A New Approach to Measuring Volatility

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Two Wrongs Can Make a Right

- This work is based on two errors:
  - A piece of research from a broker's quantitative research desk that had a fundamental flaw
  - A wrong calculation that I carried out months later
- Luck played a role (two wrongs *without any luck* certainly don't make a right)

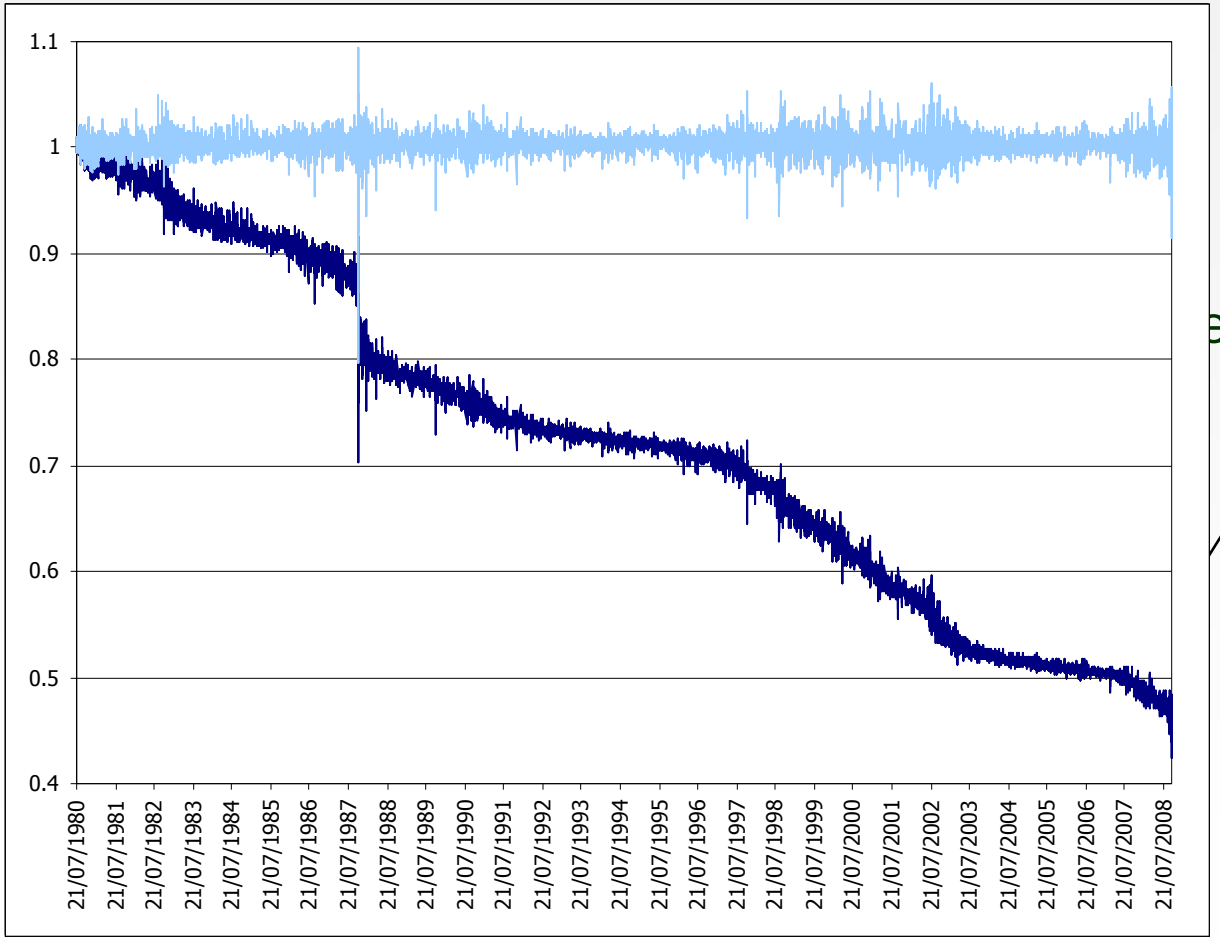
The First Mistake

- Beginning of January 2008 – the broker sends a piece of research stating that:
  - The world is in one of two states: positive one day serial correlation (P) or negative one day serial correlation (N)
  - They have a way to tell what state the world is in on a daily basis
  - They propose a trading strategy: if in state P then position investments expecting the same behaviour as yesterday; if in state N then position investments expecting the opposite behaviour
- Mid January they send a note: it was all wrong
- End of January I am still thinking about why it was wrong. *Fortunately* Jerome K of Soc Gen comes to the rescue and forces me to think about other issues

The Second Mistake

- In October 2008 I am bored. We are 100% in cash (mostly under the mattress). I start thinking about 1-day-lag correlation, but can't find the original paper – and I can't ask the broker – who has defaulted
- I start from scratch and play with time series of indices comparing each to itself with a one day lag
- I accidentally do PCA analysis. This is wrong for two reasons:
  - Obvious, meaningless output
  - I run the PCA on returns, not log-returns
- However, another unexpected help came: in "Three Men on a Bummel", Jerome K Jerome tells the story of an Englishman asking for a cushion in a German shop and receiving a kiss instead. Similarly, I performed the wrong calculation and obtained a far more interesting result than expected

What I Got



ets

Quantmare

The Model – Assumptions

- The assumptions are kept to a bare minimum
  - Daily returns are identically distributed
  - The joint distribution of the returns on two (or more) different days only depends on the lag between the days
  - If two days are far enough in time, their returns are independent
- The assumptions are intended to hold over a period in which the slope is constant

The Model – Results

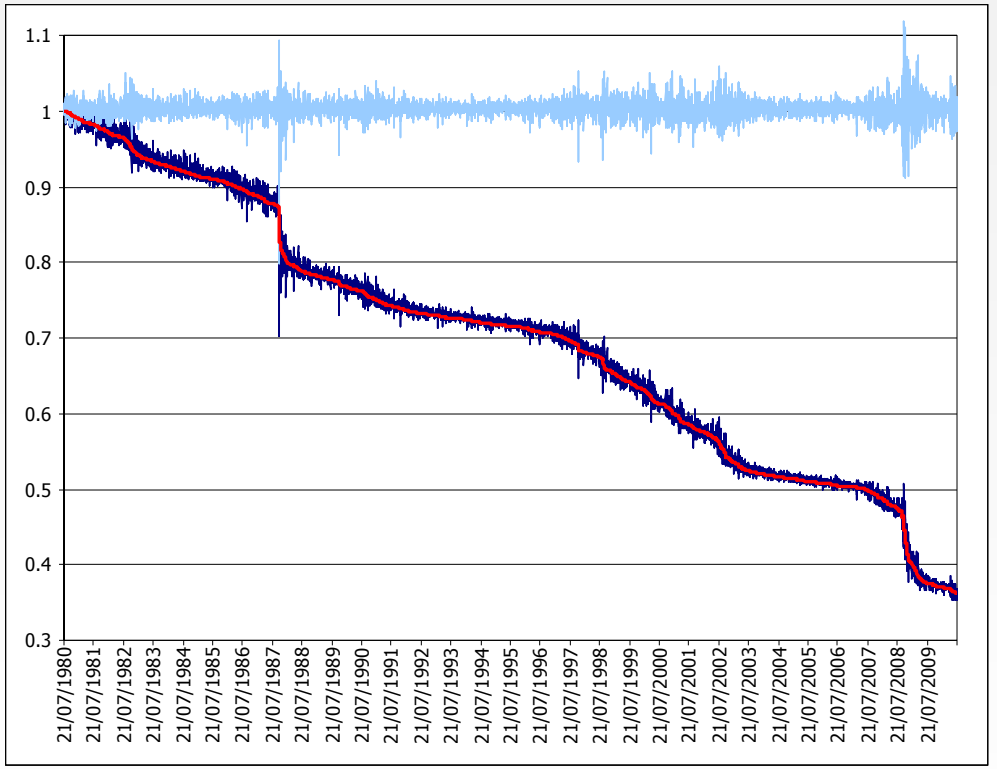
- The expected slope in the period is, in first order approximation

$$-(1-\rho)\sigma^2$$

  where σ is the standard deviation of daily returns and ρ is the one day correlation
- Higher order terms are
  - Ugly to write
  - Numerically less significant (empirical observation)

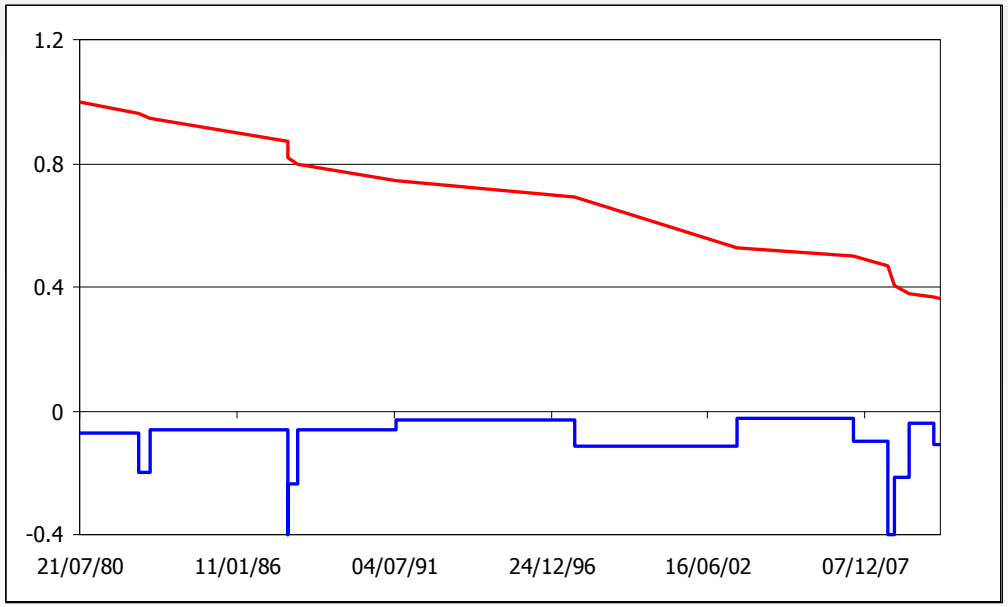
Volatility Switch Dates

- How can we find the VSD?
- The log-return line (light blue)
  - Moves around the value 1
  - Maps onto the return line (dark blue)
- We can map the constant value 1 line using this map
- We still get a wobbly



Volatility Switch Dates (2)

- A piecewise linear function
  - Has first derivative that is piecewise constant
  - Has second derivative that is zero almost everywhere
- The trick is to look for the discontinuities of the second derivative



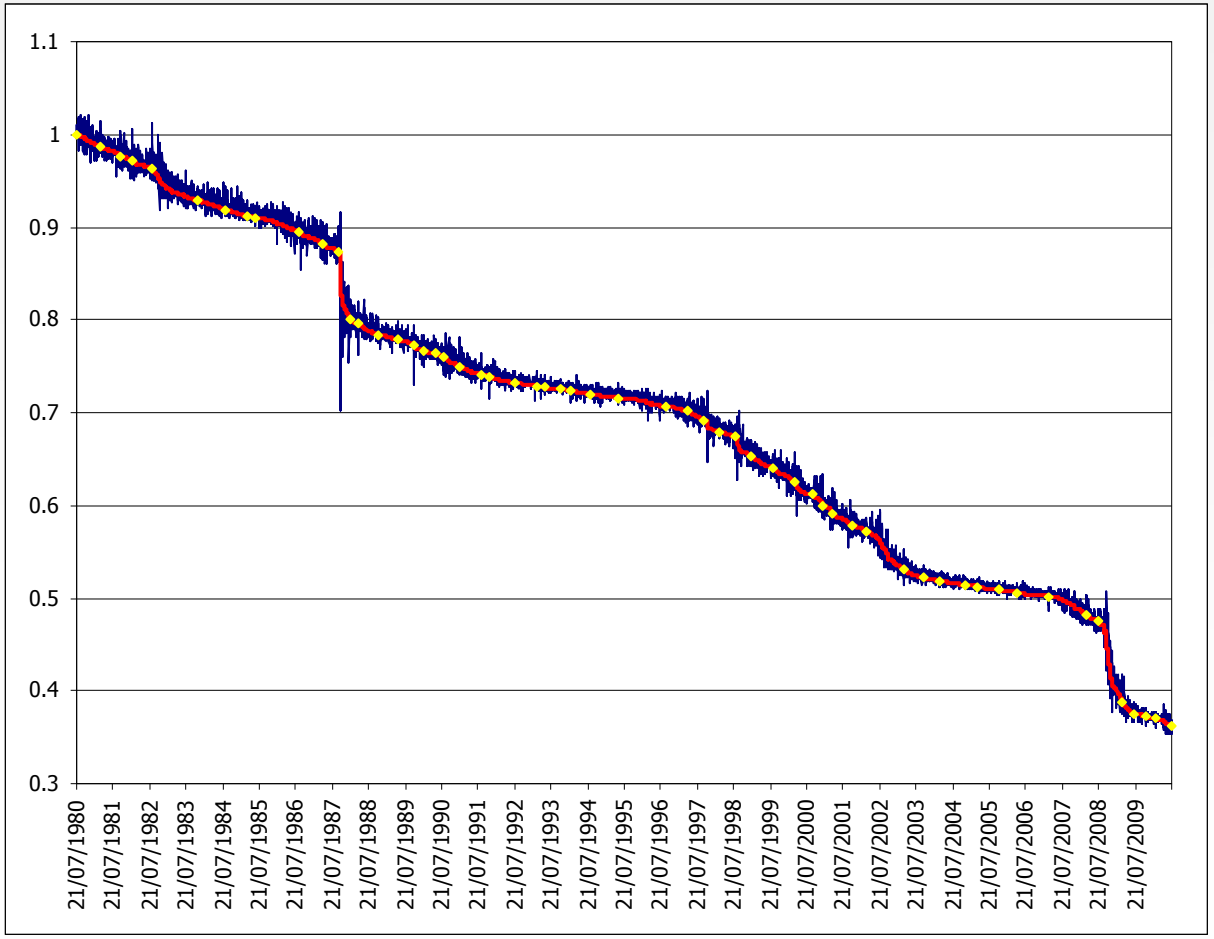
The Algorithm

```
module Calculation.VolatilityHistory

let Fill (a: float[]) maxDistance =
    let findPeaks (b: float[]) useEquals md =
        let rec auxFindPeaks i (l: int[]) =
            if i >= b.Length - 1 then (Array.append l [| b.Length - 1; b.Length - 1|])
            else
                if useEquals then
                    if b.[i - 1] <= b.[i] && b.[i] >= b.[i + 1] then auxFindPeaks (i + 1) (Array.append l [| i |])
                    else auxFindPeaks (i + 1) l
                else
                    if b.[i - 1] < b.[i] && b.[i] > b.[i + 1] then auxFindPeaks (i + 1) (Array.append l [| i |])
                    else auxFindPeaks (i + 1) l
        let c =
                if b.[0] > b.[1] then auxFindPeaks 1 [| 0 |]
                else auxFindPeaks 1 [| 0 |]
        let rec decreaseMaxDistance (arr: int[]) i m =
            if i >= arr.Length - 1 then arr
            else if arr.[i + 1] - arr.[i] > m
                 then let newArr = [| for j in 0 .. arr.Length + 1 -> match j with
                                                                      | n when n <= i -> arr.[n]
                                                                      | n when n = i + 1 -> arr.[i] + 1
                                                                      | n when n = i + 2 -> arr.[i + 1] - 1
                                                                      | n when n >= i + 3 -> arr.[n - 2]
                                                                      | _ -> 0 |]

                      decreaseMaxDistance newArr (i + 1) m
                 else decreaseMaxDistance arr (i + 1) m
        decreaseMaxDistance c 0 md
    let fillAux (a1: float[]) useEquals maxDistance =
        match a1.Length with
        | n when n <= 2 -> a1, false
        | _ -> let peaks = findPeaks a1 useEquals maxDistance
               let b = Array.create a1.Length 0.0
               let mutable j = 0
               let mutable changed = false
               for i in 0 .. b.Length - 1 do
                   b.[i] <- max a1.[i] (min a1.[peaks.[j]] a1.[peaks.[j + 1]])
                   if b.[i] <> a1.[i] then changed <- true
                   if i = peaks.[j + 1] then j <- j + 1
               b, changed
    let b1 = fst (fillAux a true maxDistance)
    let rec recFill (c1: float[]) =
        let filled, changed = fillAux c1 false maxDistance
        if changed then recFill filled
        else filled
    recFill b1
```

And What it Yields



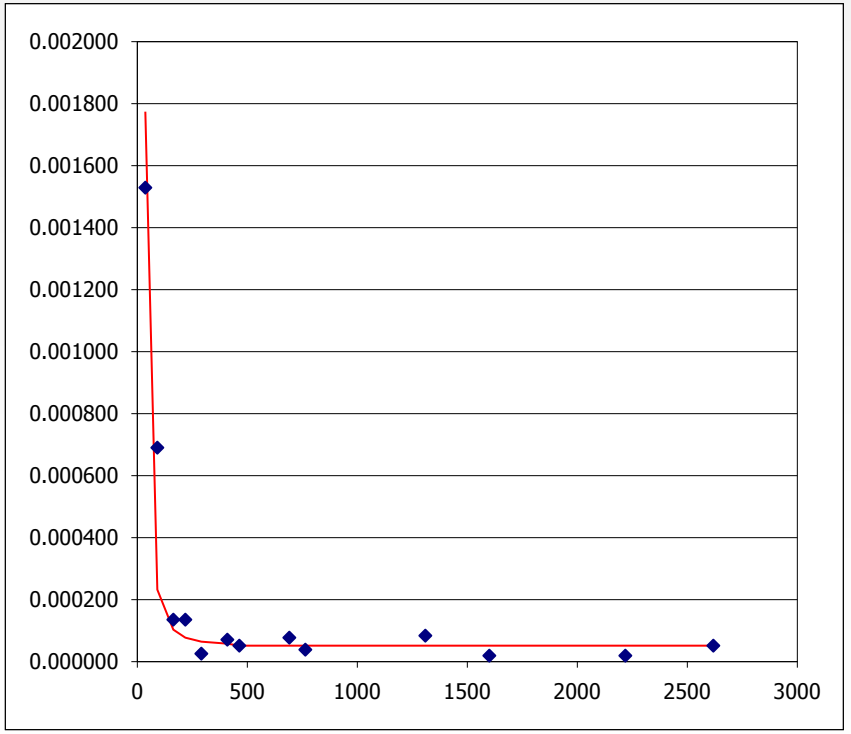
A Surprising Relation

$$s = 5 \cdot 10^{-5} \cdot p^{-2.4}$$

Where

- s is the slope
- p is the length of the period

2.4?!?



Different Asset Classes

- Equities — the larger the m-cap the better
- Indices/ETF's — works fine
- Bonds — depends on the name
- Commodities — works fine if there is enough speculation
- Currencies — exchange rates are ratios, not values!
- Hedge Funds — infrequent data…
- Broad liquidity, economic value and absence of seasonality are the key

Another Interesting Graph

- x-axis: annualized return in periods between switch dates
- y-axis: annualized volatility in periods between switch dates
- Outliers tend to correspond to short periods of time (need for a better algorithm)