INCORPORATION OF LIQUIDITY RISKS INTO EQUITY PORTFOLIO RISK ESTIMATES

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GOALS FOR THIS TALK

- Assert that liquidity of a stock is properly measured as the expected price change, given a proposed trade of particular size in a particular time frame

- Propose a conceptual framework of equity security risk that considers price movements as the accumulation of market impact over a set of transactions

- Describe our estimation of market impact from tick data, under boundary conditions

- Derive a joint measure of equity risk and liquidity risk that accounts for the greater liquidity risk of larger portfolios

A MICROSTRUCTURE VIEW OF EQUITY RISK

- The process of price discovery for stocks arises from balancing the supply available from willing sellers to the demand from willing buyers
- Security returns are just the accumulation of the price impacts of the individual trades within that time period
- The volatility of returns manifests changes in the imbalance between buyers and sellers over time, but not all observed returns are informative
- If we can forecast the potential for imbalances to arise, we can forecast equity risk more efficiently
- The larger our own portfolio is, the greater the potential for we ourselves to create imbalances and contribute to risk

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WHAT MAKES PEOPLE BUY OR SELL A PARTICULAR STOCK?

- They WANT to trade the stock
  - They believe the information that supports a valid forecast of abnormal future return
- They HAVE to trade the stock
  - They are trading to implement a change in asset allocation
  - They are trading to implement a cash versus futures arbitrage trade on a stock index
  - They are a mutual fund or ETF sponsor responding to investor cash flows in or out of the portfolio
  - They are hedge fund that is forced to transact because of a margin call
  - They are forced to cover a short position by having the stock called

THE POTENTIAL FOR "HAVE TO'S"

- We can fundamentally evaluate the potential for "have to" trades
  - Index arbitrage trades only occur with index constituents and we know the open interest in futures
  - Short interest information is published
  - We know which big hedge and mutual funds have big positions in particular stocks
  - We have somewhat out of date information on full mutual fund holdings and cash flow statistics
  - We have up to date information on ETF flows

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THE POTENTIAL FOR "WANT TO" TRADES

- Investors are responding to information, so just measure variations in the volume of information about a particular stock over time
- Capture the flow of news text coming over services such as Dow-Jones, Reuters and Bloomberg
  - Ravenpack and Thomson Reuters offer **real time statistical summaries** of the amount and content of text news distributed
  - Lexicons of popular phrases are used to score the content of text as "good news" or "bad news"
  - See diBartolomeo, Mitra and Mitra (2009)
- Judge retail investor interest directly by measuring the number of Google and Yahoo searches on ticker symbols
  - Da, Engleberg and Gao (2009)
  - Avoid company names to eliminate product or service related searches
  - Try it yourself with Google Trends

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IMPROVING SHORT HORIZON RISK ESTIMATES

- Price impact generated by “have to” trades in individual stocks is transient, but can become permanent if sufficiently widespread to impact the real economy
- Price impact generated by “want to” trades arise from informed traders and is likely to be long lasting
- We should be able to observe larger and more dispersed imbalances between buyers and sellers for stocks that are larger in potential for “have to” and “want to” trades
- We now have a new way to calibrate equity risk estimates
  - Observed variations in returns (i.e. the usual way)
  - Observed time series variation of imbalances between buyers and sellers

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MARKET IMPACT 101

- Most market impact models are of the form:

$M = \delta S^{\pi}$

Where δ is a stock specific coefficient of illiquidity
S = trade size in shares or in % expected ADV
M = expected market impact in %
$\Pi =$ exponent defining the process, almost all market impact models assume Π to be either .5 or 1

Our systems allow for a weighted combination of both processes

$M = w \delta S + (1-w) \delta S^{.5}$
$b = w \delta$
$c = (1-w) \delta$

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MEASURING BUYER/SELLER IMBALANCE

- We adopt the model of Lee and Ready (1991) for our definition of "buyer" and "seller" trading volume
  - Assumes that if a trade occurred on an uptick in price this was a "buyer" trade that was accommodated by a liquidity provider
  - If a trade occurs on a downtick in price, assume that this was a "seller" trade that was accommodated by a liquidity provider
  - If a trade occurs on a flat tick (no price change) it is assumed to be of the same character as the previous trade

- Define "imbalance"

  S = (buying volume – selling volume) / total volume

- Accumulate daily volume imbalances

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THE PROBLEM WITH TICK DATA

- Our data consists of every tick for the past fourteen years of trading on a globally representative sample of five thousand stocks classified into about one hundred thirty groups by country and market capitalization.   The data is obtained from Reuters

- It must be noted that the Reuters data is organized by RIC code, so if a stock is traded at more than one venue (e.g. multiple exchanges, ECNs), the trading at each venue will be treated separately and may require consolidation. The data set also includes every quote

- *The combined size of the entire Reuters tick data set for all securities worldwide approaches three hundred terabytes.* The computational effort associated with using the tick data method is considerable!

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TIMES SERIES COST ESTIMATION

- Estimate via three rank time series regressions where daily return is the dependent variable using a three month time window

  $R\% = b\,S + c\,S^{.5}$

  We estimate b directly. We do two separate estimations for c for positive and negative imbalances

- Use Bayes's Theorem to combine the three regression coefficients
  - Obtain w, δ, Π

- While the tick method is normally estimated based on percentage imbalance of daily volume, we can rescale the imbalance values by the relationship of average daily volume to shares outstanding to switch to cost per share units

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REFINEMENT #1: BOUNDARY CONDITIONS

- Constrain the range of the regression coefficients with boundary conditions
  - Market impact comes from information leakage. Other people figure out what you are doing and change their trading from their knowledge of your intent
  - The worst case scenario for information leakage is a hostile takeover. You are going to buy up all the shares of a firm and publicly announce you are doing it.
- The coefficients should be constrained to produce maximum buying costs similar to the expected premium in a hostile takeover for a trade of all shares outstanding (typically 25 to 70%)
- The maximum cost of a "sell" cannot exceed 100% even for a trade of all shares outstanding

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REFINEMENT #2: LARGE TRADES

- The distribution of trade size within a day often has a large degree of skew, leading to unintuitive results
  - CRH PLC (Ireland) on 14 May 2009
  - Volume imbalance -30.25% but total return is +3.3 so the relationship has the wrong sign
  - Approximately two million shares trade in 722 trades
  - Median trade size is 400 shares, average trade size is 2783 shares, *maximum trade size is 300,000 shares*
  - Large sells are much more frequent than large buys in the data
- Solution is to separate out very large institutional trades from the sample and estimate separately
  - Measure instantaneous impact as change from the previous trade price, accumulate over large trades only
  - Define "large" trades" being > 2% daily volume <u>and</u> having a Z-score > X (e.g. 2) in the log of shares traded per trade for that day

REFINEMENT #3: USE QUOTES

- Confirm the classification of "buyer" and "seller" initiated trades by comparing execution prices to the previous bid-asked quote
- Transactions occurring at or above the previous asking price are assumed to be initiated by a buyer and accommodated by a liquidity provider
- Transactions occurring at or below the previous bid price are assumed to be initiated by a seller and accommodated by a liquidity provider

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REFINEMENT #4: ADJUST FOR SHIFTS IN VOLATILITY

- Our times series estimates of coefficients b and c are based on data for the three month trailing months
  - To the extent we know believe that a given stock is expected to more volatile now than it was on average during the sample period, we should adjust the value of b and c to reflect the relationship between volatility and expected market impact

$$b_t = b * (S_t / AVERAGE[S_{t-60 \text{ to } t-1}])^K$$

$$c_t = c * (S_t / AVERAGE[S_{t-60 \text{ to } t-1}])^K$$

Where

S(t) = expected volatility of stock x at date t

K = w + .5 (1-w)

We can use this relationship to estimate costs in crisis conditions from the concurrent volatility spikes

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ALTERNATIVE ESTIMATION OF COSTS

- We can also estimate a factor model of market impact

  E_{it} = % price change / % imbalance for stock i on day t
  $E_{it} = b_{1t}\,\sigma_{it} + b_{2t} \log M_{ci}$ + other factors

  Where
  E_{it} = price elasticity of stock i on day t
  σ_{it} = forecast volatility of stock i on day t
  Mci = log of total market cap of country c of stock i

Separate information into four regressions: positive and negative imbalances each assuming $\pi = 1$ and = .5
Combine regressions using Bayes's Theorem
Take 20 day volume-weighted average across time
This model will be available daily for a fee starting July

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SAVING YOU A LITTLE MONEY

We can also use the existing Northfield cost model that has been provided free on a monthly basis with all equity risk models, since May 2009

This model estimates illiquidity for all stocks globally based on observable serial correlation in daily returns.

The higher the cost of trading a stock, the lower the number of sign changes in a daily return times series will be.

Implied serial correlation was calibrated against a database of more than 1.5 million anonymous actual trade costs

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THE NUMBERS ADD UP

- If we believe in linear factor risks and we also believe in price risk as the accumulation of market over many trades we would expect that the exponent on the market impact formula should be close to one for trading over fixed time intervals

- This means that the "square root" aspect of market impact arises from traders stretching out large trades over long time horizons to reduce costs

- Strong empirical evidence supports this hypothesis
  - See "Market Impact Monthly" research reports from JPMorgan

THE FAULT DEAR BRUTUS LIES NOT IN OUR STARS BUT IN OURSELVES

- Conventional portfolio risk calculations deal only in security weights
  - Ignores the magnitude of portfolio value
  - Ignores the potential for our own “have to” trades
  - Theoretical discussion in Acerbi and Scandolo (2007)
  - Assets have prices, but portfolios have values that are functions of a liquidity policy
- Given the nature of our own portfolio, we can estimate the potential for our own “have to” trades
- Certain types of “have to” trades are likely to highly correlated across managers of similar style and portfolio composition because they are pro-cyclical
  - October 1987 with portfolio insurance and index arbitrage
  - August 2007 with hedge fund margin calls

LIQUIDITY POLICIES AND RISK

- We can formulate a liquidity policy as:
  - We have to be able to liquidate X% of the portfolio in N trading days
- Given our models of cost, we can estimate the cost of liquidation
- To adjust our portfolio risk estimate for liquidity
  - Convert our portfolio volatility estimate to parametric Value at Risk for the length of time specified in our liquidity policy
  - Add the expected cost of fulfilling liquidation to VaR
  - Convert the new VaR value back to the equivalent volatility

QUICK EXAMPLE

- Our liquidity policy:
  - We must be able to liquidate 30% of the portfolio in 10 trading days
- Our estimated portfolio volatility is 25% per year
  - Assume 3 standard deviation VaR (covers 99.8% of normal distribution)
  - % Parametric VaR = 14.94 [25 * 3 * (10/252)^.5]
  - Assume our forecast liquidation cost is 4%
  - % Parametric VaR with Cost = 18.94 [14.94 + 4]
  - Revised portfolio volatility = 31.70 [18.94 / 3 * (252/10)^.5]
- Volatility estimate increased by more than 23%

ALTERNATIVE FORMULATION: LIQUIDITY DRAG

- As positions in a portfolio get larger and larger, executing trades at reasonable cost will take longer and longer, reducing opportunity to earn active returns
- An extreme case is fully illiquid investments such as real estate or private equity
  - You put the first dollars in the “best” deal, the next dollars in the second best deal and so on
  - The more money you allocate to an asset, the lower the expected returns become
- Experiment on a small child
  - Send them to the candy store with a large amount of money. They will buy all sorts of stuff and end up with an upset stomach
  - Send them to the candy store with a small amount of money. They will buy just their favorite candy and be happy

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LIQUIDITY RISK FROM LIQUIDITY DRAG

$U = \Sigma\, w_i\alpha_i\,(1\text{-}k_iw_i) - \sigma^2 / T$

K_i = illiquidity coefficient estimated from the cost model

α_i = absolute expected return

(we need to always adjust returns downward)

$U = \Sigma\, w_i\alpha_i - \Sigma\, k_i\alpha_i w_i^2 - \sigma^2 / T$

Note that the second term in yellow is proportional to the summation of the squared position weights, identical to the formulation of asset specific risk in our factor risk models.

In effect, we have added $k_i\alpha_i$ to each specific risk value in our risk model, providing an alternate measure of liquidity risk

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CONCLUSIONS

- Liquidity risk is a critical issue for most investors, particularly those that are either very large or are leveraged
- Estimation of trading costs associated with liquidity needs can be efficiently accomplished through our tick based model, as well as other models of cost
- Variability in buyer/seller imbalance information can be used as a metric for short horizon risk
- We have presented two methods for including liquidity risk into portfolio risk estimates