
Extending Factor Models of Equity Risk to Credit Risk, Default Correlation, and Corporate Sustainability

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Goals for this Presentation

- ✘ Illustrate how equity factor risk models and structural models of credit risk can be linked to provide consistent measures of equity risk, default risk and default correlation
- ✘ Introduce a quantitative measure of the “sustainability” of firms
- ✘ Describe results in an empirical analysis of all US listed equities from 1992 to present
- ✘ Show that common conception of “sustainable” investing is confirmed in these results
- ✘ Illustrate an alternative use of this method as a way to define the level of systemic risk to developed economies

Basic Contingent Claims Literature

- ✘ Merton (1974) poses the equity of a firm as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt
 - + Alternatively, lenders are short a put on the firm assets
 - + Default can occur only at debt maturity
- ✘ Black and Cox (1976) provide a "first passage" model
 - + Default can occur before debt maturity
 - + Firm extinction is assumed if asset values hit a boundary value (i.e. specified by bond covenants)
- ✘ Leland (1994) and Leland and Toft (1996)
 - + Account for the tax deductibility of interest payments and costs of bankruptcy
 - + Estimate boundary value as where equity value is maximized subject to bankruptcy

Default Correlations

- ✘ Hull and White (2001) and Overbeck and Schmidt (2005)
 - + You can estimate default correlation if you knew the (unobservable) true interdependence between firms
- ✘ Estimate default correlation from asset correlation
 - + Zhou (2001) derives default correlations from asset correlation
 - + Frey, McNeil and Nyfeler (2005) use a factor model to describe asset correlations
- ✘ Include effect of correlation of changes in default boundary to asset correlations
 - + Giesecke (2003, 2006)
- ✘ Take the easy way out: assume asset correlation is equal to equity return correlation
 - + DeSerigny and Renault (2002) provide negative empirical results
 - + CreditMetrics, Hull and White (2004)
 - + Close if leverage levels are low and horizons are short

Equity Return Properties Help Out

- ✘ Defaults are usually rare events so it's impossible to directly observe default correlations over time
- ✘ The book value of firm assets is a very incomplete measure of firm assets, so observing asset volatility and asset correlations across firms are very weak estimates
- ✘ Equity return volatility and correlation are readily observable
- ✘ Zeng and Zhang (2002) shows asset correlations must arise from correlation of both equity and debt components
- ✘ Qi, Xie, Liu and Wu (2008) provide complex analytical derivation of asset correlations given equity return correlation

Bring on the Factor Models

- ✘ If you have an “equity only” factor model
 - + Estimate pair-wise correlations for equity returns
 - + See diBartolomeo 1998 for algebra
 - + Convert to asset correlation using method of Qi, Xie, Liu and Wu
- ✘ If you have a “multi-asset class” factor model you can use the fundamental accounting identity to get a factor representation of asset volatility and equity
 - + $\text{Assets} = \text{Liabilities} + \text{Equity}$
 - + Asset volatility is just equity volatility de-levered, adjusted for covariance with the market value of debt
 - + When interest rates rise equity values usually drop, but market value of debt definitely declines, reducing leverage
 - + Convert to pair-wise asset correlation values

In Theory, We're Ready to Go

- ✘ With asset volatility and correlations estimated we can use our preferred structural model to estimate default probability of a firm
- ✘ Use method from Zhou to convert asset correlations to default correlations
- ✘ We can now produce joint default probabilities across firms
- ✘ However there are some pretty restrictive assumptions
 - + Firm must have debt today
 - + Firm must have positive book value today
 - + Balance sheet leverage must stay fixed in the future

Reverse the Concept: Sustainability

- ✘ Instead of trying to estimate how likely it is that firm goes bankrupt, let's reverse the logic
- ✘ We will actually estimate the “market implied expected life” of firms using contingent claims analysis
- ✘ Firms with no debt can now be included since it is possible that they get some debt in the future and default on that
- ✘ A quantitative measure of the fundamental and “social” concept of *sustainability*

Our Basic Option Pricing Exercise

- ✘ Underlying is the firm's assets with asset volatility determined from the factor model as previously described
- ✘ Solve numerically for the “implied expiration date” of the option that equates the option value to the stock price
 - + *Market implied expected life of the firm*
- ✘ Include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately
- ✘ If you choose Black-Scholes as your option model, then you can solve BS for the implied time to expiration using a Taylor series approximation
- ✘ More complex option models allow for stochastic interest rates

Filling in with “Distance to Run”

- ✘ For firm’s with no debt or negative book value, we simply assume that non-survival will be coincident with stock price to zero, since a firm with a positive stock price should be able to sell shares to raise cash to pay debt
 - + If you have a stock with 40% a year volatility you need a 2.5 standard deviation event to get a -100 return
 - + Convert to probability under your distributional assumption
- ✘ We convert both measures to the median of the distribution of future survival in years
 - + What is the number of years such that the probability of firm survival to this point in time is 50/50
 - + Highly skewed distribution so we upper bound at 300 years
- ✘ Z-score the “median of life” for both measures and map the distance to run Z-scores into the “option method” distribution for firms with no debt

Empirical Study Design

- ✘ Use a simple Merton model (Black-Scholes European put)
- ✘ Use equity volatilities from Northfield US Fundamental Model
 - + One year horizon for risk forecast
 - + Near horizon” model are more suitable but less history available
- ✘ Estimate monthly for all firms in Northfield US equity universe from December 31, 1991 to March 31, 2010
- ✘ Study three samples:
 - + All
 - + Financial firms
 - + Non-financial firms
- ✘ Sources of Time series variation
 - + Stock prices, debt levels, Northfield risk forecasts
 - + Mix of large and small firms, $4660 \leq N \leq 8309$

Let's Start at the End (March 31, 2010)

- ✘ Current life expectations for all (5068) firms in years
 - + Median 23, Mean 22.18, Cap Weighted 25.71
- ✘ Financials firms only (1132)
 - + Median 24, Mean 21.69, Cap Weighted 18.95
 - + Surprising (or maybe not) cap-weighted is a lot lower
- ✘ Non-Financials (3936)
 - + Median 23, Mean 22.33, Cap Weighted 27.36
- ✘ Highlights:
 - + AIG 7, Citicorp 6, GS 6
 - + IBM 30, MSFT 32
 - + RD 30/39, XOM 54

Time Series Properties Full Sample

- ✘ Calculate the cross-sectional mean, cap weighted mean and median for 220 months, average sample = 6587
 - + Time series average of the monthly medians, 21.63 years
 - + Time series average of the monthly means, 24.42
 - + Time series average of cap weighted means 22.66

- ✘ Lowest expectations, January 1992
 - + median 10, mean 13.20, cap weighted mean 11.05

- ✘ Highest expectations, January 2005
 - + median 30, mean 41.09, cap weighted mean 32.36

Time Series Properties Sub Samples

- ✘ Financials (average sample size = 1630)
 - + Time series average of the monthly medians, 31.03
 - + Time series average of the monthly means, 31.51
 - + Time series average of cap weighted means 24.09

- ✘ Non Financials (average sample size = 4955)
 - + Time series average of the monthly medians, 20.03
 - + Time series average of the monthly means, 22.13
 - + Time series average of cap weighted means 22.23

- ✘ Note that for the full time series, financial firms were expected to survive about 50% longer than non-financials
 - + At last date, financials have slightly lower expected lives

Another Angle on Default Correlations

- ✘ Once the time series of expected lives have been calculated, we can estimate default correlation as the correlation of percentage changes in expected lives across firms
- ✘ As expected lives shorten, changes of a given magnitude become larger percentage changes
 - + Since correlation is a bounded function (-1 to +1) larger events drive the correlation values toward the extreme value
 - + Two bonds that have one day of expected life each will have a very high default correlation
- ✘ Better than trying to correlate OAS spreads since bond prices are driven by liquidity effects

Quantifying “Sustainability”

- ✘ FTSE/KLD DSI 400 index of US large cap firms considered socially responsible, 20 year history
 - + Typically about 200 firms in common with the S&P 500

- ✘ July 31, 1995
 - + DSI 400, Median 17, Average 17.91, Standard Deviation 9.93
 - + S&P 500, Median 14, Average 15.40, Standard Deviation 9.28
 - + Difference in Means is statistically significant at 95% level

- ✘ March 31, 2010
 - + DSI 400, Median 30, Average 26.39, Standard Deviation 11.45
 - + S&P 500, Median 30, Average 24.93, Standard Deviation, 10.92
 - + Difference in Means is statistically significant at 90% but not 95%

- ✘ Testing on Disjoint Sets (DSI NOT S&P, S&P NOT DSI)
 - + Statistically significant difference in means for every time period tested

A Measure of Systemic Risk?

- ✘ Obviously, if the market thinks public companies are not going to be around very long, the economy is in a bad way
- ✘ Low equity valuations and high leverage equate to short life expectancy
 - + Higher leverage can be sustained with higher growth rates that cause higher equity valuations
We propose “revenue weighted” expected average life as a measure of systemic stress on an economy
 - + By revenue weighting we capture the stress in the real economy
 - + Avoids bias of cap weighting since failing firm’s have small market capitalization and don’t count as much
- ✘ Full sample low values are in the 6-7 range (1997-1998) with high value above 30.
 - + From July 2007 to July 2008, went from over 29 to below 12.

Next Steps

- ✘ Use more sophisticated option pricing model that allows for stochastic interest rates and possibly stochastic volatility

- ✘ Use expected life data at the firm level to predict changes in credit ratings
 - + We have hand collected (copied from Barron's week by week) every credit rating down grade and upgrade since 1991
 - + Relate changes in expected life to subsequent rating changes
 - + Relate expected life values that are outliers within their rating category to subsequent rating changes
 - + Adjust credit risk expectations for bond issuers and financial counterparties in our fixed income risk model

Conclusions

- ✘ Combining factor models and structural models of credit risk allows for consistent estimation of equity risk, credit risk and default correlation
- ✘ Structural models based on contingent claims methods are a direct and informative approach to assessing the expected survival of firms
- ✘ Comparison of SRI and conventional US stock indices reveals a positive and significant difference in expected lives, confirming the existence of “sustainability”
- ✘ We believe this technology will have usefulness as a measure of systemic risks in developed economies