Northfield Portfolio Optimization

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Talk Objective

- Intended to:
  - Illustrate optimization inputs
  - Provide Northfield specific functionality
  - Provide a lot of reference resources
    - Talk with links will be provided
- Not intended to:
  - Teach Modern Portfolio Theory
  - Critique optimization techniques
  - Provide in-depth research (since it covers an exhaustive list of functionality)

Overview

- Review Modern Portfolio Theory
  - Risk-return trade off
  - Efficient frontier
- Utility function
  - Measure of portfolio optimization
  - Inputs
- Conveying portfolio preferences
  - Constraints
- Optimization process

Risk-Return Tradeoffs

- Maximize Return
  - $E(r_P) = w_1 * E(r_1) + (1 - w_1) * E(r_2)$
- Minimize Risk
  - $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho$
- Combining assets with correlations (ρ) < 1, risk is diversified
  - Provides a more efficient tradeoff of risk
  - Example: A combination of Assets 1 & 2 exist that could have the same risk as Asset 1 with a greater level of return

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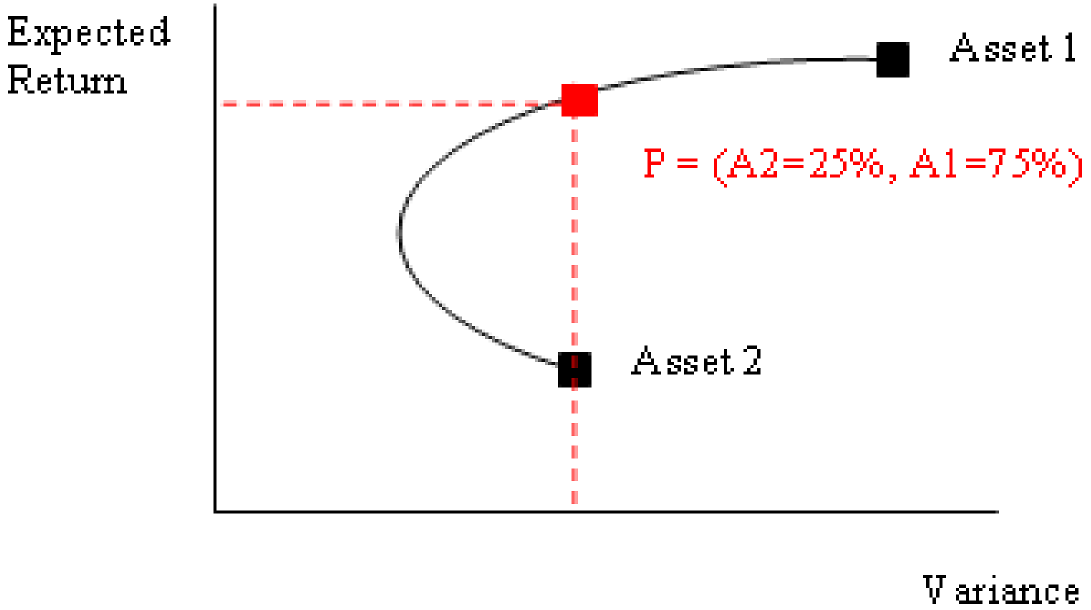
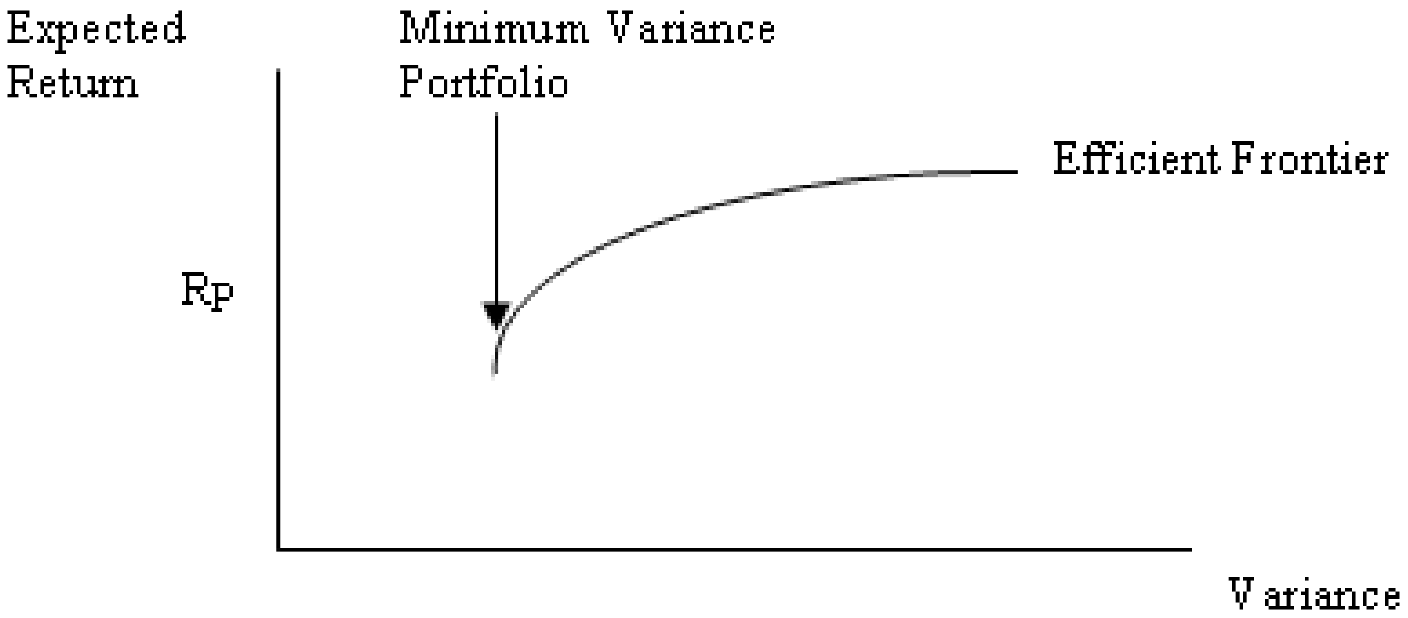
Efficient Diversification



Efficient Portfolios

- Consider entire investable universe of assets
- Goal: weight assets to construct most efficient portfolios
  - Greatest return for any given level of risk
  - Lowest risk for any given level of return
- Minimum Variance Portfolio
  - Portfolio with lowest possible risk given investable universe
- These portfolios defines the Efficient Frontier

Efficient Frontier



Optimization

- Determines combination of assets to best suit investor preferences on Efficient Frontier
  - Mean-Variance Utility function measures optimal trade-off
    - $Utility \sim f(+return, -risk, -\cos t)$
  - Constant utility line measures investor's indifference between the risk free asset and risky asset paying a premium which compensates for risk
    - $Utility = R_f = R_p - R_{Risk\ \Pr emium}$
  - Optimal portfolio is intersection between highest attainable constant utility curve and Efficient Frontier

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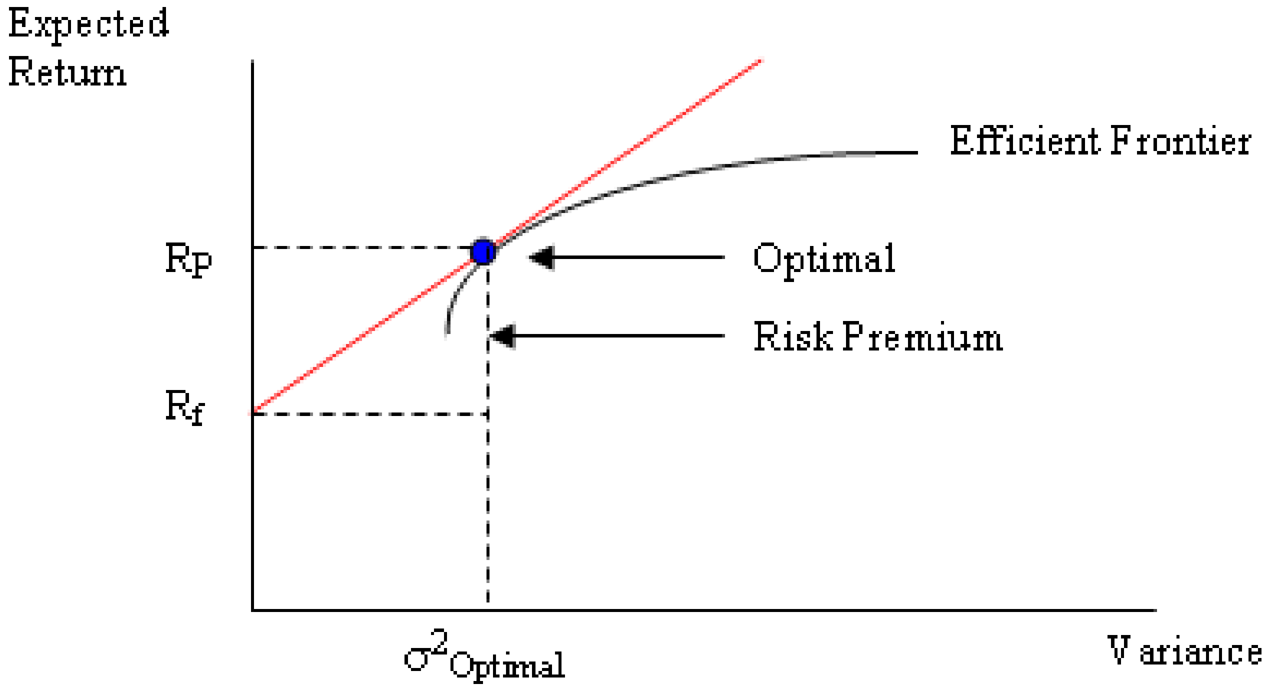
Optimal Portfolio



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Mean-Variance Utility Function

$$\text{Max } U = \alpha - (\sigma_s^2 / RAP_s) - (\sigma_u^2 / RAP_u) - ((C + T) * A) - P$$

- α = Forecasted portfolio return
- σ_s^2 = Portfolio variance due to common factors and correlation
- σ_u^2 = Portfolio variance due to stock specific risk
- RAP = Risk acceptance parameter
- C = Transaction costs for the optimization
- T = Capital gain taxes for the optimization
- A = Amortization constant
- P = Quadratic penalty cost

Caveat, Already?

- Frequent criticism of mean-variance optimization is the possibility of error maximization
    - see “Portfolio Optimization: The Robust Solution”, http://www.northinfo.com/documents/45.pdf
  - Inputs treated as certain values
  - Incorrect estimates
  - Unrealistic estimates (30% alphas)
- Northfield has introduced a series of functionalities to mitigate the impact of misspecified inputs
  - Adjusts inputs to account for estimation errors
  - Shrinks inputs to more realistic values

Alpha

- Index portfolio, E(R) = 0 => Risk minimization
- Active portfolio, E(R) scale benchmark-relative
  - Huge financial literature exist on alpha construction involving strategy dependent subtleties
- Reshape as Cross-Sectional Forecast:
    - See Technical Support Tip: "Reshaping Alpha as a Cross-Sectional Forecast", http://www.northinfo.com/documents/343.pdf
  - Converts forecast to standard units compatible to the optimizer
    - $\alpha = IC \times CS\_VOL \times STD\_SCORE$

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Misspecified Alpha

- Northfield provides tools blunting the impact of misspecified alpha using Bayesian adjustments:
  - Bayes Adjust combines user supplied returns with an equilibrium portfolio based on a universe of assets provided
    - See Technical Support Tip: "Bayes Adjust", http://www.northinfo.com/documents/345.pdf
  - Bayes-Stein Shrinkage combines user supplied returns with the expected return of a minimum variance portfolio constructed from the universe of provided assets
    - See Technical Support Tip: "Bayes-Stein Return Covariance (Return Shrinkage)", http://www.northinfo.com/documents/408.pdf

Risk

- Northfield risk models provide variance numbers
  - Systematic risk is the market risk measured by the Northfield common factors
  - Unsystematic risk is the risk specific to a security
  - For shorter term risk predictions Northfield provides near-term risk models, which incorporates recent market variability
    - See "Adaptive Near Horizon Risk Models", http://www.northinfo.com/documents/356.pdf
- Users may choose to use their own models
    - For calculations see Technical Support Tip: "Calculating Risk Using Northfield Flat Text Files", http://www.northinfo.com/documents/348.pdf)

Errors In Risk Estimates

- Estimation Error Adjustments can mitigate errors:
  - Bayes-Stein Shrinkage combines individual asset risk with the risk of a minimum variance portfolio constructed from the universe of provided assets
    - See Technical Support Tip: “Bayes-Stein Return Covariance (Return Shrinkage)”, http://www.northinfo.com/documents/408.pdf
  - Blend Covariance Matrix is a combination of covariance matrix with up to three more structured version of itself
    - See Technical Support Tip: “Estimation Error Adjustment-Covariance Blend”, http://www.northinfo.com/documents/347.pdf
    - Single Index, Constant Correlation, Constant Covariance

Risk Acceptance Parameter (RAP)

- RAP is investor’s willingness to trade between risk and return
  - The slope of the constant utility curve
  - Smaller/larger numbers provide more/less weight to risk
  - $RAP = 1/\lambda$
- Rational range of values: 0 < RAP < 200
  - Zero would represent a totally risk averse investor (hide the money under the mattress)
  - 200 represents an investor that only cares about maximizing long term growth
- Default value = 100
  - Historically induced investors to be invested at the long term total risk/return (RAP = 133 ~ λ = .0075)

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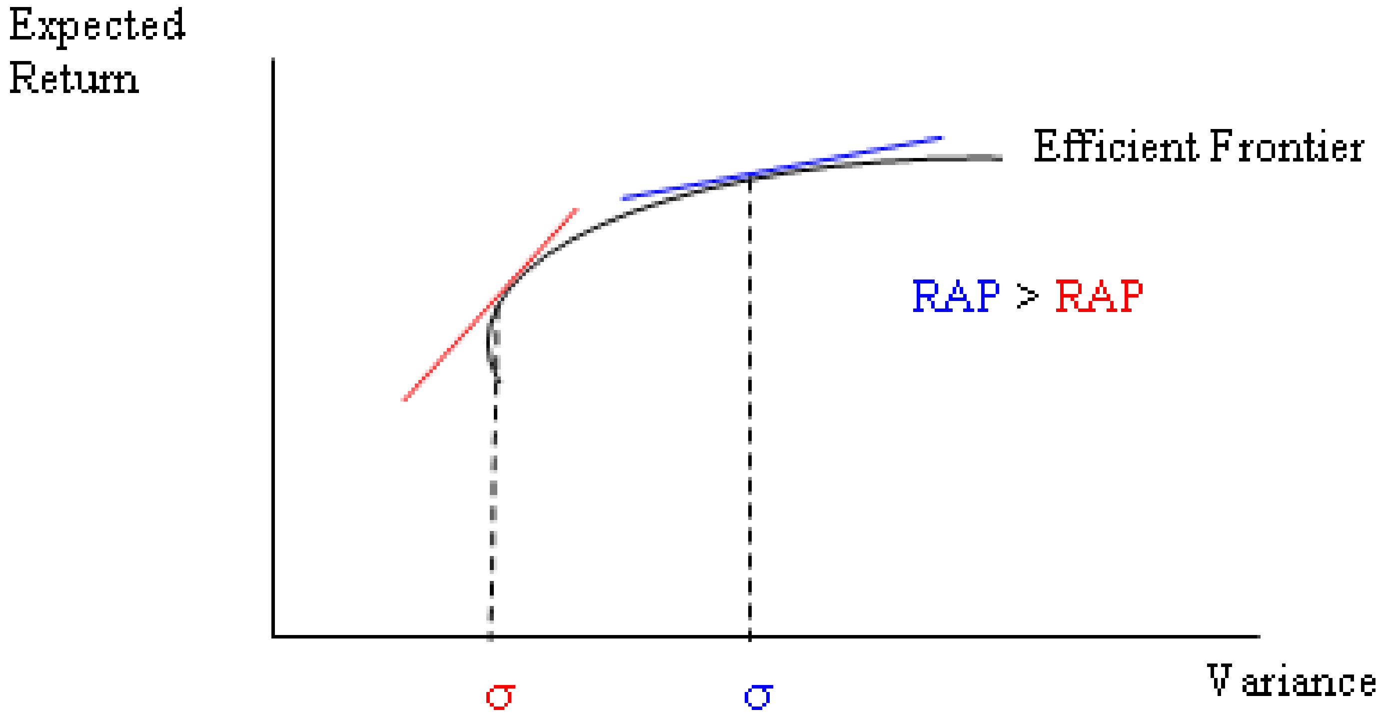
RAP &The Efficient Frontier



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Quantifying RAP

- Two good rules of thumb
  - In absolute risk terms, RAP approximately 2 times investor's net worth as a percentage of total assets. This will vary over the investor life cycle
    - $RAP = 2 \times (A - L) / A$
  - In benchmark relative terms, RAP approximately 6 times the desired tracking error
    - $RAP = 6 \times E(TE)$
  - For additional information see:
    - Technical Support Tip: "Choosing Risk Acceptance Parameter", http://www.northinfo.com/documents/16.pdf
    - Tech Support Tip: "Risk Acceptance Parameter (RAP)", http://www.northinfo.com/documents/413.pdf

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Transaction Cost

- Transaction costs negatively impact utility
  - See "Transaction Cost Model", http://www.northinfo.com/Documents/354.pdf
  - Increased utility must outweigh cost incurred
- Such cost can be broken into the
  - Linear: per-share cost known by the manager (i.e. $.05/trade)
  - Non-Linear/Market Impact: cost movement due to trades by the manager and other market participants perception of information due to trades in the assets
  - Cross Sectional: cost impact on similar assets

Transaction Cost Function

$T = A + (B_i/G^R)\ S + (C_i/(G^{(0.5*R)}))\ S^{0.5} + D * \text{Max}\ [S\text{-}L, 0] + \text{Max}\ [Z, -A]$

- A = per-share cost
- B_i = the coefficient on the linear process
- C_i = the coefficient on the square root process
- S_i = the number of shares to be traded
- G = number of days required for the trade (note fractional days are permissible)
- R = the proportion of temporary versus permanent market impact is the impact decay and is usually R≤1. Empirical evidence suggests .71.
- D = the coefficient on the quadratic process
- L = threshold of traded assets at which the quadratic process is invoked
- Z = cross market impact coefficient

- Northfield provides monthly Market Impact Model (B and C)

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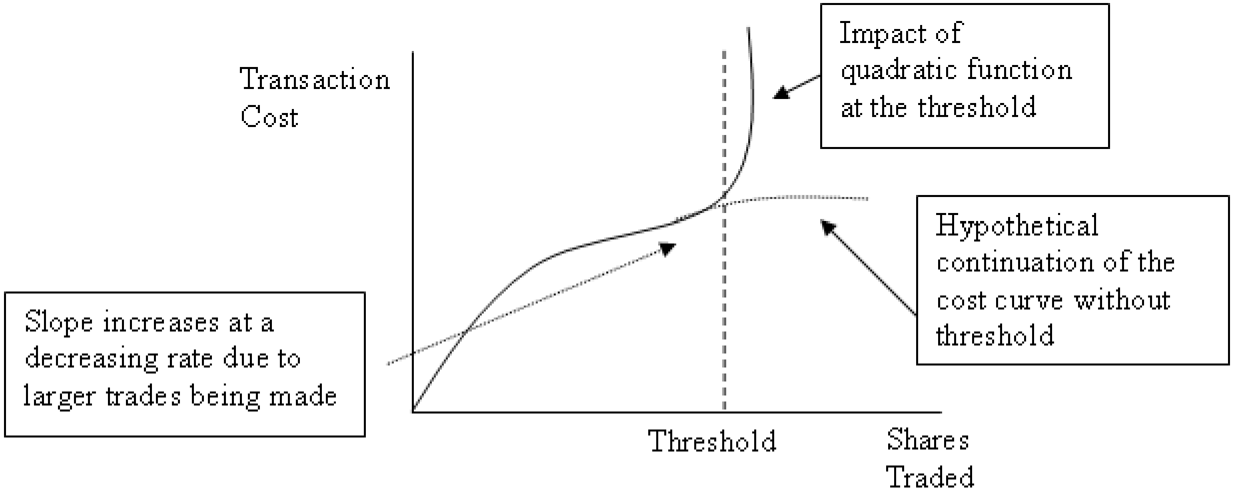
Non-Linear Transaction Cost



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Taxes

- Tax cost is incorporated into the utility function as an additional cost allowing users to extract “tax alpha”
    - See “Everything You Wanted to Know about Asset Management for High Net Worth Investors”, http://www.northinfo.com/Documents/186.pdf
  - Losses increase an assets relative utility to sell
  - Gains decrease an assets relative utility to sell
  - Asset lots are traded in order from least taxed to most taxed lots, unless prohibited by wash sale rules
  - Long-term and short-term gains are offset by long-term and short-terms losses

Amortization

- Amortization measures how quickly cost is recognized
  - The default is 100% of transaction cost is incurred within a year
  - For two years, the amortization should be 50
  - For six months, the amortization should be 200
- Increase/decrease the rate of amortization for tax strategy
  - Set a higher year end amortization rate to harvest losses
- The multi-period approximation functionality
    - See “Technical Support Tip: Multiperiod Approximation”, http://www.northinfo.com/documents/390.pdf
  - Relaxes the assumption of a one period buy and hold by dynamically changing the Amortization parameter for a more cost efficient transition from the initial to the optimal portfolio over multiple periods

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Quadratic Penalty Cost

- Quadratic penalties are accounted for in the utility function
  - See Technical Support Tip: “How to Use Penalties”, http://www.northinfo.com/documents/190.pdf
  - TBD, next newsletter
- The further away a portfolio moves from a goal the greater the penalty cost
  - $PenaltyCost = Scale \times (Goal - Current)^2$
- Penalties may be applied to:
  - Industries/Sector where goals are the benchmark weight
  - Any Attributes for which a user can supply data where goals are user defined

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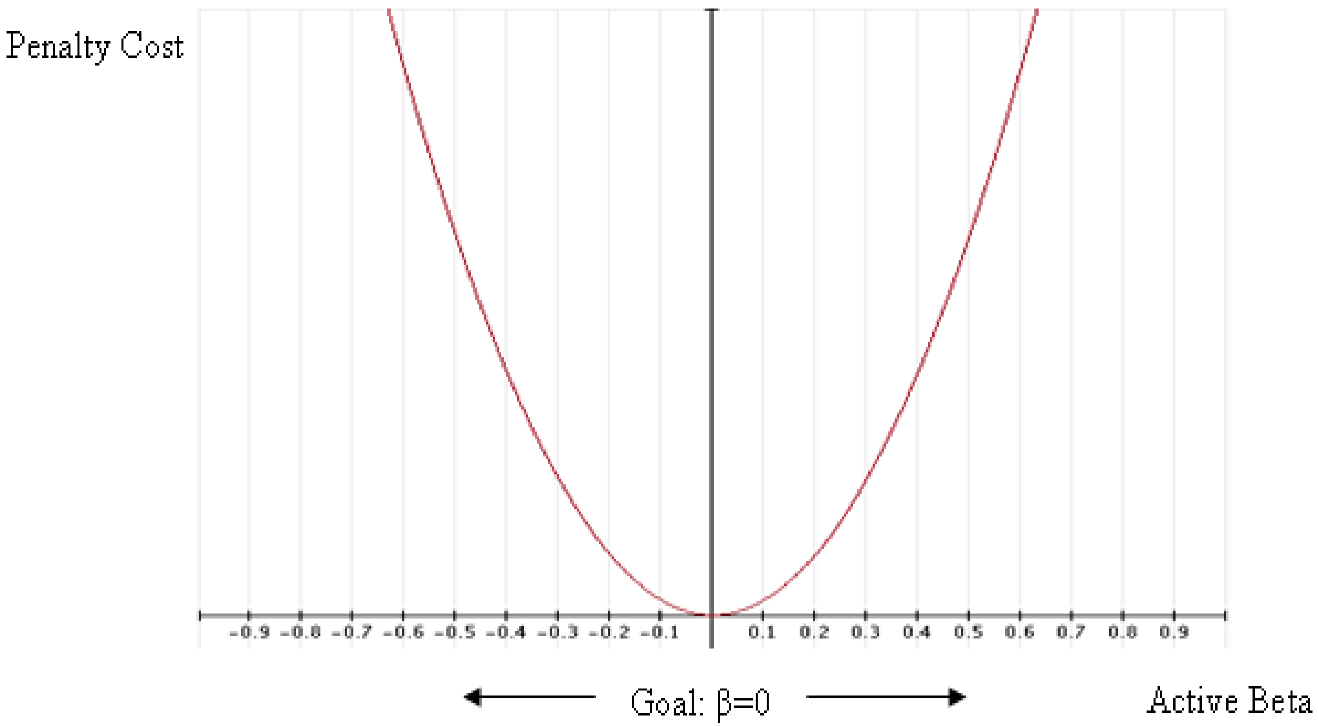
Quadratic Penalty



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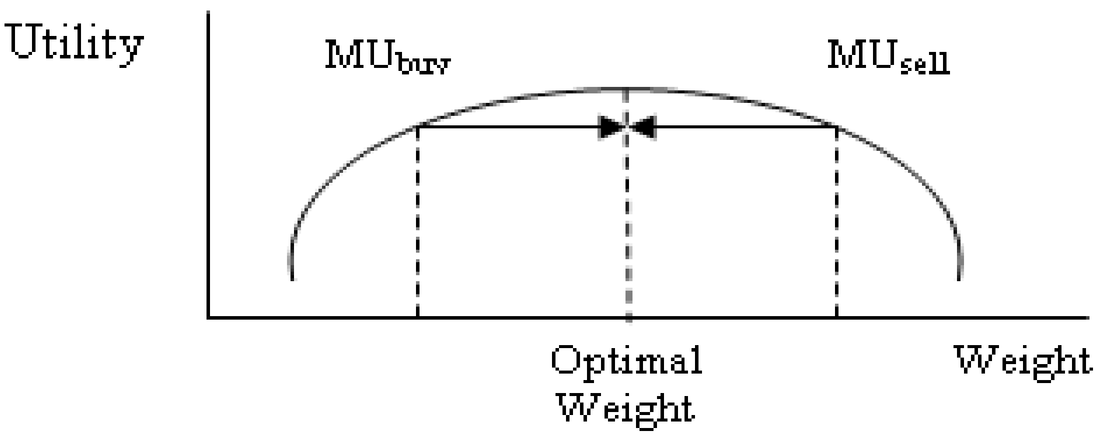
Utility Maximization

- Assets are ranked according to Marginal Utility (MU)
  - MU measures the relative utility improvement of each asset
- Trades are made in an iterative process in the direction of increasing utility
  - Assets with highest MU are purchased
  - Assets with lowest MU are sold
  - MU is recalculated after each trade
- Trades continue until maximum utility is reached
  - At this point no one asset can add additional utility (subject to constraints)
  - Optimal portfolio

Marginal Utility



Increase weights for assets on the left MU_{buy}
Decreases weights for assets on the right MU_{sell}

Subject to Constraints

- Optimization can be run with a variety of constraints
  - Constraint misuse is the largest contributor to failed optimizations
- Hierarchy of constraint classes:
  - Class I constraints are those that are linear combinations of security weights
    - Max asset weight = 0.5%, anywhere between 0-.5%
  - Class II constraints are cardinality constraints applied to the optimal portfolio
    - No longer linear combination, but applied to individual asset(s). Such as max assets.
    - Max asset = 20, but the optimal portfolio has 21. An entire asset is removed despite weight.

Constraint Priority

- Class I constraints:
  1. Position limits
  2. Industry and sector limits
  3. Factor Exposure
  4. Attribute File Variables
- Class II constraints:
  - Maximum number of assets
  - Maximum turnover
  - Minimum trade sizes
  - Maximum realized capital gain
  - Holding threshold

Optimization in Steps

- Step 1, start from a feasible position
  - If conflicts exist among the constraints, the portfolio is "infeasible"
  - The optimizer attempts to find a portfolio that meets all Class I constraints, with the least amount of turnover from the initial portfolio if one exists
  - This feasible portfolio is the real starting point of the process
- Step 2, the main optimization loop occurs
  - Goal is to maximize the utility function, MU
  - Subject to class I constraints according to priority or class II stopping criteria (max turnover)

Optimization in Steps, cont.

- Step 3, post-optimal constraints (threshold loop)
  - Class I constraints still have priority
  - Class II constraints are applied after maximum utility is achieved
  - There is no specific priority among Class II constraints
    - If constraint conflict exist, the optimizer seeks a reasonable compromise based on the objective function inputs
  - Class II constraints is where most frequent violations occur
    - Can be violated if they drive the portfolio very far away from the unconstrained optimal state
  - No matter which method is used the further away from the unconstrained optimal portfolio the resulting portfolio is the more likely it may be sub-optimal
  - **Optimal!!!!**

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Support Issues

- Most support issues are related to optimizations setups, especially constraints
- Review output reports:
  - Exceptions
  - Warnings in Runtime Messages
- Getting help:
  - Northfield Optimizer Online HELP is very extensive
  - Northfield's website, www.northinfo.com
  - Contact Support

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Concluding Remarks

- A lot of information has been presented
  - Included links expand on some of the ideas
  - Additional resources:
    - Client Information Guide, TBD
    - Optimizer formula description manual: http://www.northinfo.com/documents/12.pdf
- Future Newsletters:
  - Constraints Priority/Quadratic Penalties
  - Marginal Utility and Optimality
  - Optimization talk

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Feedback

- Feedback welcome:
  - Input on this talk
  - Future Newsletter articles of interest
  - Other Webinar interest
- Email support at:
  - US: support@northinfo.com
  - Europe: support-europe@northinfo.com
  - Asia: support-asia@northinfo.com

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Questions & Responses

- Try to get to as many as possible, if unable I will email a response
- Feel free to contact your local support representative for more in-depth questions

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