State Street Global Markets®

STATE STREET ASSOCIATES

Carrier Portfolios and Exotic Index Replication

23rd Annual Northfield Research Conference

Colorado Springs, CO

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Outline

- **Part I**
  - **Introduction to the Carrier Portfolio (CP) and its approximation**
  - **Comparing the CP and mean-variance techniques**

- **Part II**
  - **Applications**
  - **Indexes, Data and Results**
  - **Conclusions**

The Carrier Portfolio

- ***(What it is)*** **An innovation in portfolio construction -- based on developments in inverse problems and contemporary signal processing -- that can be employed to optimally track any market index or composite or benchmark.**
- ***(What it does)*** **Minimizes the number of (many potential) investable assets while simultaneously replicating the performance of the index (benchmark) over time.**
- ***(How it performs)*** **Robust, computationally efficient, and used in support of over 100 specialized client and internal projects.**

Advantages of the Carrier Portfolio

- ***(Fundamental)*** **A filter-based technology, not a statistical sampling method -- no parameter or model tuning is required.**
- ***(Simple)*** **Inputs are security and index returns -- no estimates of covariances are required nor used.**
- ***(Flexible)*** **Based on a linear program that is less computationally intensive and more stable than quadratic ones, e.g., mean-variance optimization.**
- ***(Scalable)*** **Amenable and aimed to treat very large-scale optimization problems involving (literally) thousands (or more) of (highly correlated) assets.**

Motivation

$$\text{Tracking Portfolio} = \{w_1, w_2, \cdots, w_N\} \in R^N$$

$$\underset{w \in \Omega \subset R^N}{\arg\min} \left(r_t - E[r_t] - (R_t w - E[R_t w])\right)^t \left(r_t - E[r_t] - (R_t w - E[R_t w])\right),$$

where :

1) $\sum_{j=1}^{N} w_j = B$

2) $a_j \le w_j \le b_j, \quad j = 1, 2, \cdots, N$

3) $A_j \le \sum_{i=1}^{N_G} \delta_{i,j} w_i \le B_j, \ j = 1, 2, \cdots, G, \quad \delta_{i,j} : \{1,2,\ldots,N\} \to [0, B]$

Motivation

$$\text{Tracking Portfolio} = \{w_1, w_2, \cdots, w_N\} \in R^N$$

$$\underset{w \in \Omega \subset R^N}{\arg\min} \; \left(r_t - E[r_t] - (R_t w - E[R_t w])\right)^t \left(r_t - E[r_t] - (R_t w - E[R_t w])\right)$$

$$+ \lambda \|w\|_{\ell^p}, \; 0 \le p \le 1$$

Penalty/concentration term

where :

$$1) \sum_{j=1}^{N} w_j = B$$

$$2) \; a_j \le w_j \le b_j, \quad j = 1, 2, \cdots, N$$

$$3) \; A_j \le \sum_{i=1}^{N_G} \delta_{i,j} w_i \le B_j, \; j = 1, 2, \cdots, G$$

Motivation

$$\text{Tracking Portfolio} = \{w_1, w_2, \cdots, w_N\} \in R^N$$

$$\underset{w \in \Omega \subset R^N}{\arg\min} \left(r_t - E[r_t] - (R_t w - E[R_t w])\right)' \left(r_t - E[r_t] - (R_t w - E[R_t w])\right) + \lambda \|w\|_{\ell^p}$$

$R_t w = r_t$??

where :

1) $\sum_{j=1}^{N} w_j = B$

2) $a_j \le w_j \le b_j, \quad j = 1, 2, \cdots, N$

3) $A_j \le \sum_{i=1}^{N_G} \delta_{i,j} w_i \le B_j, \; j = 1, 2, \cdots, G$

Motivation – Why not simply solve...?

$$\text{Tracking Portfolio} = \{w_1, w_2, \cdots, w_N\}$$

$$\underset{w \in \Omega \subset R^N}{\arg\min} \; \|w\|_{\ell^p}, \;\; 0 \le p \le 1$$

where :

$$1) \sum_{j=1}^{N} R_{j,t} w_j = r_t, \;\; t = 1, 2, \ldots, T$$

$$2) \sum_{j=1}^{N} w_j = B$$

$$3) \;\; a_j \le w_j \le b_j, \quad j = 1, 2, \cdots, N$$

$$4) \; A_j \le \sum_{i=1}^{N_G} \delta_{i,j} w_i \le B_j, \; j = 1, 2, \cdots, G$$

The Algorithm

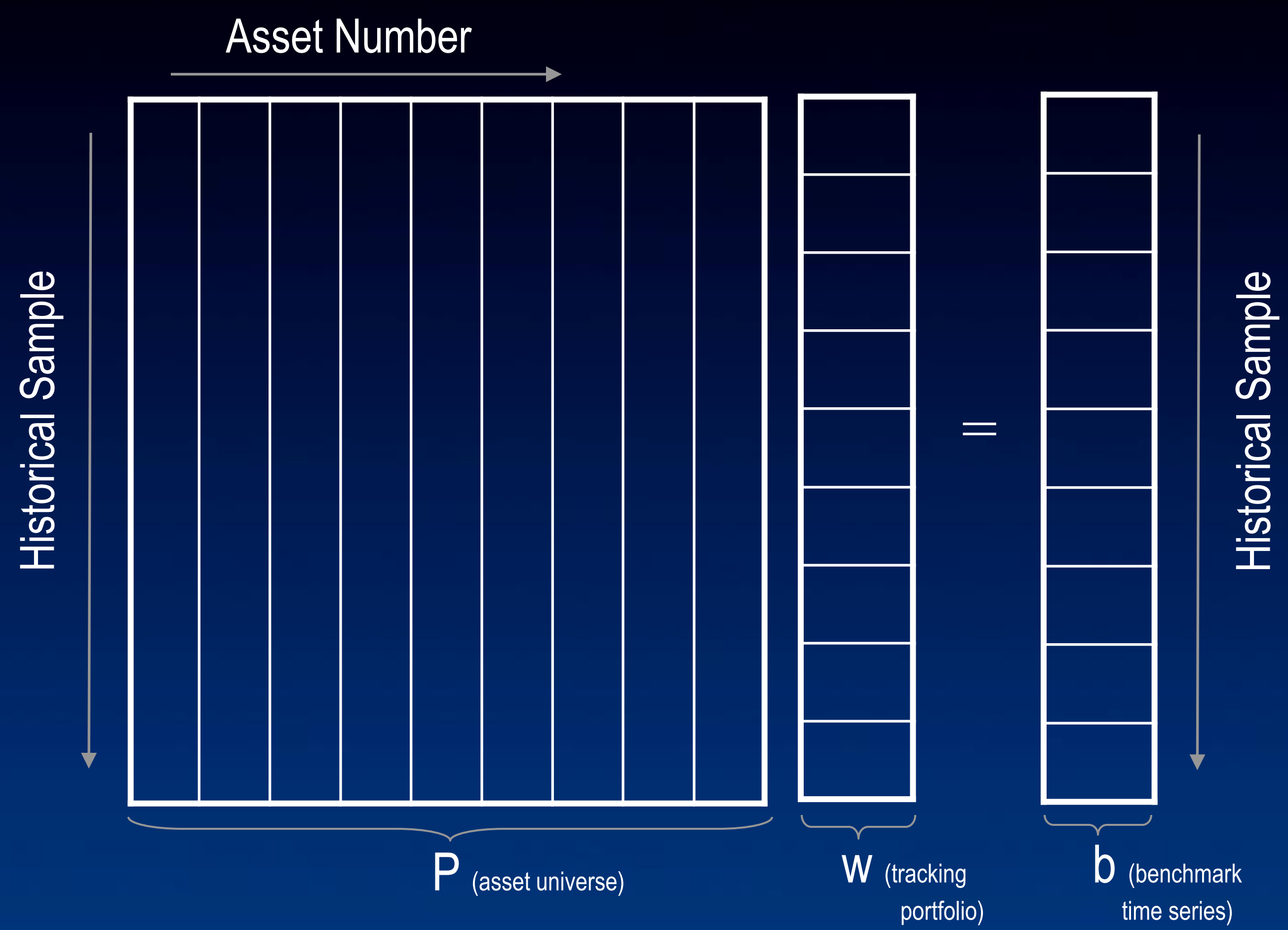
$$\text{Carrier Portfolio Approximation} = \{w_1, w_2, \cdots, w_N\}$$

that: minimizes total absolute market exposure

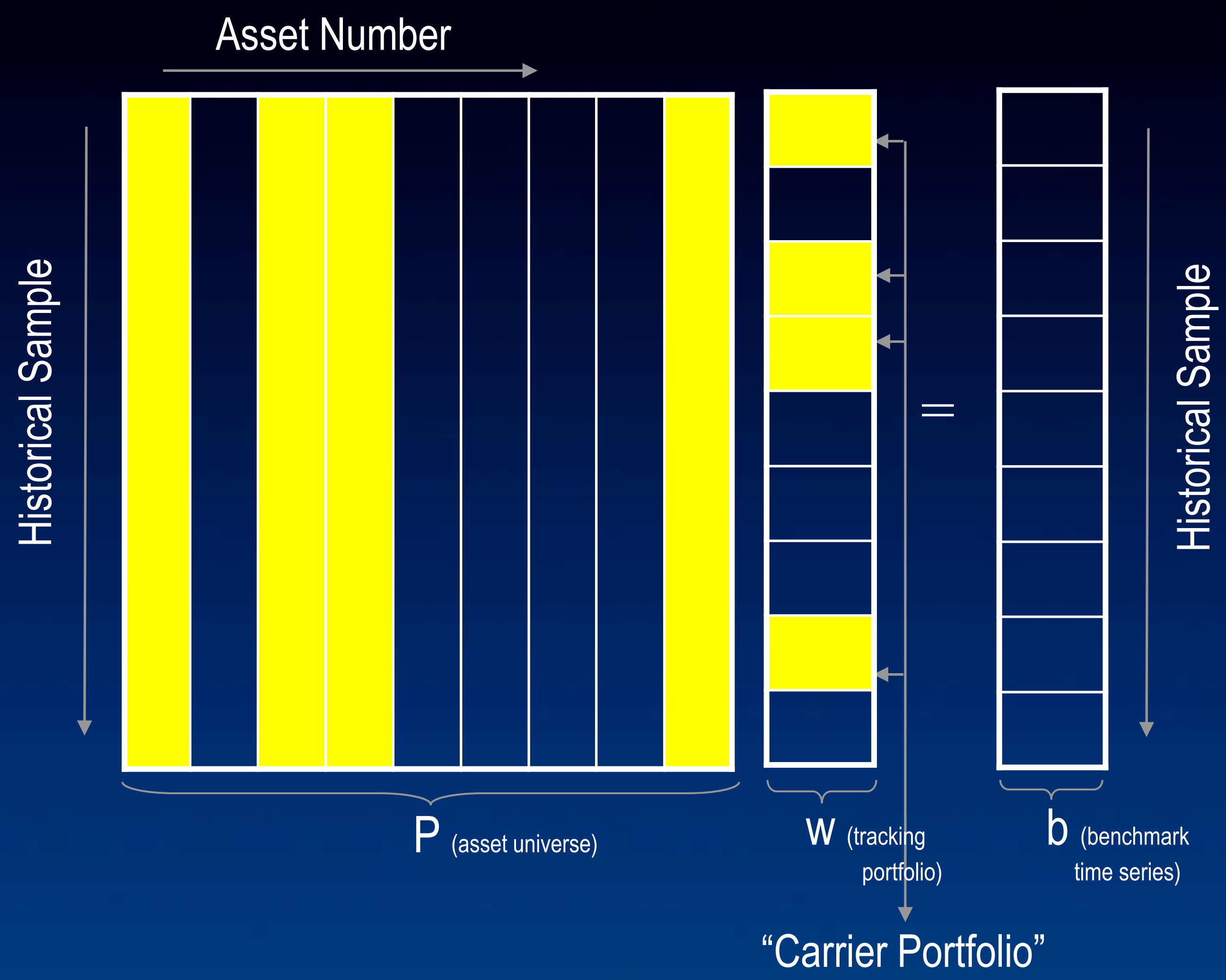
where (in-sample):

1) The weighted asset returns = the index returns over time
2) The asset weights add to a targeted budget
3) Each asset weight makes financial/ portfolio sense
4) Asset weights appropriately cover sectors and meet turnover requirements

Origins of the CP – Sparse Approximation



Origins of the CP – Sparse Approximation



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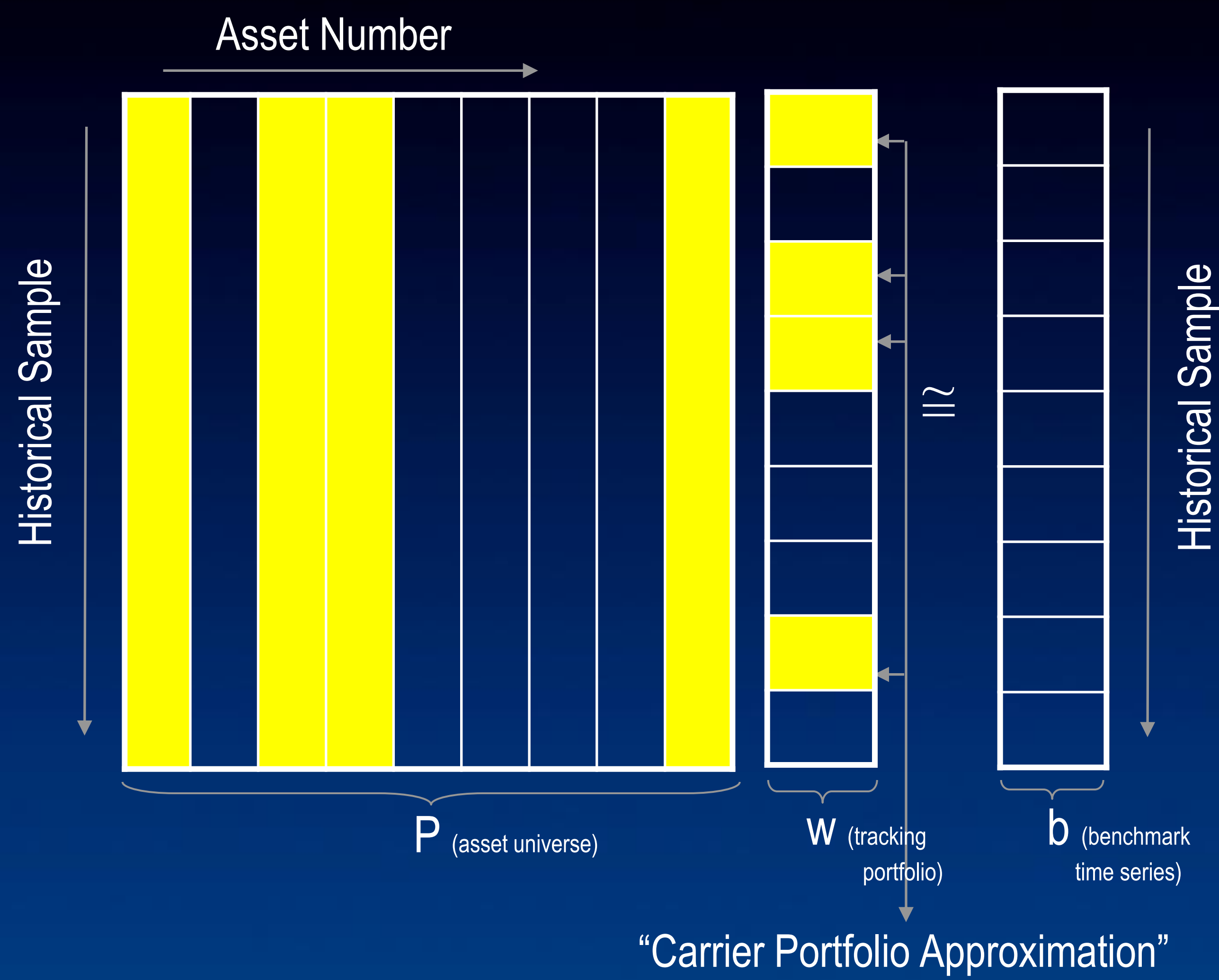

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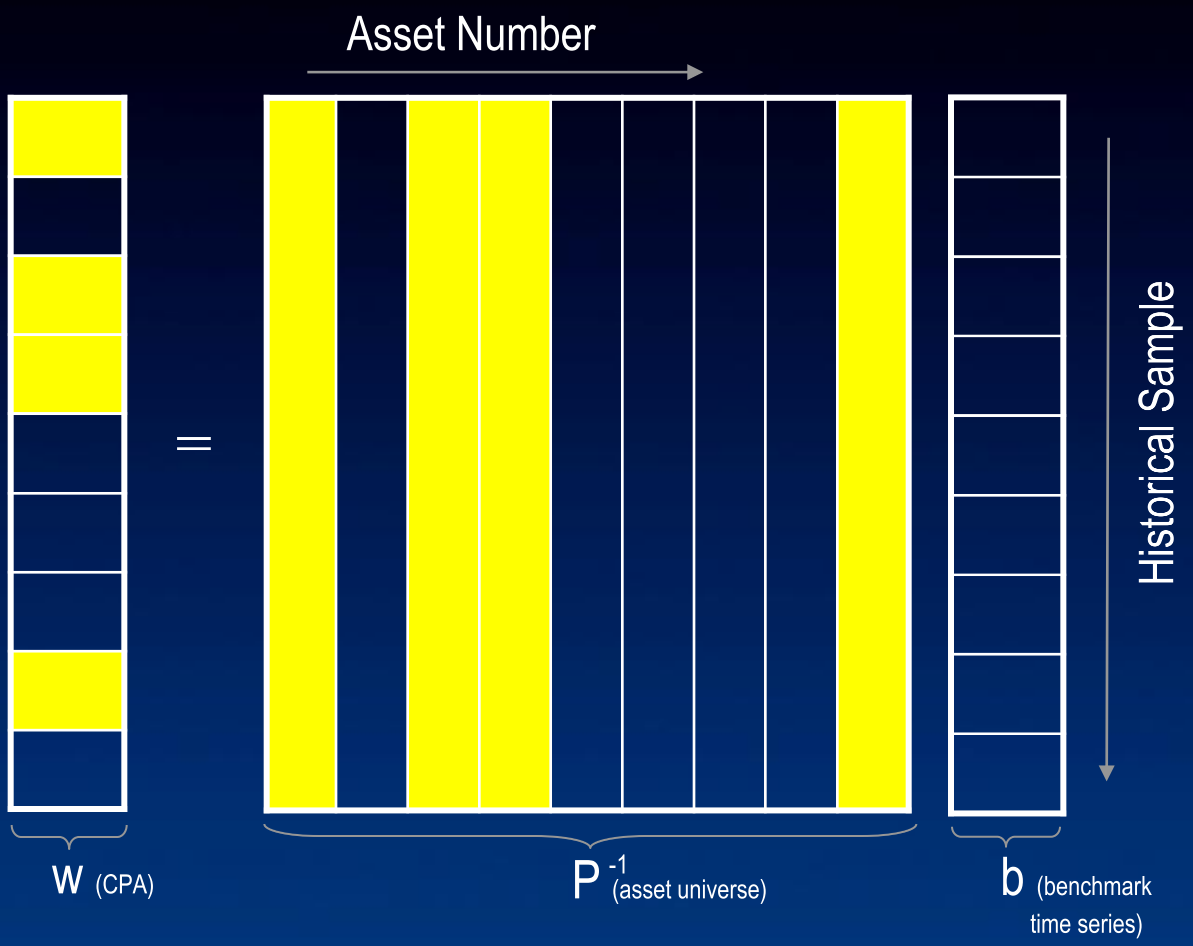
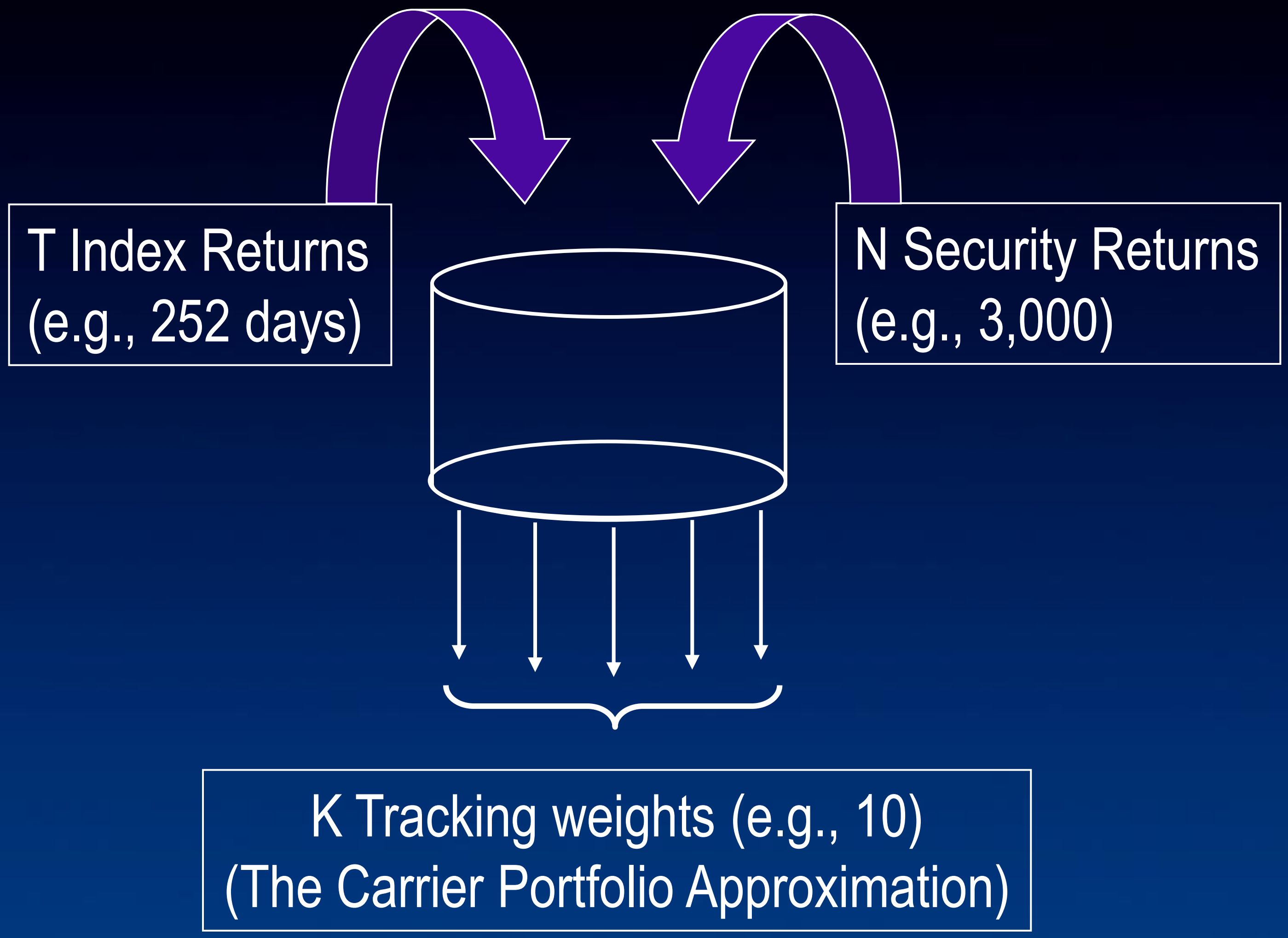


Filtering Securities



History: Why the name "Carrier Portfolio?"

Scattering Support

Smallest "equivalent" support of a source to produce observed scattering data.

[Kusiak, Sylvester 2003]

↓

Carrier Support

Smallest "equivalent" support of a source to produce observed diffusion, scattering, electrostatic data.

[Kusiak, Sylvester, Weatherwax 2007]

[Hanke, Hyvonen, Reusswig 2008]

↓

Carrier Portfolio

Smallest "equivalent" collection of securities to produce observed composite (index) data

[Kusiak 2008]

Time

Motivation: Comparing the CPA and Quadratic Programming Solutions

Three Asset Index Tracking Example

Asset returns (%)

Asset 1	Asset 2	Asset 3
-4.0	-2.0	3.0
-1.0	-1.0	4.0

Index returns (%)

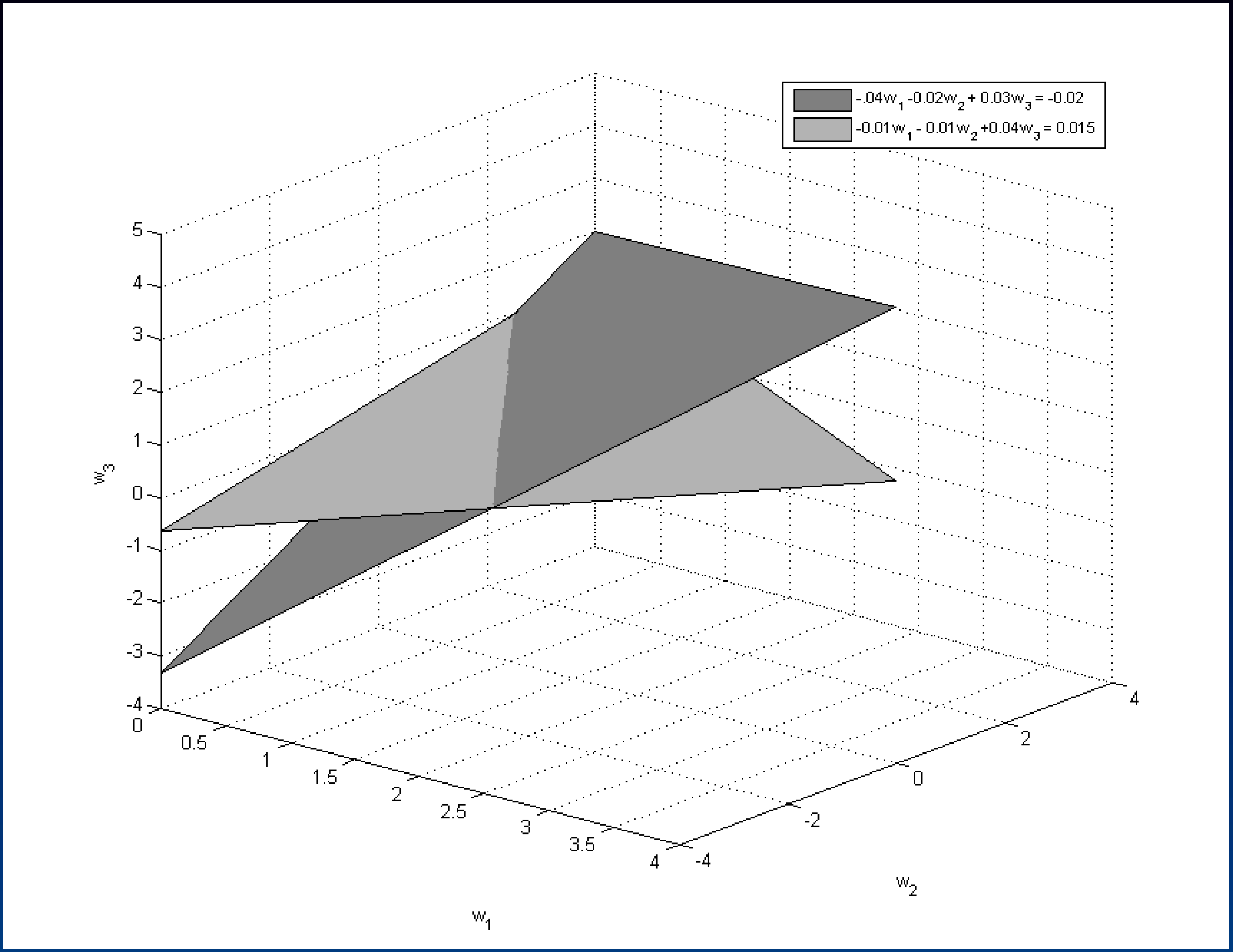
-2.0
1.5

Mission: $\min \; |w_1| + |w_2| + |w_3|$

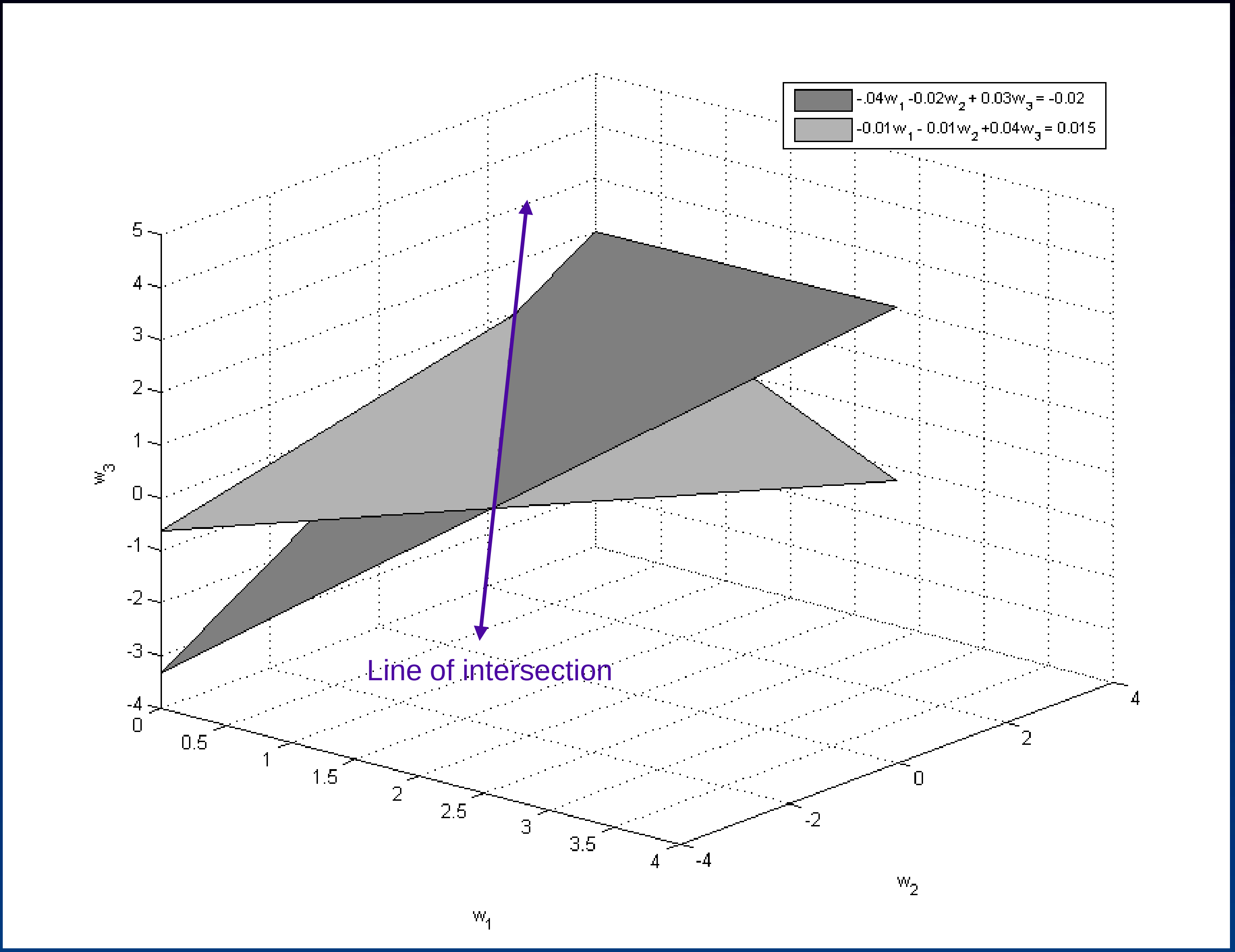
such that

$$R_{1,1}w_1 + R_{1,2}w_2 + R_{1,3}w_3 = r_1$$
$$R_{2,1}w_1 + R_{2,2}w_2 + R_{2,3}w_3 = r_2$$

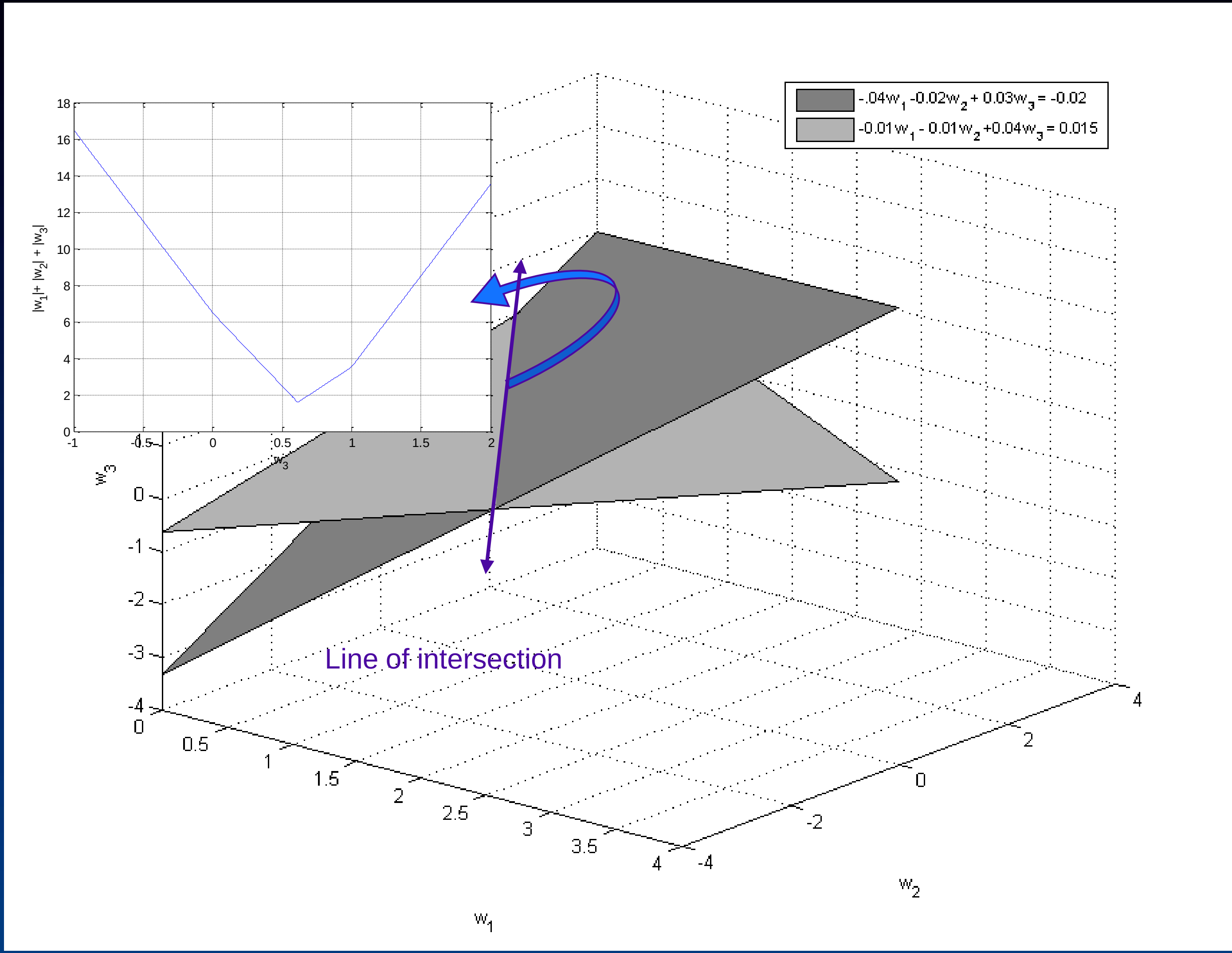
Three Asset Example: Return Constraint Geometry



Three Asset Example: Return Constraint Geometry



Three Asset Example: Constraint Geometry



Three Asset Example: Alternative Covariance Approach

Asset returns (%)

Asset 1	Asset 2	Asset 3
-4.0	-2.0	3.0
-1.0	-1.0	4.0

Index returns (%)

-2.0
1.5

$$Rw = r \quad \Rightarrow \quad R^T Rw = R^T r, \ R^T R = (T-1)\Sigma$$

Hence,

$$w = \left(R^T R\right)^{-1} R^T r = \frac{1}{T-1} \Sigma^{-1} R^T r$$

Three Asset Example Summary

	w_1	w_2	w_3	L1 Norm
Covariance-Based Portfolio	1.36	-0.56	0.51	2.43
Carrier Portfolio	0.96	0	0.62	1.58

Source: State Street Bank Europe Limited

> The Carrier Portfolio results in smaller total absolute market exposure of the underlying assets to the index, i.e., CP total market exposure = 1.58 < 2.43 = covariance total market exposure, which is 54% more exposure

>The Carrier Portfolio perfectly replicates the (in-sample) index returns, i.e., TE = 0 bps, while the covariance result has 37 bps of TE

Is this (or are we) MAD?

"…if I remember right the Mean Absolute Deviation Problem was solved over 40 years ago. …There is nothing particularly special about the 'Carrier' Portfolio other than the name."

-Anonymous

Is this (or are we) MAD?

$$\text{Mean Absolute Deviation} \equiv \|P_t - \mu\|_{\ell^1(R^T)} = \sum_{t=1}^{T} \left| \sum_{j=1}^{N} w_j R_{j,t} - \mu \right|, \quad \mu = E[P_t] = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} w_j R_{j,t}$$

Hence,

$$\min_w \|P_t\|_{\ell^1(R^T)} = \sum_{t=1}^{T} \left| \sum_{j=1}^{N} w_j R_{j,t} - \mu \right| \neq \min_w \|w\|_{\ell^1(R^N)} = \min_w \sum_{j=1}^{N} |w_j|, \quad R_t w = P_t$$

Commonality :

$$\text{MAD}: \min_w \|P_t - \mu\|_{\ell^1(R^T)}$$

$$\text{CP}: \min_w \|w\|_{\ell^1(R^N)}, \quad \textit{such that } P_t = R_t w$$

Three Asset Example Summary

	w_1	w_2	w_3	L1 Norm
Minimum MAD Portfolio*	0.30	0.74	1.86	2.90
Carrier Portfolio	0.96	0	0.62	1.58

Source: State Street Bank Europe Limited

* This is ***exactly*** the minimum mean-variance TE solution, as it should be (under normally-distributed returns) according to Konno and Yamazaki (1991)!

Three Asset Example Summary

	w_1	w_2	w_3	L1 Norm
Minimum MAD Portfolio*	0.30	0.74	1.86	2.90
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> Again, the Carrier Portfolio results in smaller total absolute market exposure of the underlying assets to the index, i.e., CP total market exposure = 1.58 < 2.90 = min MAD total market exposure, which is 84% more exposure!!

* This is ***exactly*** the minimum mean-variance TE solution, as it should be (under normally-distributed returns) according to Konno and Yamazaki (1991)!

Three Asset Example Summary

	w_1	w_2	w_3	L1 Norm
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> Again, the Carrier Portfolio results in smaller total absolute market exposure of the underlying assets to the index, i.e., CP total market exposure = 1.58 < 2.90 = min MAD total market exposure, which is 84% more exposure!!

> Finally, we are not MAD.

* This is ***exactly*** the minimum mean-variance TE solution, as it should be (under normally-distributed returns) according to Konno and Yamazaki (1991)!

For T+1 < N In-Sample TE = 0*

$$
\begin{aligned}
w_1 &= f_1(w_2, w_3, \cdots, w_N) \\
w_2 &= f_2(w_3, w_4, \cdots, w_N) \\
&\vdots \\
w_T &= f_T(w_{T+1}, w_{T+2}, \cdots, w_N) \\
&\vdots \\
w_{T+1} &= f_{T+1}(w_{T+2}, w_{T+3}, \cdots, w_N)
\end{aligned}
$$

> Return hyperplanes of dimension N intersect on another hyperplane (manifold) of at most dimension $N - T + 1$

* We add 1 to include the common budget constraint

For T > N In-Sample TE = ?

$$\text{Carrier Portfolio Approximation} = \{w_1, w_2, \cdots, w_N\}$$

$$= \underset{w \in R^N}{\arg\min} \; \|w\|_{l^1(R^N)},$$

where :

$$1)\ \sum_{j=1}^{N} r_{j,t}^{Universe} w_j = r_t^{Benchmark} + \lambda_t, \quad t = 1, 2, \cdots, T$$

$$2)\ \sum_{j=1}^{N} w_j = B$$

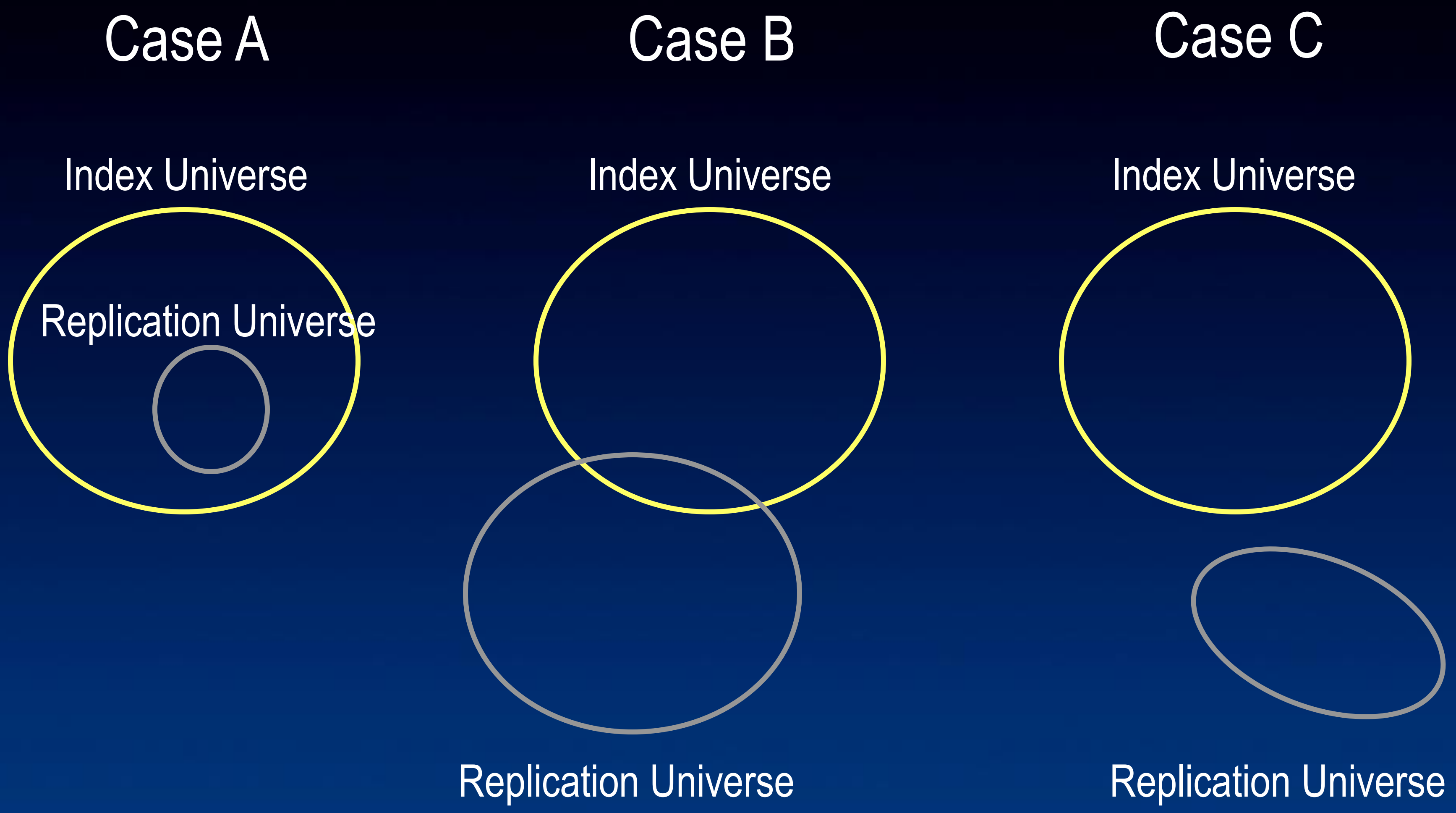
$$3)\ a_j \le w_j \le b_j, \quad j = 1, 2, \cdots, N$$

$$4)\ A_j \le \sum_{i=1}^{N_G} \delta_{i,j} w_i \le B_j,\ j = 1, 2, \cdots, G$$

> Use slack variables, define linear program on a hypercube of dimension N rather than a hyperplane

Part II
Applications

Universes of Securities



Applications

- **Replicating a large index with a small number of securities; especially when a risk model is not available (A)**
- **Constructing hedging portfolios or factor-mimicking baskets (A,B,C)**
- **Tracking illiquid indices, e.g., private equity, using liquid assets (B,C)**
- **Tracking products or indices with unknown weights or holdings (C)**

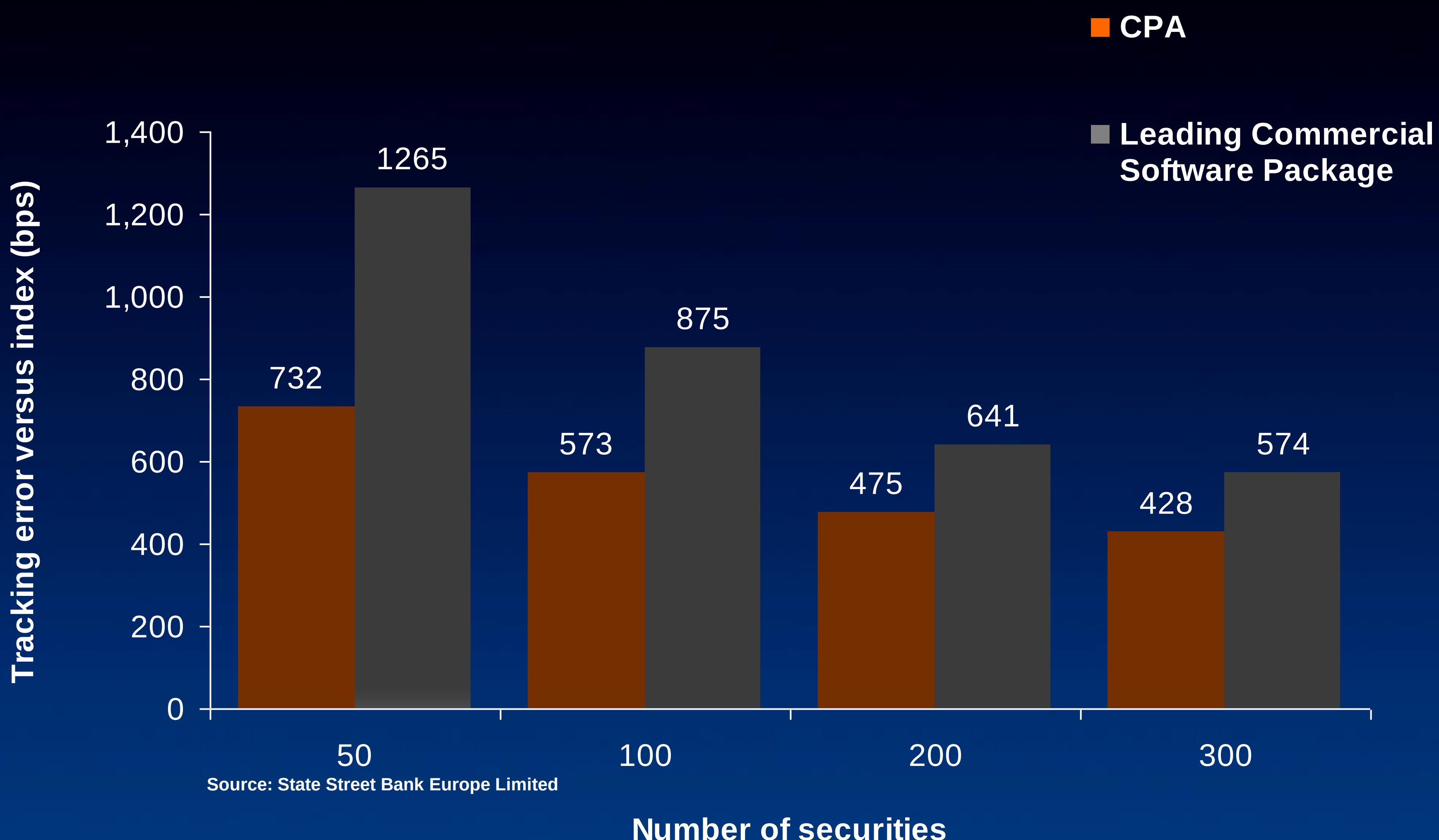
A Short List of Tracking Projects

1) S&P 500
2) Frank Russell 3000
3) FTSE RAFI US 1000 Equity
4) Dow Jones Euro Stoxx 50
5) S&P/ASX 200
6) Nikkei 225
7) MSCI US Small Cap Equity + 10 Underlying Sectors
8) MSCI US Equity
9) MSCI EMU
10) MSCI Pacific Ex Japan Equity — 1
11) MSCI Pacific Small Cap Equity
12) MSCI Global All Country Equity
13) MSCI Global EM Equity
14) MSCI Global Aggregate Debt
15) EAFE
16) JP Morgan Global EM Debt
17) U.S. Inflation (CPI) — 2
18) U.S. Risk (VIX)
19) U.S. Growth (ISM)
20) U.S./EUR FX rates
21) Real Estate (CPPI)
22) Private Equity
23) HFRX — 3

Some Noteworthy Contemporary Comparisons

MSCI Pacific Small Cap Index

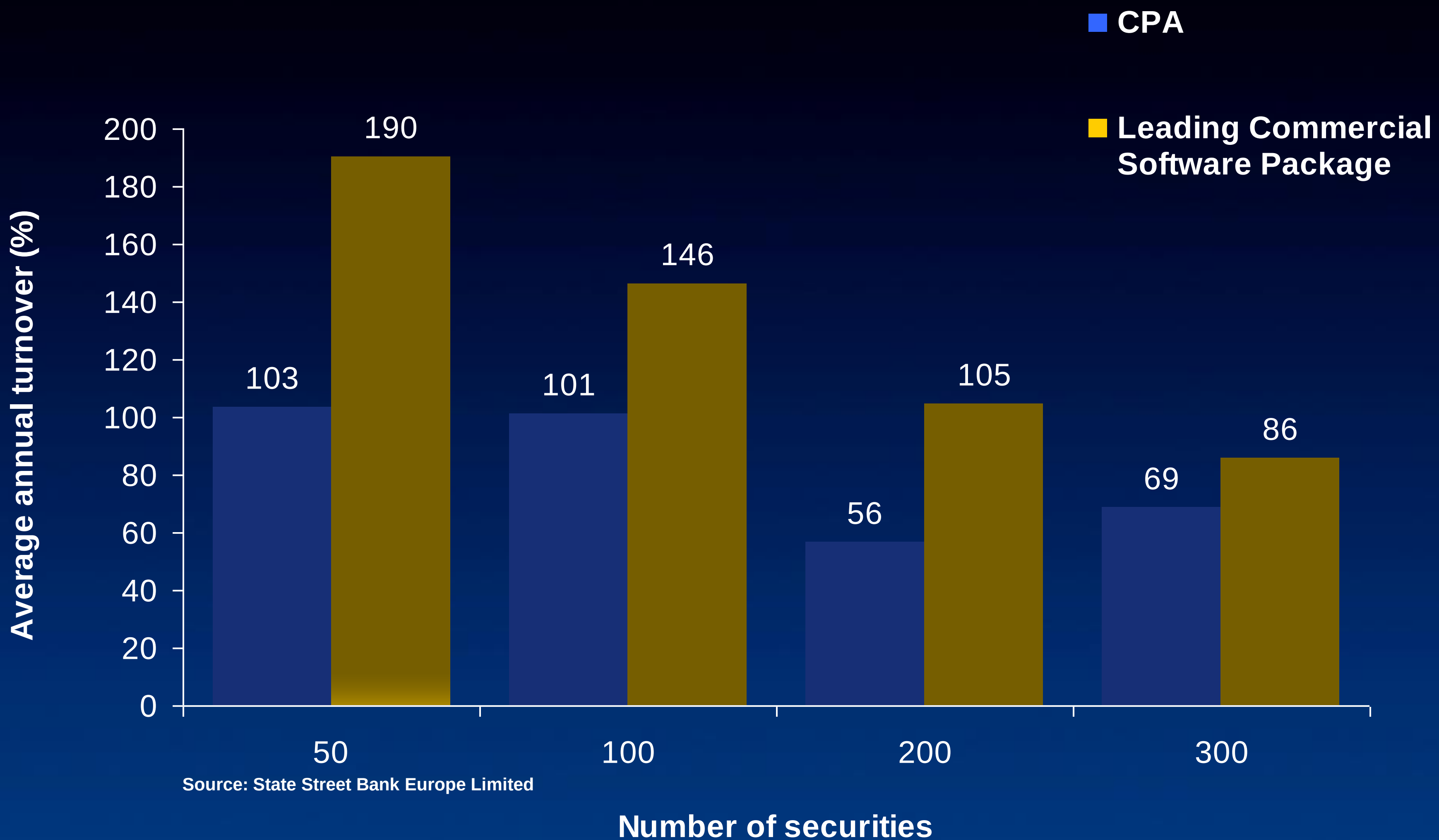
Out-of-Sample Results: MSCI Pacific Small Cap*



Source: State Street Bank Europe Limited

* Dec 2002 – Aug 2009, quarterly rebalancing, 252 days of historical returns.

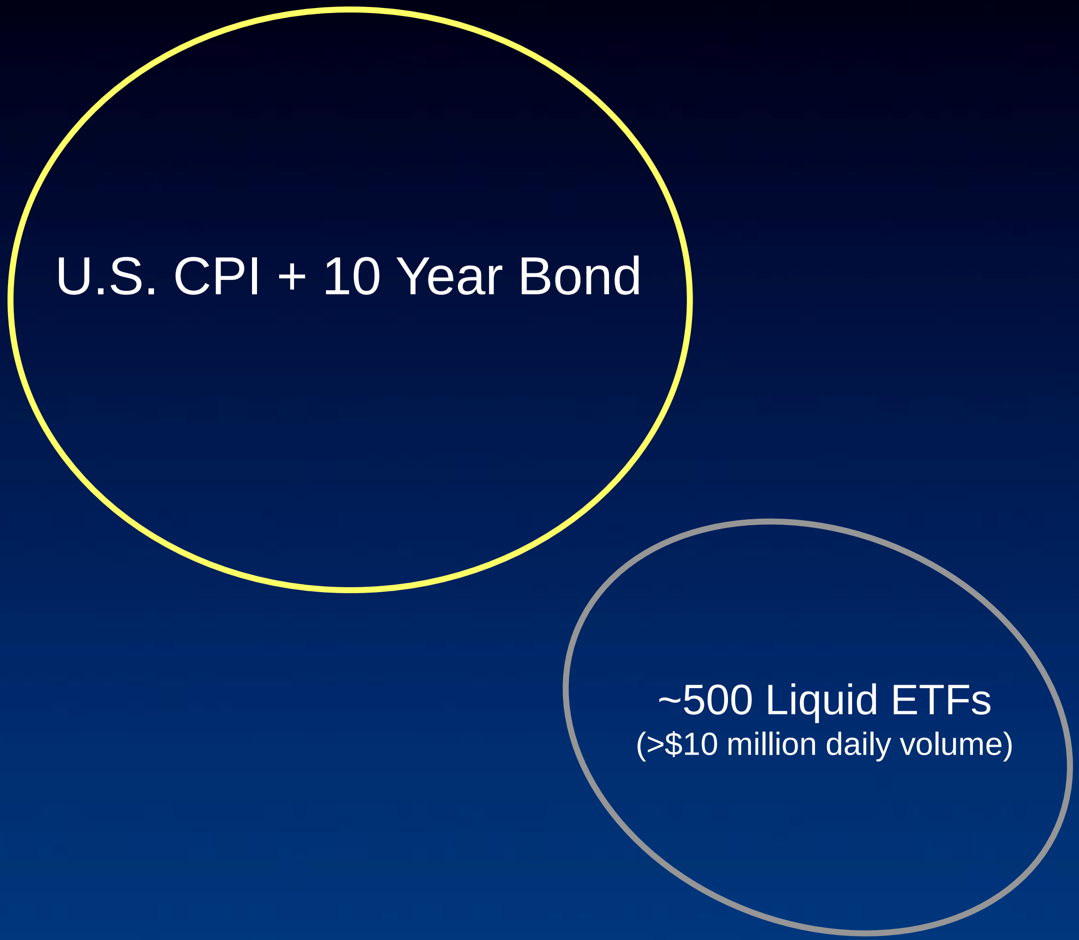
Out-of-Sample Results: MSCI Pacific Small Cap*



Source: State Street Bank Europe Limited

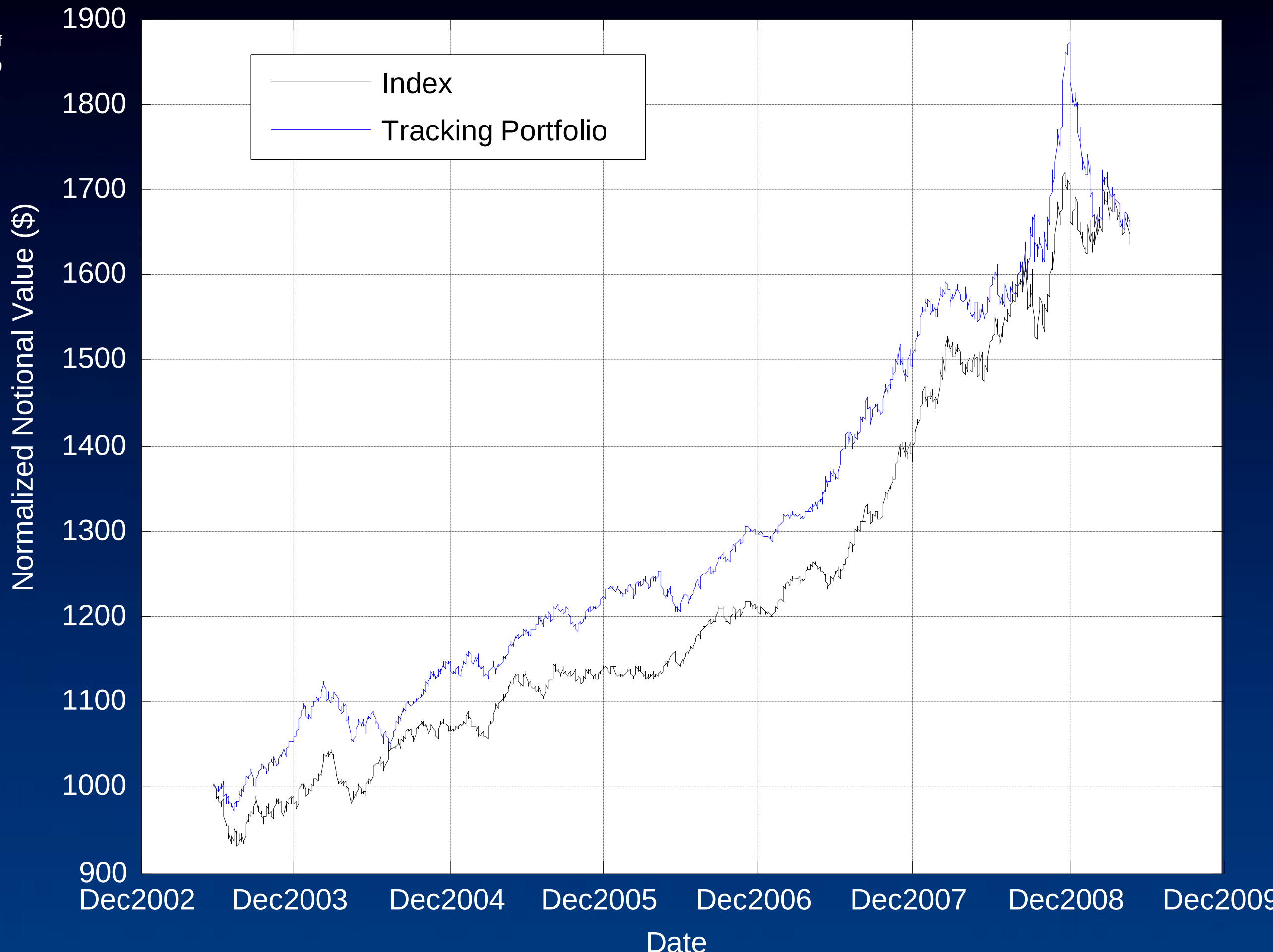
* Dec 2002 – Aug 2009, quarterly rebalancing, 252 days of historical returns, average annual Index turnover is ~100%.

U.S. CPI + 10 Year U.S. Treasury Bond



Out-of-Sample Results: Jul 2003 – May 2009 U.S. CPI + 10 Year Bond: Monthly Rebalancing

ETF	Weights as of 25 May, 2009
ISHARES BARCLAYS AGENCY BOND	18.83%
SPDR BARCLAYS CAPITAL LONG	8.83%
ISHARES BARCLAYS 1-3 YEAR TR	14.41%
POWERSHARES DYNAMIC OTC PORT	3.87%
HORIZONS BETAPRO S&P TSX CAP	7.69%
HORIZONS BETAPRO S&P/TSX CAP	3.04%
POWERSHARES DYN FINANCIAL	2.44%
SPDR METALS & MINING ETF	2.84%
FIRST TRUST LARGE CAP VALUE	4.35%
WISDOMTREE SMALLCAP DVD FUND	2.23%
WISDOMTREE LARGECAP VALUE FU	2.20%
DB X-TRACKERS QUIRIN WEALTH	2.40%
ETFS LVRG SUGAR	3.46%
DB X-TRACKERS S&P/ASX 200	2.36%
XACT DERIVAT BULL	2.85%
ISHARES EBREXX JUM PFANDB DE	7.36%
ISHARES EB REXX MONEY MARK-A	3.26%
KSMFN	2.63%
EASYETF FTSE ET50 ENVIRONMNT	2.41%
NEXT FUNDS TOPIX-17 WHOLESAL	2.54%
Sum of weights	100.00%
Max weight	18.83%
Min weight	2.20%
Tracking Error (bps)	567
Index Correlation	0.62
Alpha	275
Beta	0.76



Source: State Street Bank Europe Limited

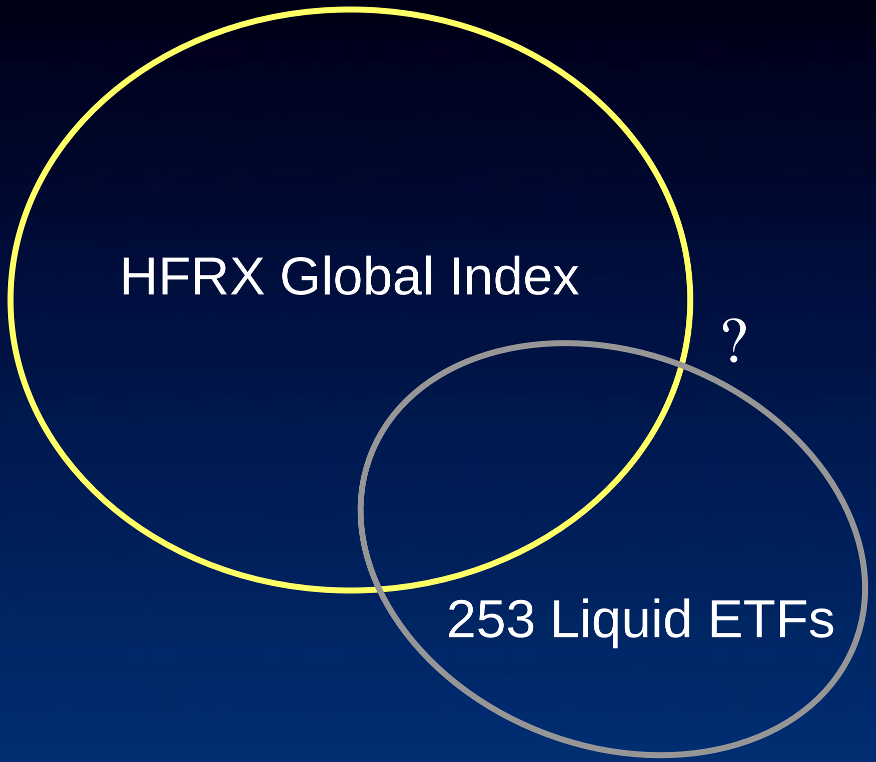
Key Application Advantages

- **Carrier portfolio grows (at least) with the rate of the CPI, i.e., it can efficiently insure against inflation**

- **Long-term (upper yield curve) returns are attainable without long-term capital commitment, i.e., investment are highly liquid**

- **Investments are fully transparent**

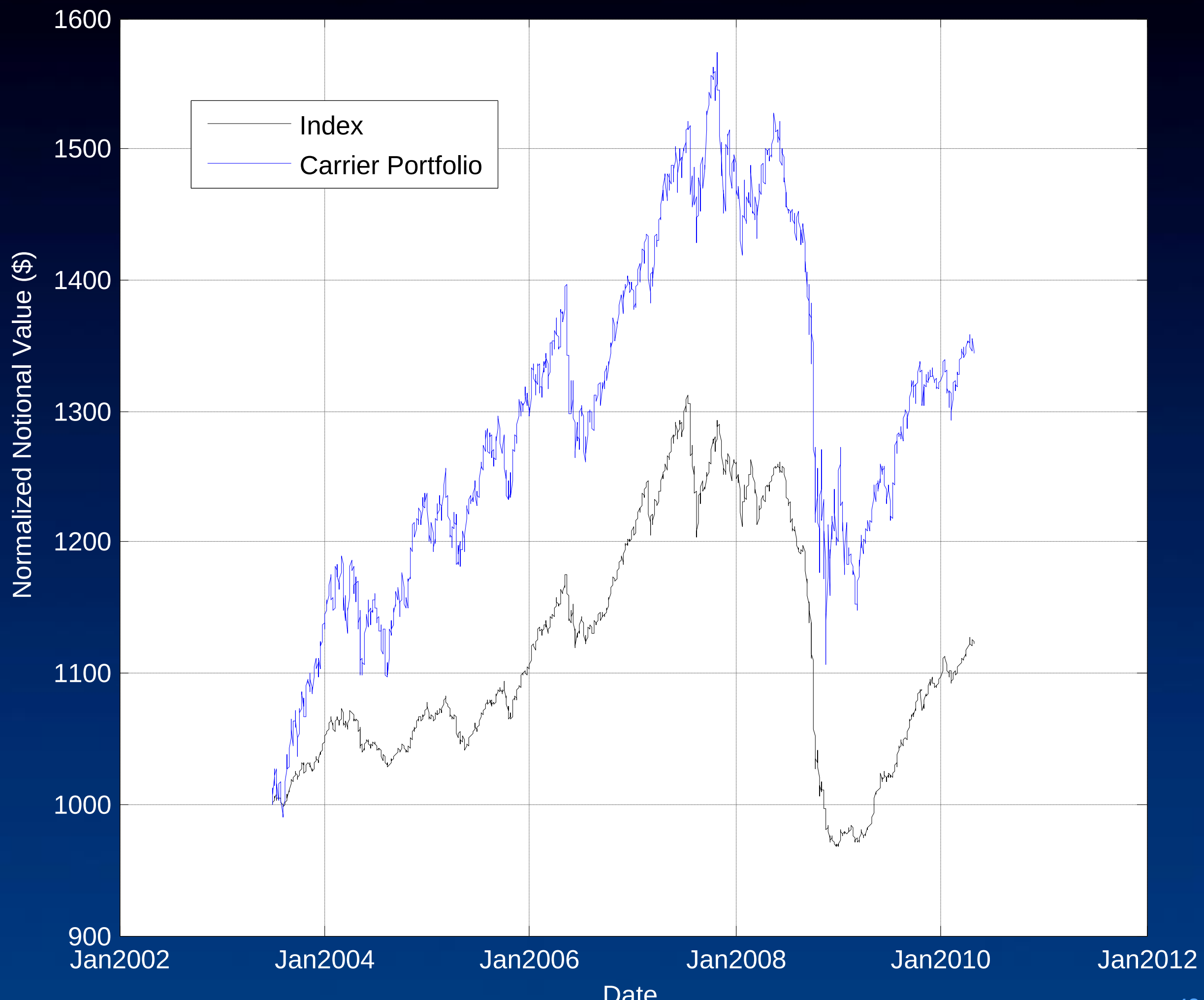
The HFRX Global



Out-of-Sample Results: Jul 2003 – April 2010
HFRX Global Index: Monthly Rebalancing

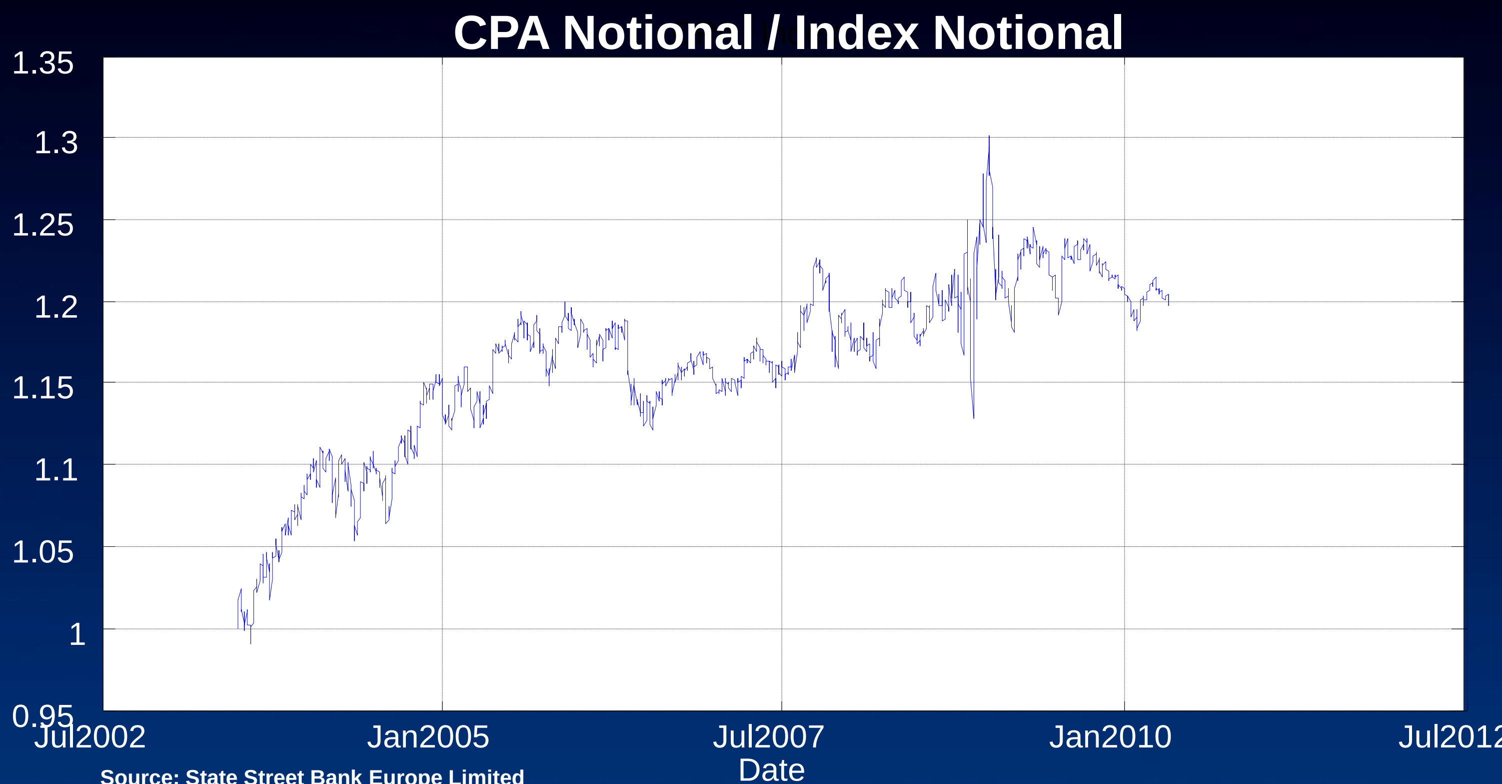
April 30, 2010

Asset Names	Weights (%)
iShares Barclays Short Treasur	13.18
PowerShares DB USD Index Bulli	13.18
SPDR Lehman 1-3 Month T-Bill E	10.18
iShares Barclays Credit Bond F	8.11
WisdomTree Dreyfus Chinese Yua	7.61
iShares Dow Jones Select Divid	6.87
PowerShares DB Commodity Index	5.65
SPDR Lehman Municipal Bond	5.52
iShares Morningstar Mid Value	5.12
iShares MSCI Japan Index Fund	4.83
iShares MSCI Germany Index Fun	4.48
iShares Nasdaq Biotechnology	3.97
Vanguard Long-Term Bond ETF	3.84
SPDR KBW Insurance ETF	3.84
HOLDRS Merrill Lynch Market Oi	3.63
Sum of Weights	100.00



Source: State Street Bank Europe Limited

Out-of-Sample Results: Jul 2003 – May 2010
HFRX Global Index: Monthly Rebalancing



Source: State Street Bank Europe Limited

Tracking Error (bps)	Correlation	Beta	Alpha (bps)	Mean Return	Volatility	IR	Hit Ratio
597	0.75	1.07	230	4.03	8.83	0.43	1.13
				1.48 **(Index)**	*6.34* **(Index)**	*0.23* **(Index)**	

Source: State Street Bank Europe Limited

Key Application Advantages

- **Carrier portfolio grows (at least) with the rate of the HFRX Global.**

- **Hedge fund-like returns are attainable without large entrance and management fees, no gating of invested funds ,i.e., investment process is less restrictive and more efficient.**

- **Carrier portfolio is fully transparent and liquid.**

Conclusions

- **The Carrier Portfolio Approximation, an innovation in portfolio construction, can be employed to perform large-scale optimizations involving thousands of assets spanning all asset classes and independent of a risk model.**

- **Client projects have demonstrated the efficacy, novelty and superiority of the CPA technology for numerous US, European and Asian Equity Indexes, fixed-income (debt) indexes, and macro economic factors and emerging markets.**