Northfield Portfolio Optimization

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Talk Objective

- Intended to:
  - Illustrate optimization inputs
  - Provide Northfield specific functionality
  - Provide a lot of reference resources
    - Talk with links will be provided

- Not intended to:
  - Teach Modern Portfolio Theory
  - Critique optimization techniques
  - Provide in-depth research (since it covers an exhaustive list of functionality)
Overview

- Review Modern Portfolio Theory
  - Risk-return trade off
  - Efficient frontier
- Utility function
  - Measure of portfolio optimization
  - Inputs
- Conveying portfolio preferences
  - Constraints
- Optimization process
Risk-Return Tradeoffs

- Maximize Return
  \[ E(r_p) = w_1 \cdot E(r_1) + (1 - w_1) \cdot E(r_2) \]

- Minimize Risk
  \[ \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho \]

- Combining assets with correlations (\( \rho < 1 \)), risk is diversified
  - Provides a more efficient tradeoff of risk
  - Example: A combination of Assets 1 & 2 exist that could have the same risk as Asset 1 with a greater level of return
Efficient Diversification

Expected Return

P = (\text{A2}=25\%, \text{A1}=75\%)

Asset 1

Asset 2

Variance
Efficient Portfolios

- Consider entire investable universe of assets
- Goal: weight assets to construct most efficient portfolios
  - Greatest return for any given level of risk
  - Lowest risk for any given level of return
- Minimum Variance Portfolio
  - Portfolio with lowest possible risk given investable universe
- These portfolios defines the Efficient Frontier
Efficient Frontier

- Expected Return
- Minimum Variance Portfolio
- Efficient Frontier

\[ \text{Expected Return} \quad \text{Minimum Variance Portfolio} \quad \text{Efficient Frontier} \]

\[ \text{Rp} \quad \text{Variance} \]
Optimization

- Determines combination of assets to best suit investor preferences on Efficient Frontier
  - Mean-Variance Utility function measures optimal trade-off
    - \( Utility \sim f(+\text{return}, -\text{risk}, -\text{cost}) \)
  - Constant utility line measures investor’s indifference between the risk free asset and risky asset paying a premium which compensates for risk
    - \( Utility = R_f = R_p - R_{Risk \ Pr emium} \)
  - Optimal portfolio is intersection between highest attainable constant utility curve and Efficient Frontier
Optimal Portfolio

- Expected Return
- Efficient Frontier
- $\sigma^2_{\text{Optimal}}$
- $R_p$
- $R_f$
- Optimal
- Risk Premium
- Variance
Mean-Variance Utility Function

Max U = \( \alpha - \left( \frac{\sigma_s^2}{\text{RAP}_s} \right) - \left( \frac{\sigma_u^2}{\text{RAP}_u} \right) - ((C + T) \times A) - P \)

- \( \alpha \) = Forecasted portfolio return
- \( \sigma_s^2 \) = Portfolio variance due to common factors and correlation
- \( \sigma_u^2 \) = Portfolio variance due to stock specific risk
- \( \text{RAP} \) = Risk acceptance parameter
- \( C \) = Transaction costs for the optimization
- \( T \) = Capital gain taxes for the optimization
- \( A \) = Amortization constant
- \( P \) = Quadratic penalty cost
Caveat, Already?

- Frequent criticism of mean-variance optimization is the possibility of error maximization
    - Inputs treated as certain values
    - Incorrect estimates
    - Unrealistic estimates (30% alphas)
- Northfield has introduced a series of functionalities to mitigate the impact of misspecified inputs
  - Adjusts inputs to account for estimation errors
  - Shrinks inputs to more realistic values
Alpha

- Index portfolio, \( E(R) = 0 \Rightarrow \) Risk minimization
- Active portfolio, \( E(R) \) scale benchmark-relative
  - Huge financial literature exist on alpha construction involving strategy dependent subtleties
- Reshape as Cross-Sectional Forecast:
  - Converts forecast to standard units compatible to the optimizer
    - \( \alpha = IC \times CS\_VOL \times STD\_SCORE \)
Misspecified Alpha

- Northfield provides tools blunting the impact of misspecified alpha using Bayesian adjustments:
  - Bayes Adjust combines user supplied returns with an equilibrium portfolio based on a universe of assets provided
  - Bayes-Stein Shrinkage combines user supplied returns with the expected return of a minimum variance portfolio constructed from the universe of provided assets
Risk

- Northfield risk models provide variance numbers
  - Systematic risk is the market risk measured by the Northfield common factors
  - Unsystematic risk is the risk specific to a security
  - For shorter term risk predictions Northfield provides near-term risk models, which incorporates recent market variability

- Users may choose to use their own models
Errors In Risk Estimates

- Estimation Error Adjustments can mitigate errors:
  - Bayes-Stein Shrinkage combines individual asset risk with the risk of a minimum variance portfolio constructed from the universe of provided assets
  - Blend Covariance Matrix is a combination of covariance matrix with up to three more structured version of itself
    - Single Index, Constant Correlation, Constant Covariance
Risk Acceptance Parameter (RAP)

- RAP is investor’s willingness to trade between risk and return
  - The slope of the constant utility curve
  - Smaller/larger numbers provide more/less weight to risk
  - \( RAP = 1/\lambda \)

- Rational range of values: \( 0 < RAP < 200 \)
  - Zero would represent a totally risk averse investor (hide the money under the mattress)
  - 200 represents an investor that only cares about maximizing long term growth

- Default value = 100
  - Historically induced investors to be invested at the long term total risk/return (RAP = 133 ~ \(\lambda = .0075\))
RAP & The Efficient Frontier
Quantifying RAP

- Two good rules of thumb
  - In absolute risk terms, RAP approximately 2 times investor’s net worth as a percentage of total assets. This will vary over the investor life cycle
    \[ RAP = 2 \times \left( \frac{A - L}{A} \right) \]
  - In benchmark relative terms, RAP approximately 6 times the desired tracking error
    \[ RAP = 6 \times E(TE) \]
  - For additional information see:
Transaction Cost

- Transaction costs negatively impact utility
  - Increased utility must outweigh cost incurred

- Such cost can be broken into the
  - Linear: per-share cost known by the manager (i.e. $.05/trade)
  - Non-Linear/Market Impact: cost movement due to trades by the manager and other market participants perception of information due to trades in the assets
  - Cross Sectional: cost impact on similar assets
Transaction Cost Function

\[ T = A + \left( \frac{B_i}{G^R} \right) S + \left( \frac{C_i}{(G^{0.5R})} \right) S^{0.5} + D \times \text{Max} \left[ S - L, 0 \right] + \text{Max} \left[ Z, -A \right] \]

- \( A \) = per-share cost
- \( B_i \) = the coefficient on the linear process
- \( C_i \) = the coefficient on the square root process
- \( S_i \) = the number of shares to be traded
- \( G \) = number of days required for the trade (note fractional days are permissible)
- \( R \) = the proportion of temporary versus permanent market impact is the impact decay and is usually \( R \leq 1 \). Empirical evidence suggests .71.
- \( D \) = the coefficient on the quadratic process
- \( L \) = threshold of traded assets at which the quadratic process is invoked
- \( Z \) = cross market impact coefficient

- Northfield provides monthly Market Impact Model (B and C)
Non-Linear Transaction Cost

- Impact of quadratic function at the threshold
- Hypothetical continuation of the cost curve without threshold

Slope increases at a decreasing rate due to larger trades being made

Transaction Cost vs. Shares Traded

Threshold
Taxes

- Tax cost is incorporated into the utility function as an additional cost allowing users to extract “tax alpha”
  - Losses increase an asset’s relative utility to sell
  - Gains decrease an asset’s relative utility to sell
  - Asset lots are traded in order from least taxed to most taxed lots, unless prohibited by wash sale rules
  - Long-term and short-term gains are offset by long-term and short-term losses
Amortization

- Amortization measures how quickly cost is recognized
  - The default is 100% of transaction cost is incurred within a year
  - For two years, the amortization should be 50
  - For six months, the amortization should be 200

- Increase/decrease the rate of amortization for tax strategy
  - Set a higher year end amortization rate to harvest losses

- The multi-period approximation functionality
  - See “Technical Support Tip: Multiperiod Approximation”,
  - Relaxes the assumption of a one period buy and hold by
dynamically changing the Amortization parameter for a more cost
efficient transition from the initial to the optimal portfolio over
multiple periods
Quadratic Penalty Cost

- Quadratic penalties are accounted for in the utility function
  - TBD, next newsletter

- The further away a portfolio moves from a goal the greater the penalty cost
  - $PenaltyCost = Scale \times (Goal - Current)^2$

- Penalties may be applied to:
  - Industries/Sector where goals are the benchmark weight
  - Any Attributes for which a user can supply data where goals are user defined
Quadratic Penalty

Penalty Cost

Goal: $\beta=0$       Active Beta
Utility Maximization

- Assets are ranked according to Marginal Utility (MU)
  - MU measures the relative utility improvement of each asset
- Trades are made in an iterative process in the direction of increasing utility
  - Assets with highest MU are purchased
  - Assets with lowest MU are sold
  - MU is recalculated after each trade
- Trades continue until maximum utility is reached
  - At this point no one asset can add additional utility (subject to constraints)
  - Optimal portfolio
Marginal Utility

Increase weights for assets on the left $MU_{buy}$
Decreases weights for assets on the right $MU_{sell}$
Subject to Constraints

- Optimization can be run with a variety of constraints
  - Constraint misuse is the largest contributor to failed optimizations

- Hierarchy of constraint classes:
  - Class I constraints are those that are linear combinations of security weights
    - Max asset weight = 0.5%, anywhere between 0-.5%
  - Class II constraints are cardinality constraints applied to the optimal portfolio
    - No longer linear combination, but applied to individual asset(s). Such as max assets.
    - Max asset = 20, but the optimal portfolio has 21. An entire asset is removed despite weight.
Constraint Priority

- **Class I constraints:**
  1. Position limits
  2. Industry and sector limits
  3. Factor Exposure
  4. Attribute File Variables

- **Class II constraints:**
  - Maximum number of assets
  - Maximum turnover
  - Minimum trade sizes
  - Maximum realized capital gain
  - Holding threshold
Optimization in Steps

- **Step 1**, start from a feasible position
  - If conflicts exist among the constraints, the portfolio is “infeasible”
  - The optimizer attempts to find a portfolio that meets all Class I constraints, with the least amount of turnover from the initial portfolio if one exists
  - This feasible portfolio is the real starting point of the process

- **Step 2**, the main optimization loop occurs
  - Goal is to maximize the utility function, MU
  - Subject to class I constraints according to priority or class II stopping criteria (max turnover)
Step 3, post-optimal constraints (threshold loop)
- Class I constraints still have priority
- Class II constraints are applied after maximum utility is achieved
- There is no specific priority among Class II constraints
  - If constraint conflict exist, the optimizer seeks a reasonable compromise based on the objective function inputs
- Class II constraints is where most frequent violations occur
  - Can be violated if they drive the portfolio very far away from the unconstrained optimal state
- No matter which method is used the further away from the unconstrained optimal portfolio the resulting portfolio is the more likely it may be sub-optimal
- Optimal!!!!
Support Issues

- Most support issues are related to optimizations setups, especially constraints.
- Review output reports:
  - Exceptions
  - Warnings in Runtime Messages
- Getting help:
  - Northfield Optimizer Online HELP is very extensive
  - Northfield’s website, [www.northinfo.com](http://www.northinfo.com)
  - Contact Support
Concluding Remarks

- A lot of information has been presented
  - Included links expand on some of the ideas
  - Additional resources:
    - Client Information Guide, TBD

- Future Newsletters:
  - Constraints Priority/Quadratic Penalties
  - Marginal Utility and Optimality
  - Optimization talk
Feedback

Feedback welcome:
- Input on this talk
- Future Newsletter articles of interest
- Other Webinar interest

Email support at:
- US: support@northinfo.com
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Questions & Responses

- Try to get to as many as possible, if unable I will email a response
- Feel free to contact your local support representative for more in-depth questions