

Implied kurtosis

R. J. Barlow (1988): “Kurtosis is not used much by physicists, chemists, or indeed by anyone else. It is a really obscure and arcane quantity whose main use is inspiring awe in demonstrators, professors, or anyone else you need to impress.”

A Linear v Non-Linear Experiment

Regression of eight hedge funds

Fund 1, Fund 2, Fund 3, Fund 4, Fund 5, Fund 6, Fund 7, Fund 8

with four factors

equity markets, equity market spread, bond markets, credit spread

The funds

- are mainly exposed to equities and bonds
- have long time series
- have time series that are stable over time

A Few Caveats

- Four linear factors yield ten quadratic ones. (Four are of the form $X_i^2 - 1$ and six are of the form $X_i X_j$.)

- The experiment compares the regression using the four linear factors with that using the four linear factors and the ten quadratic ones.

- The consistency of the funds’ time series over time is fundamental as longer time series are necessary for the linear+quadratic case.

The Steps Taken

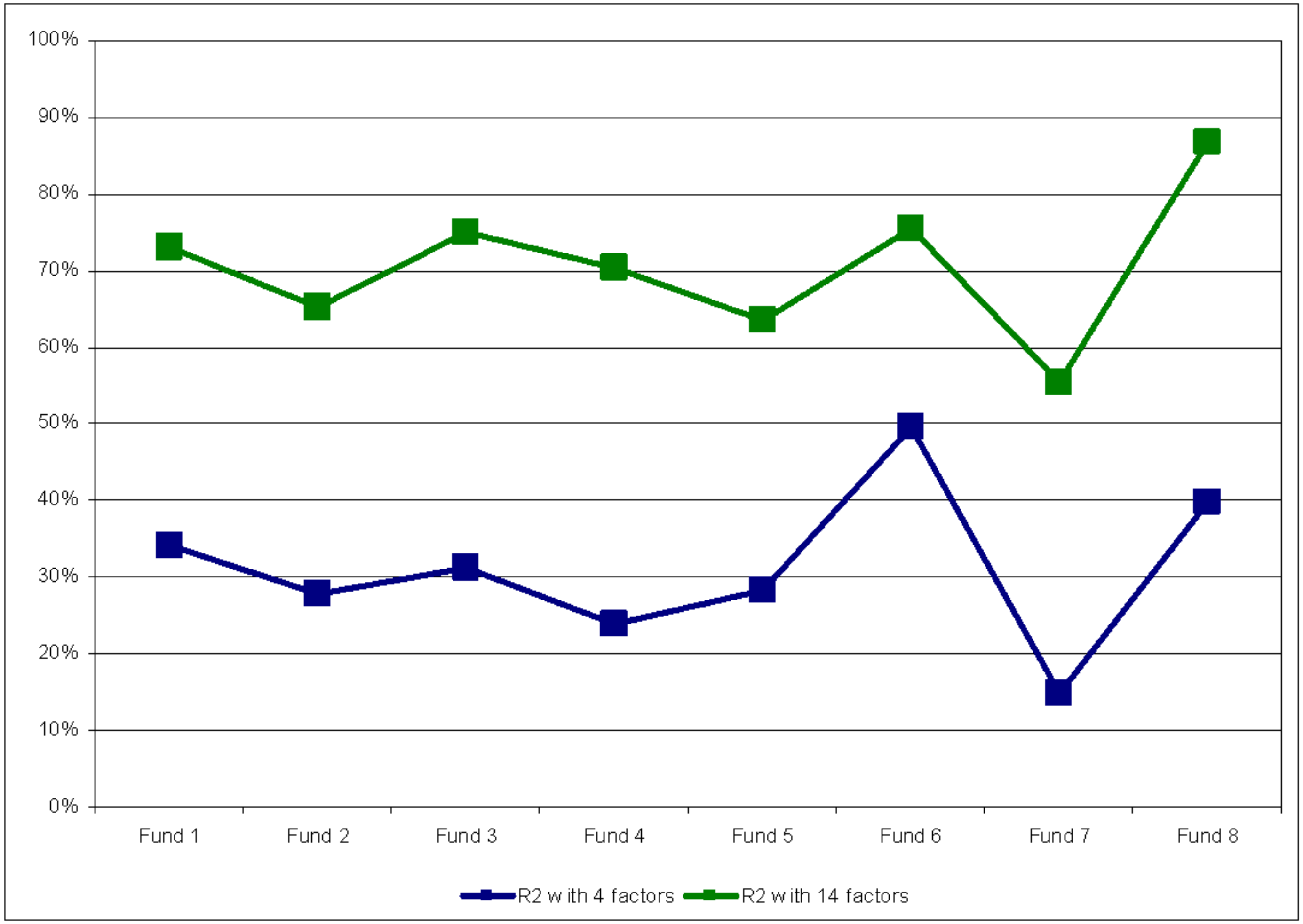
•Normalizing the factors

•Diagonalizing the factors

•Carrying out the regressions with the four linear terms over several time horizons of length 24m

•Carrying out the regressions with the fourteen linear and quadratic terms over several horizons of lengths 24m and 48m

Results of the Experiment



A Few Facts

- Funds 1-6 have relatively large exposure to commodities and other asset classes excluded from the factors
- Fund 7 has a massive exposure to the Euro (80% of its assets!)
- Fund 6 does a “buy and hold” strategy consistently more than the others
- Fund 8 only invests in R3000 stocks and US corporate bonds
- The average R-squared goes from 30% to 70%
- The t-stats remain virtually unchanged

Handle with Care!

- Raising the degree raises the number of regressors enormously!
  - Long, stable time series
  - Fewer initial factors
- What is the economic meaning of the Hermite polynomials?
- The exposure to the linear factors and higher order terms need to be translated back into the original factors: choose a “smart” normalization and diagonalization!