

Personal Asset/Liability Management: Using the Discretionary Wealth Hypothesis within an Equilibrium Term Structure

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The Need for Individual Asset/Liability Management

- In most countries, responsibility for provision of retirement income is shifting away from governments and corporate entities to individuals
- Many defined benefit retirement systems are being replaced with defined contribution plans where the burden of intelligently investing contributed funds to the individual
- The recent global financial crisis has raised obvious doubts about the soundness and ethics of many financial institutions.
- Households are advised to adopt the more sophisticated techniques of large pension schemes in thinking about the balance of current assets and liabilities arising from planned future consumption

The Financial Crisis and You

- ❑ The recent financial crisis included great volatility brought about by the failure of major financial institutions
- ❑ Values for both securities and real property declined dramatically, but have since recovered substantially
- ❑ Individual investors are largely uncertain how to prudently respond to generally negative changes in the personal financial circumstances, while remaining ready to take advantage of good investment opportunities when available
- ❑ The volatile crisis period underscores the need for investor to think separately about “what are my expectations for returns and risks of available investments?” and “what level of aggressiveness is appropriate to my current financial circumstances?”

Goals for this Discussion

- Describe the a new approach to asset-liability management that combines four key elements, one of which is new to the finance literature
 - The key benefit of this technique is that it dynamically reallocates assets over time in a precise way which maximizes the median, rather than the expected value of surplus
- Introduce the “Discretionary Wealth Hypothesis” (DWH) from Wilcox (2003) and illustrate how it is derived for ALM purposes
- Show how the combined technique is equally suitable as an ALM technique for both institutions and households
- Introduce a new approximation to Resolve the conflict between use of the DWH and the single period assumption in Markowitz

Key Elements of the Method

- Use the approach from diBartolomeo (1995) to forecast the entire distribution of the surplus between assets and liabilities for all future periods
 - diBartolomeo, Dan. “An Equilibrium Term Structure Approach to Asset Liability Management, Northfield Working Paper #49, 1995.
- In each possible future state, use the “Discretionary Wealth Hypothesis” from Wilcox (2003) to determine the optimal degree of mean-variance risk aversion for the investor
- Apply traditional Markowitz mean-variance optimization to find the optimal asset allocation for each future state
 - Use a new approximation to modify the “single period” assumption in Markowitz to more precisely incorporate transaction costs in a multi-period Markowitz approach

Equilibrium Term Structure Approach 1

- ▣ Traditional actuarial procedures assume a single rate for discounting future cash outflows to present value
 - Asset cash flows are priced in financial markets by a “yield curve” or term structure of interest rates that reflects investor preferences for maturities and expectations about future changes in interest rates
 - Due to this conflict of methods, a fund can have riskless incoming cash flows that *exactly meet all required outflows* but still appear to have a substantial surplus or deficit
 - Potential correlation between asset values and the present value of liabilities are often ignored, or addressed in a primitive fashion by assuming some statistical correlation

Equilibrium Term Structure Approach 2

- Assume a lognormal interest rate process in discrete time, and model the evolution of short term interest rates as a binomial tree
 - The interest rate for any maturity subsequent to any point in the tree can be calculated as the cumulative interest rate over all possible paths
 - The present value of any future cash flow at any time point in the tree can be calculated by discounting the cash flow over all possible paths
- Key Step
 - Calibrate the tree, by changing the probabilities of upward or downward moves in the short term interest rate until all riskless bonds have a present value at the root of the tree equal to their market value
 - The interest rate process is now in equilibrium as arbitrage transactions are impossible

Equilibrium Term Structure Approach 3

- ▣ Represent the price process for assets as a second binomial tree that is correlated with interest rate process
- ▣ The correlation between the two trees can be represented geometrically as in:
 - Margrabe, W. “The Value of an Option to Exchange one Asset for Another” Journal of Finance, 1978.
- ▣ Asset returns in each period are the sum of a drift term (risk premium), a effect from correlation with the interest rate process and a noise term

Let's See if We Can Do Better

- *Assuming a fixed asset allocation, we can now get a expected value for surplus at any particular node of the tree, and hence the expected value of the surplus distribution at any point in time*
- What if the asset allocation were not fixed?
 - Since we can project asset returns all along the binomial tree, we can change asset allocation at each node of the tree with no loss of generality
 - **We just have to start projecting asset values at the root of the tree and work outward, just as we work from the branches inward to estimate the present value of liabilities**
- We propose to change asset allocation dynamically over time, using Markowitz mean-variance optimization but allowing state dependent risk aversion that varies with both time and the relationship between assets and liabilities

To a Man with a Hammer, Everything Looks Like a Nail

- ▣ Our key assumption is that all investment assets may be liquidated to fund consumption, but are subject to non-zero transaction costs
- ▣ This implies that liabilities can be treated as portfolio assets in negative quantities that are not available for trading
- ▣ The general concepts of Markowitz mean-variance efficiency hold, but are subject to a multi-period process

Review of Modern Portfolio Theory

- ▣ Markowitz, Harry. "Portfolio Selection," Journal of Finance, 1952, v7(1), 77-91.
- ▣ This theory says that an investor can form an efficient frontier of differently composed portfolios
 - Each portfolio has the maximum return for a given level of risk
 - Each portfolio has the minimum risk for a given level of return
 - Time is defined as a single long period
- ▣ But what did Markowitz say about how an investor should choose which efficient portfolio to hold?
 - **Nothing**

Twenty Seven Years Later

- ▣ Levy, H. and H. M. Markowitz. "Approximating Expected Utility By A Function Of Mean And Variance," American Economic Review, 1979, v69(3), 308-317.
- ▣ Assumes investors want to maximize the expectation of the log of their wealth
- ▣ The mean-variance formulation is derived from a Taylor series approximation to the log of wealth. It's just the first two terms

$$U = \alpha - \sigma^2 / T \quad \text{or} \quad U = \alpha - \lambda \sigma^2 \quad \lambda = 1 / T$$

λ is just the slope of the tangent line to the frontier

Ponderable

- ▣ Have you ever asked a passer-by on the street “What is your risk tolerance parameter in mean-variance space?”
 - My grandmother would have taken great offense at an impertinent question and slapped me
- ▣ Investors constantly use terms of art such as “conservative” or “aggressive” to describe their posture without actually understanding what that means
- ▣ Portfolio compositions change for a muddle of two reasons:
 - Expectations have changed about the risk or return of various assets
 - The investor’s risk aversion has changed, probably without being recognized or consciously done

The Discretionary Wealth Hypothesis

- ▣ Wilcox, Jarrod. “Harry Markowitz and the Discretionary Wealth Hypothesis”, Journal of Portfolio Management, 2003.
- ▣ Simply put it says that investors should not put more money at risk than they can afford to lose
- ▣ Equally applicable to institutional ALM or households
- ▣ *Think of your life as a balance sheet*, including the present value of future savings and the present value of the liability for expected expenditures
 - Low discount rates for important liabilities, higher discount rates for non-essential expenditures
 - **What is the debt/equity ratio for your life?**

More on Discretionary Wealth

- Wilcox derives that the optimal risk aversion for an investor:
 - Let L = total assets / net worth (surplus)
 - Optimal risk aversion $\lambda = L / 2$
 - Allowing time variation in λ maximizes the expected median of future wealth, rather than the mean

- This implies that optimal risk aversion varies in both unpredictable and predictable ways
 - Market volatility will change our net worth in unpredictable ways
 - Getting closer to retirement age, or having a child graduate university changes our balance sheet in predictable ways

- This implies that we have some ability to forecast our optimal asset allocation for times in the future, requiring a multi-period framework

Even More on Discretionary Wealth

- Following the Discretionary Wealth Hypothesis is similar to constant proportion portfolio insurance for investors
 - You are increasing aggressiveness when you can afford to do so
 - You are taking a more conservative posture when you must
- These changes only impact your risk tolerances
 - Changes in portfolio composition must also reflect changes in capital market expectations
- The DWH approach is now included in CFA Institute curriculum
- DWH approach can also incorporate uncertainty in the balance sheet formation
 - We don't know future inflation will impact liabilities of institutions
 - Individuals don't know how long we'll live
 - Will our children require financial support for education or not

Moving to A Multi-Period View

- Use of the DWH requires a change to the traditional Markowitz assumption of a time being a single long period
- If we know that our portfolio will be changing over time, and those changes require transaction costs, we need to weigh the benefits of improvements in expected utility against the trading costs in the right way
- Traditional optimization procedures that assume a single period can be substantially improved by incorporating a simple approximation

The Multi-period Approximation

- ▣ Imagine I have a portfolio, P_1 with return α (net of fees and expenses) and standard deviation σ . Our usual utility function would say:

$$U_1 = \alpha - \sigma^2 / T$$

Where T is equal to our risk acceptance parameter

- ▣ Now let's imagine there is another portfolio, P_2 that has a higher utility, because either the return is higher or the standard deviation is lower
 - This portfolio has completely different positions than the initial portfolio. Let's assume that this portfolio has a higher return by increment Δ , so

$$U_2 = (\alpha + \Delta) - \sigma^2 / T$$

More Approximating

- Since U_2 is greater than U , we should be willing to pay some transaction costs to switch from P_1 to P_2 .
- Now let's consider a different way to improve our returns
 - We go back to the manager of Portfolio 1 and ask them to reduce their fees by Δ , so now our revised utility on P_1 is $U_{1,L}$ for "lowered fees"
 - Notice that $U_{1,L}$ and U_2 are equal. So if we invest our money in either P_2 or P_1 (after lowering the fees), the expected value of wealth at the end of time is the same.
 - This suggests that we should be willing to pay the manager an upfront fee to lower his management fees that is equal to the trading costs we would be willing to pay to switch from the initial portfolio. **As long as conditions never change, this is valid**

The Key Concept

- ▣ Since P_2 and P_{1L} have different securities, the performance will be different from month to month
 - Even if the long term average return and volatility are identical
 - So over any finite time horizon, we cannot be sure which of the portfolios will perform better
- ▣ P_{1L} will always perform better than P_1 , over all time horizons, as it is just the same portfolio with lower fees
 - For P_{1L} the probability of outperforming P_1 is always 1
 - P_2 is guaranteed to be better than P_1 in the long run if conditions don't change, but the probability that P_2 will actually outperform P_1 over any finite horizon is between .5 and 1
 - **Amortizing transaction costs over a single period is equivalent to assuming that this value is always one**

Implementation

- ▣ Let's assume a single period optimization for a strategy with expected turnover of $M\%$ per annum
 - In this case we want to amortize the cost of each transaction over a four year expected holding period, or $M\%$ per annum
 - In our multi-period world, we want to amortize by M divided by the probability that the revised portfolio will actually realize a better risk adjusted return over the finite holding period

- ▣ We use the tracking error (expected volatility of the return difference) between the original portfolio P_1 and the modified portfolio P_2 , as the standard error on the increase in expected utility between U_2 and U_1
 - **The tracking error between P_1 and P_{1L} is zero**
 - Just pick your distribution assumption, convert to a T-stat and calculate the probability value by which to adjust amortization

Conclusions

- ❑ Traditional ALM has substantial limitations arising from the distortions caused by overly simplistic methods for discounting liabilities, which our ETS approach resolves
- ❑ The Discretionary Wealthy Hypothesis can be utilized to rationally quantify risk tolerance, **and even more importantly optimally provide required changes in risk tolerance in both response to and in anticipation of changes in surplus**
- ❑ Implementing the DWH requires converting to a multi-period framework in which the classical treatment of transaction costs in Markowitz optimization must be modified

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