Extending Factor Models of Equity Risk to Credit Risk and Default Correlation

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Goals for this Presentation

- Illustrate how equity factor risk models and structural models of credit risk can be linked to provide consistent measures of equity risk, default risk and default correlation
- Introduce a quantitative measure of the “sustainability” of firms
- Describe results in an empirical analysis of all US listed equities from 1992 to present
- Show that the common conception of “sustainable” investing is confirmed in these results
- Illustrate an alternative use of this method as a way to define the level of systemic risk to developed economies

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Basic Contingent Claims Literature

- Merton (1974) poses the equity of a firm as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt
  - Alternatively, lenders are short a put on the firm assets
  - Default can occur only at debt maturity
- Black and Cox (1976) provide a "first passage" model
  - Default can occur before debt maturity
  - Firm extinction is assumed if asset values hit a boundary value (i.e. specified by bond covenants)
- Leland (1994) and Leland and Toft (1996)
  - Account for the tax deductibility of interest payments and costs of bankruptcy
  - Estimate boundary value as where equity value is maximized subject to bankruptcy

Default Correlations

- Hull and White (2001) and Overbeck and Schmidt (2005)
  - You can estimate default correlation if you knew the (unobservable) true interdependence between firms
- Estimate default correlation from asset correlation
  - Zhou (2001) derives default correlations from asset correlation
  - Frey, McNeil and Nyfeler (2005) use a factor model to describe asset correlations
- Include effect of correlation of changes in default boundary to asset correlations
  - Giesecke (2003, 2006)
- Take the easy way out: assume asset correlation is equal to equity return correlation
  - DeSerigny and Renault (2002) provide negative empirical results
  - CreditMetrics, Hull and White (2004)
  - Close if leverage levels are low and horizons are short

Equity Return Properties Help Out

- Defaults are usually rare events so it's impossible to directly observe default correlations over time
- The book value of firm assets is a very incomplete measure of firm assets, so observing asset volatility and asset correlations across firms are very weak estimates
- Equity return volatility and correlation are readily observable
- Zeng and Zhang (2002) shows asset correlations must arise from correlation of both equity and debt components
- Qi, Xie, Liu and Wu (2008) provide complex analytical derivation of asset correlations given equity return correlation

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Bring on the Factor Models

- If you have an "equity only" factor model
  - Estimate pair-wise correlations for equity returns
  - See diBartolomeo 1998 for algebra
  - Convert to asset correlation using method of Qi, Xie, Liu and Wu
- If you have a "multi-asset class" factor model you can use the fundamental accounting identity to get a factor representation of asset volatility and equity
  - Assets = Liabilities + Equity
  - Asset volatility is just equity volatility de-levered, adjusted for covariance with the market value of debt
  - When interest rates rise equity values usually drop, but market value of debt definitely declines, reducing leverage
  - Convert to pair-wise asset correlation values

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In Theory, We're Ready to Go

- With asset volatility and correlations estimated we can use our preferred structural model to estimate default probability of a firm
- Use method from Zhou to convert asset correlations to default correlations
- We can now produce joint default probabilities across firms
- However there are some pretty restrictive assumptions
  - Firm must have debt today
  - Firm must have positive book value today
  - Balance sheet leverage must stay fixed in the future

Reverse the Concept: Sustainability

- Instead of trying to estimate how likely it is that firm goes bankrupt, let's reverse the logic
- We will actually estimate the "market implied expected life" of firms using contingent claims analysis
- Firms with no debt can now be included since it is possible that they get some debt in the future and default on that
- A quantitative measure of the fundamental and "social" concept of *sustainability*

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Our Basic Option Pricing Exercise

- Underlying is the firm's assets with asset volatility determined from the factor model as previously described
- Solve numerically for the "implied expiration date" of the option that equates the option value to the stock price
  - *Market implied expected life of the firm*
- Include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately
- If you choose Black-Scholes as your option model, then you can solve BS for the implied time to expiration using a Taylor series approximation
- More complex option models allow for stochastic interest rates

Filling in with “Distance to Run”

- For firm’s with no debt or negative book value, we simply assume that non-survival will be coincident with stock price to zero, since a firm with a positive stock price should be able to sell shares to raise cash to pay debt
  - If you have a stock with 40% a year volatility you need a 2.5 standard deviation event to get a -100 return
  - Convert to probability under your distributional assumption
- We convert both measures to the median of the distribution of future survival in years
  - What is the number of years such that the probability of firm survival to this point in time is 50/50
  - Highly skewed distribution so we upper bound at 300 years
- Z-score the “median of life” for both measures and map the distance to run Z-scores into the “option method” distribution for firms with no debt

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Empirical Study Design

- Use a simple Merton model (Black-Scholes European put)
- Use equity volatilities from Northfield US Fundamental Model
  - One year horizon for risk forecast
  - Near horizon” model are more suitable but less history available
- Estimate monthly for all firms in Northfield US equity universe from December 31, 1991 to March 31, 2010
- Study three samples:
  - All
  - Financial firms
  - Non-financial firms
- Sources of Time series variation
  - Stock prices, debt levels, Northfield risk forecasts
  - Mix of large and small firms, 4660 <= N <= 8309

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Let’s Start at the End (March 31, 2010)

- Current life expectations for all (5068) firms in years
  - Median 23, Mean 22.18, Cap Weighted 25.71
- Financials firms only (1132)
  - Median 24, Mean 21.69, Cap Weighted 18.95
  - Surprising (or maybe not) cap-weighted is a lot lower
- Non-Financials (3936)
  - Median 23, Mean 22.33, Cap Weighted 27.36
- Highlights:
  - AIG 7, Citicorp 6, GS 6
  - IBM 30, MSFT 32
  - RD 30/39, XOM 54

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Time Series Properties Full Sample

- Calculate the cross-sectional mean, cap weighted mean and median for 220 months, average sample = 6587
  - Time series average of the monthly medians, 21.63 years
  - Time series average of the monthly means, 24. 42
  - Time series average of cap weighted means 22.66
- Lowest expectations, January 1992
  - median 10, mean 13.20, cap weighted mean 11.05
- Highest expectations, January 2005
  - median 30, mean 41.09, cap weighted mean 32.36

Time Series Properties Sub Samples

- Financials (average sample size = 1630)
  - Time series average of the monthly medians, 31.03
  - Time series average of the monthly means, 31.51
  - Time series average of cap weighted means 24.09

- Non Financials (average sample size = 4955)
  - Time series average of the monthly medians, 20.03
  - Time series average of the monthly means, 22.13
  - Time series average of cap weighted means 22.23

- Note that for the full time series, financial firms were expected to survive about 50% longer than non-financials
  - At last date, financials have slightly lower expected lives

Another Angle on Default Correlations

- Once the time series of expected lives have been calculated, we can estimate default correlation as the correlation of percentage changes in expected lives across firms
- As expected lives shorten, changes of a given magnitude become larger percentage changes
  - Since correlation is a bounded function (-1 to +1) larger events drive the correlation values toward the extreme value
  - Two bonds that have one day of expected life each will have a very high default correlation
- Better than trying to correlate OAS spreads since bond prices are driven by liquidity effects

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Quantifying "Sustainability"

- FTSE/KLD DSI 400 index of US large cap firms considered socially responsible, 20 year history
  - Typically about 200 firms in common with the S&P 500

- July 31, 1995
  - DSI 400, Median 17, Average 17.91, Standard Deviation 9.93
  - S&P 500, Median 14, Average 15.40, Standard Deviation 9.28
  - Difference in Means is statistically significant at 95% level
- March 31, 2010
  - DSI 400, Median 30, Average 26.39, Standard Deviation 11.45
  - S&P 500, Median 30, Average 24.93, Standard Deviation, 10.92
  - Difference in Means is statistically significant at 90% but not 95%

- Testing on Disjoint Sets (DSI NOT S&P, S&P NOT DSI)
  - Statistically significant difference in means for every time period tested

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A Measure of Systemic Risk?

- Obviously, if the market thinks public companies are not going to be around very long, the economy is in a bad way
- Low equity valuations and high leverage equate to short life expectancy
  - Higher leverage can be sustained with higher growth rates that cause higher equity valuations
- We propose "revenue weighted" expected average life as a measure of systemic stress on an economy
  - By revenue weighting we capture the stress in the real economy
  - Avoids bias of cap weighting since failing firm's have small market capitalization and don't count as much

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Next Steps

- Use more sophisticated option pricing model that allows for stochastic interest rates and possibly stochastic volatility

- Use expected life data at the firm level to predict changes in credit ratings
  - We have hand collected (copied from Barron's week by week) every credit rating down grade and upgrade since 1991
  - Relate changes in expected life to subsequent rating changes
  - Relate expected life values that are outliers within their rating category to subsequent rating changes
  - Adjust credit risk expectations for bond issuers and financial counterparties in our fixed income risk model

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Conclusions

- Combining factor models and structural models of credit risk allows for consistent estimation of equity risk, credit risk and default correlation
- Structural models based on contingent claims methods are a direct and informative approach to assessing the expected survival of firms
- Comparison of SRI and conventional US stock indices reveals a positive and significant difference in expected lives, confirming the existence of "sustainability"
- We believe this technology will have usefulness as a measure of systemic risks in developed economies

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