Northfield's 26th Annual Research Conference

The Decision to Lever

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CENTER FOR
RISK MANAGEMENT
RESEARCH

October 8, 2013

Outline

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Outline

1. What drives the cumulative return to a levered strategy?
   - Source plus magnifier plus covariance
   - Transaction costs
   - Volatility drag
2. What is the right benchmark for a levered strategy?

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Outline

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Illustration

We'll show how leverage works in the context of risk parity

- Two asset classes: US Equity and US Treasury Bonds
- Rebalance monthly to:
  - Equalize estimated volatility of the two asset classes
  - Lever to a target
- Volatility is estimated with a 36-month rolling window
- Horizon: January 1929–December 2012

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However, risk parity is merely an illustration

Our results hold in great generality

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The elements of a levered strategy

Source portfolio S

the fully invested portfolio to be levered

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Borrowing rate r^b

not the risk-free rate in practice

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Target T

that determines...

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Target T

that determines...

Leverage λ

assumed to be at least 1 for convenience

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Unconditional levered risk parity (UVT)

- Source: fully invested risk parity strategy consisting of US Equity and US Treasury Bonds

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- The target is the annualized volatility of a 60/40 strategy over the horizon, 1929–2012

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Unconditional levered risk parity (UVT)

- Source: fully invested risk parity strategy consisting of US Equity and US Treasury Bonds
- The target is the annualized volatility of a 60/40 strategy over the horizon, 1929–2012
- Leverage is dynamically determined by rebalancing to a constant annualized volatility target of 11.59%

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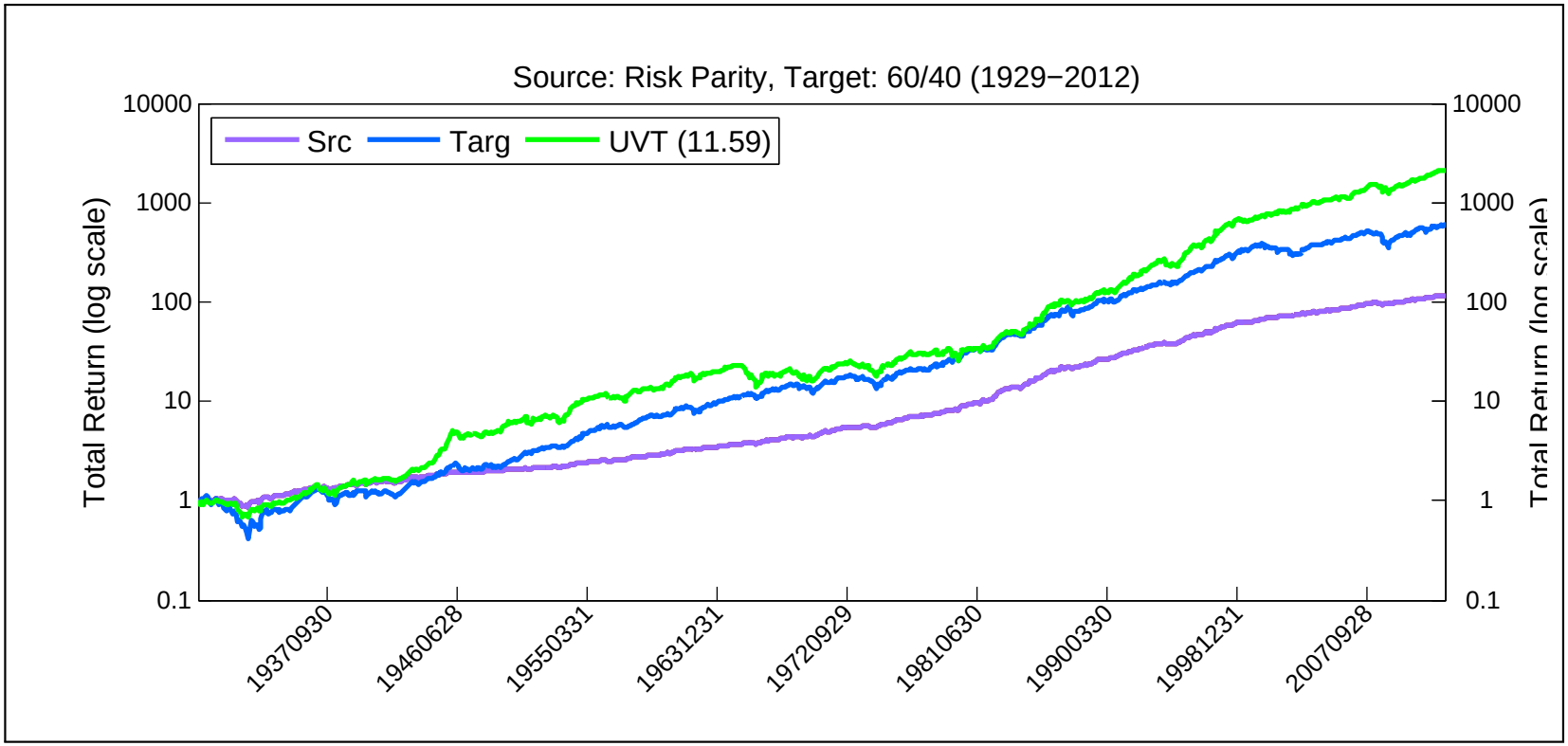
Cumulative returns over an 84-year period



borrowing at the risk-free rate, no penalty for trading

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Return to a levered portfolio in a single period

$$\begin{aligned} r^L &= \lambda r^S - (\lambda - 1) r^b \\ &= r^S + (\lambda - 1)\left(r^S - r^b\right) \end{aligned}$$

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One effect of leverage on return over many periods

Return to a levered portfolio in a single period...

$$r^L = r^S + (\lambda - 1)(r^S - r^b)$$

One effect of leverage on return over many periods

Return to a levered portfolio in a single period...

$$r^L = r^S + (\lambda - 1)(r^S - r^b)$$

...looks different from expected return over many periods

$$\begin{aligned} E\left[r^L\right] &= E\left[r^s\right] + E\left[(\lambda - 1)(r^S - r^b)\right] \\ &= E\left[r^s\right] + E\left[\lambda - 1\right] E\left[r^S - r^b\right] + cov\left(\lambda, r^S - r^b\right) \end{aligned}$$

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Nevertheless, the covariance term can make a large contribution to the return of a levered strategy

$E\left[r^s\right]$	5.75
$E\left[\lambda - 1\right] E\left[r^S - r^b\right]$	6.10
$cov\left(\lambda, r^S - r^b\right)$	-2.04
$E\left[r^L\right]$	9.81

- UVT (11.59%)
- Risk parity levered to match 60/40
- 1929–2012
- annualized percent

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Types of transaction costs

Financing

Eurodollar deposit rate

Types of transaction costs

Financing

Eurodollar deposit rate

Trading costs

A linear model

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Liquidation due to drawdown

Beyond the scope of this presentation

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Market impact

Once again, beyond our scope

Financing and trading costs preferentially degrade the return to a levered strategy...

	Before	After
$E\left[r^s\right]$	5.75	5.68
$E\left[\lambda-1\right]E\left[r^S-r^b\right]$	6.10	3.79
$cov\left(\lambda, r^S-r^b\right)$	-2.04	-1.83
r^{TC}	0.00	-.78
$E\left[r^L\right]$	9.81	6.86

- UVT (11.59%)
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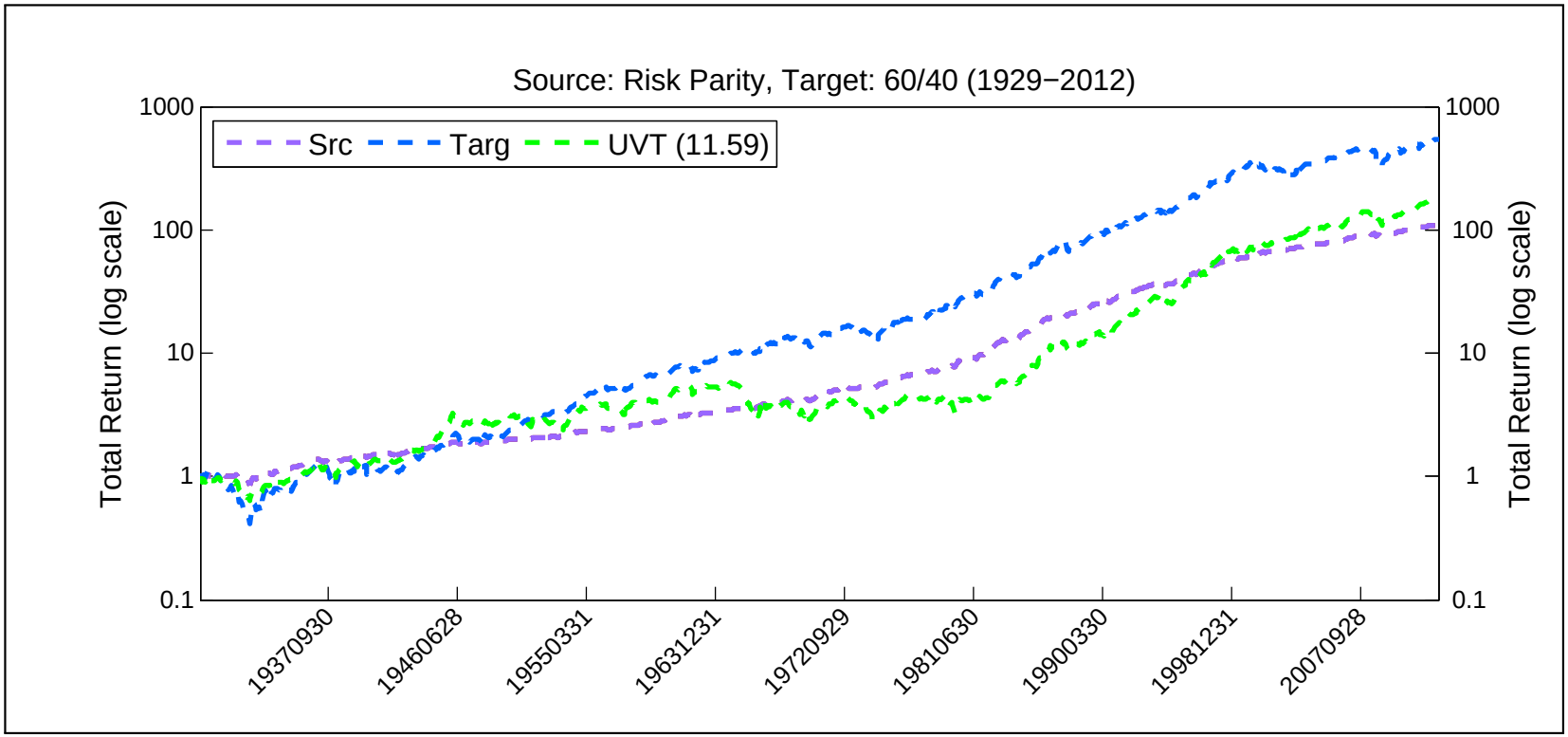
... and they can interchange the order of strategies ranked by cumulative return



borrowing at the Eurodollar deposit rate, penalty for trading

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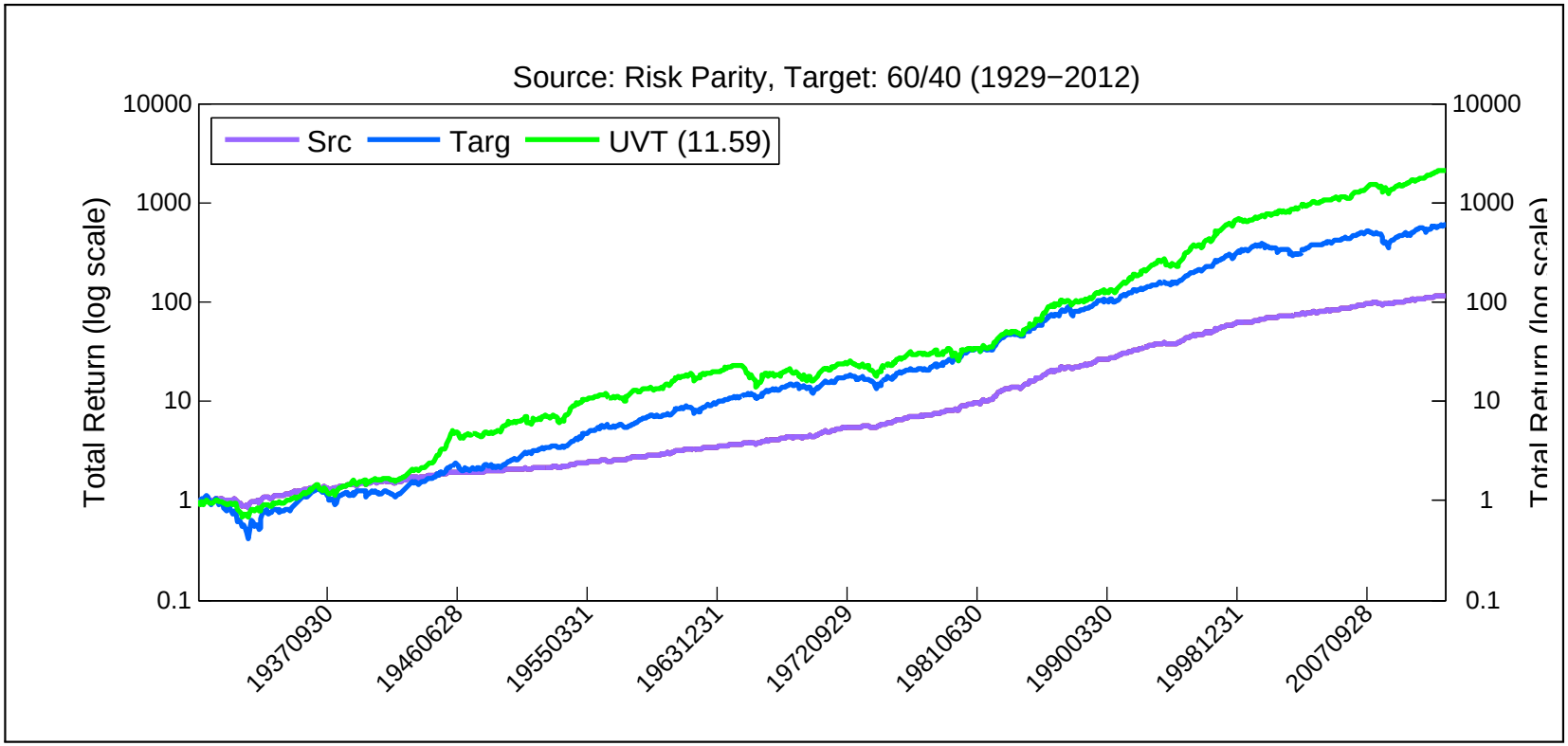
Cumulative returns over an 84-year period



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Geometric return (after transaction costs)

$$G\left[r^L\right] \sim \left(1 + E\left[r^L\right]\right) e^{-\frac{var\left(r^L\right)}{2}} - 1$$

	Arithmetic	Geometric
Source	5.68	5.74
Target	8.18	7.77
UVT ($\sigma = 11.59\%$)	6.86	6.37

- UVT (11.59%)
- Risk parity levered to match the volatility of 60/40
- 1929–2012
- annualized percent

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What drives the cumulative return to a levered strategy?

$$E\left[r^L\right] = E\left[r^s\right] + E\left[\lambda - 1\right] E\left[r^S - r^b\right] + cov\left(\lambda, r^S - r^b\right) - r^{TC}$$

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$$G\left[r^L\right] \sim \left(1 + E\left[r^L\right]\right) e^{-\frac{var\left(r^L\right)}{2}} - 1$$

- Volatility drag

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Candidate benchmarks

Source

Why lever a source if it only makes things worse?

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Target

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Fixed leverage (FLT)

Eradicates the covariance term, a big source of uncertainty

- Expected leverage matched (and volatility higher)
- Volatility matched (and expected leverage lower)

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Fixed leverage (FLT)

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Conditional volatility target (CVT)

Incorporates market conditions

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Lookahead bias

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- CVT can be implemented without lookahead bias

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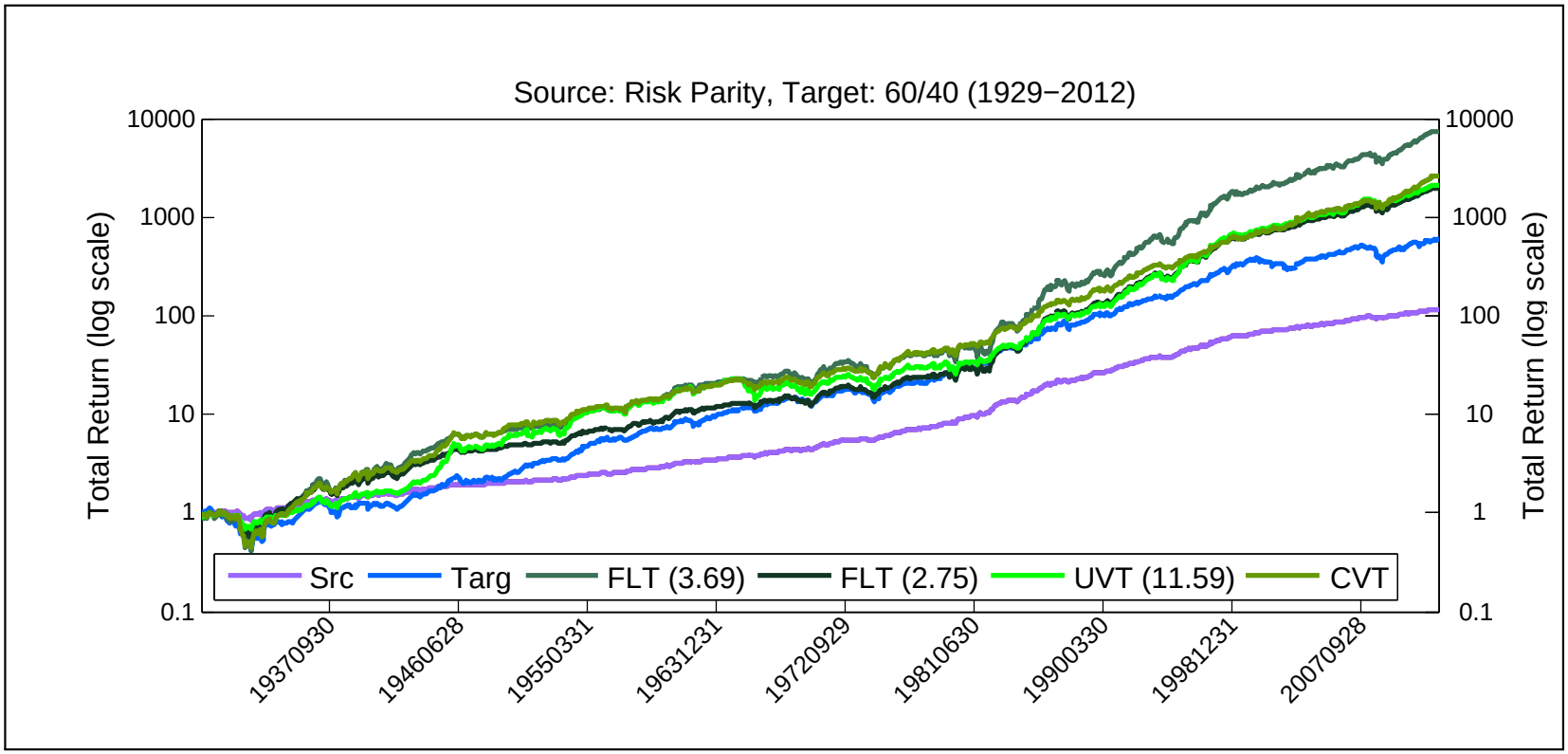
Cumulative returns over an 84-year period—before transaction costs



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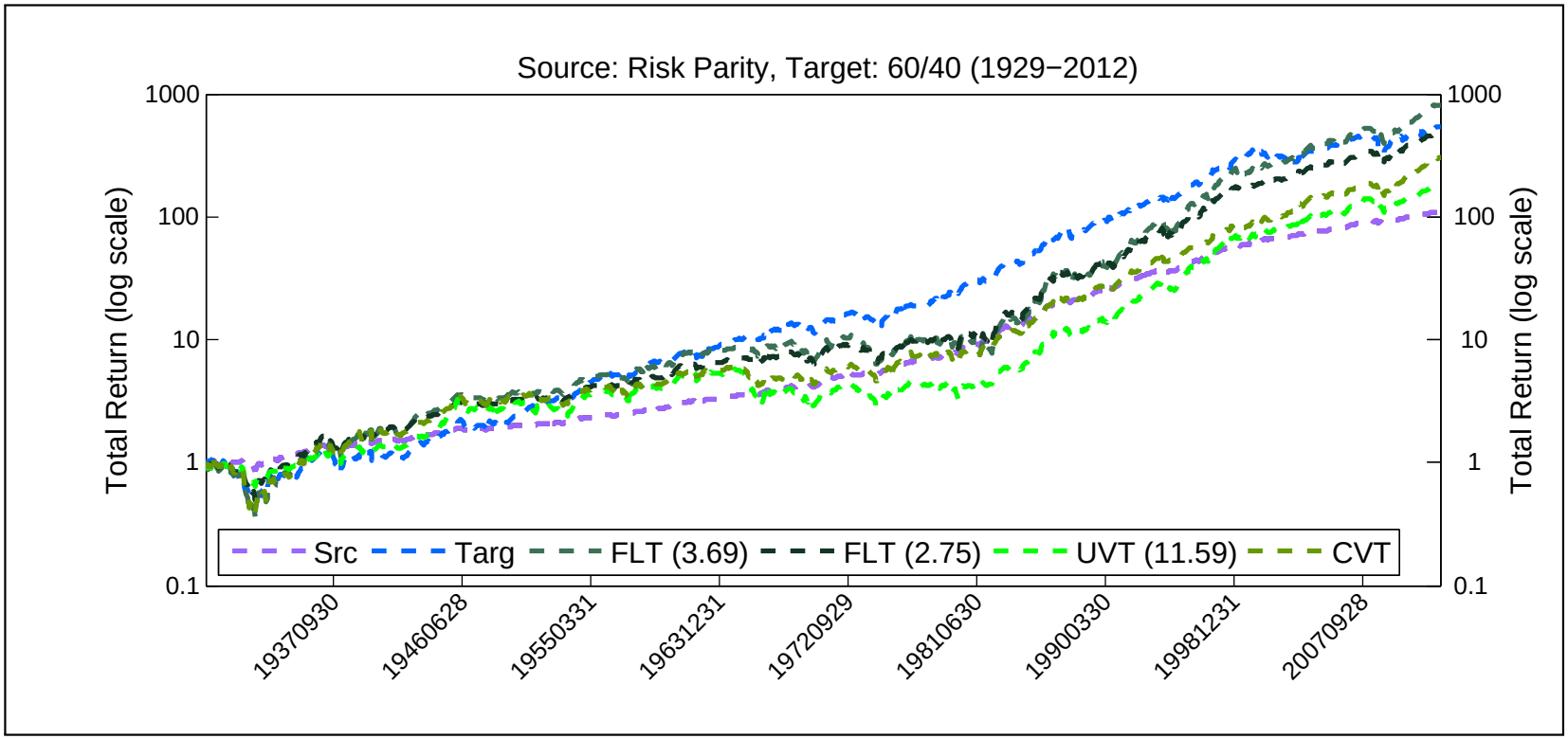
Cumulative returns over an 84-year period—after transaction costs



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Transaction costs scrambled the cumulative return ranking of the strategies we looked at (1929–2012)

	Before	After
Source (Risk Parity)	6	6
Target (60/40)	5	2
UVT ($\sigma = 11.59$)	3	5
FLT ($\lambda = 3.69$)	1	1
FLT ($\lambda = 2.75$)	4	3
CVT (60/40)	2	4

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Performance metrics

Cumulative return over a long horizon

That has been our focus up until now but it neglects important features of investment strategies, such as:

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- Risk
- Return mandate
- Investment horizon
- Normalization

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Performance (after transaction costs, 1929–2012)

	Geometric Return	Sharpe Ratio	P(Meeting 8% Return Target)
Source (Risk Parity)	6	1	6
Target (60/40)	2	2	2
UVT ($\sigma = 11.59$)	5	6	5
FLT ($\lambda = 3.69$)	1	4	1
FLT ($\lambda = 2.75$)	3	3	3
CVT (60/40)	4	5	4

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Conclusions

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- ... but is zero in a fixed leverage strategy

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More conclusions

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More conclusions

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- ...but since we can't see into the future, it may not be clear how to compare these strategies ex ante
- However, fixed leverage strategies eliminate a substantial source of uncertainty and they tend to dominate dynamically levered strategies
- 60/40 looks pretty good too

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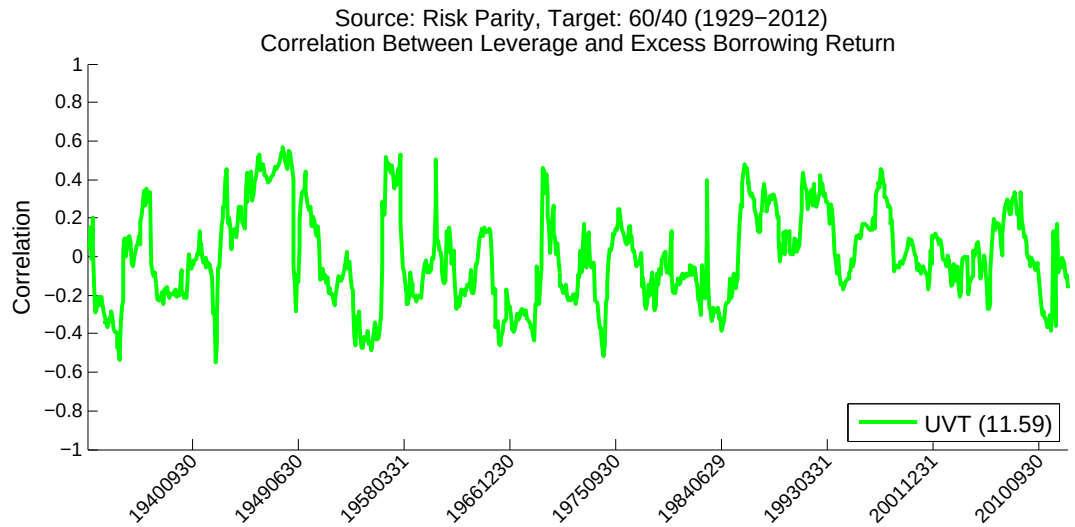
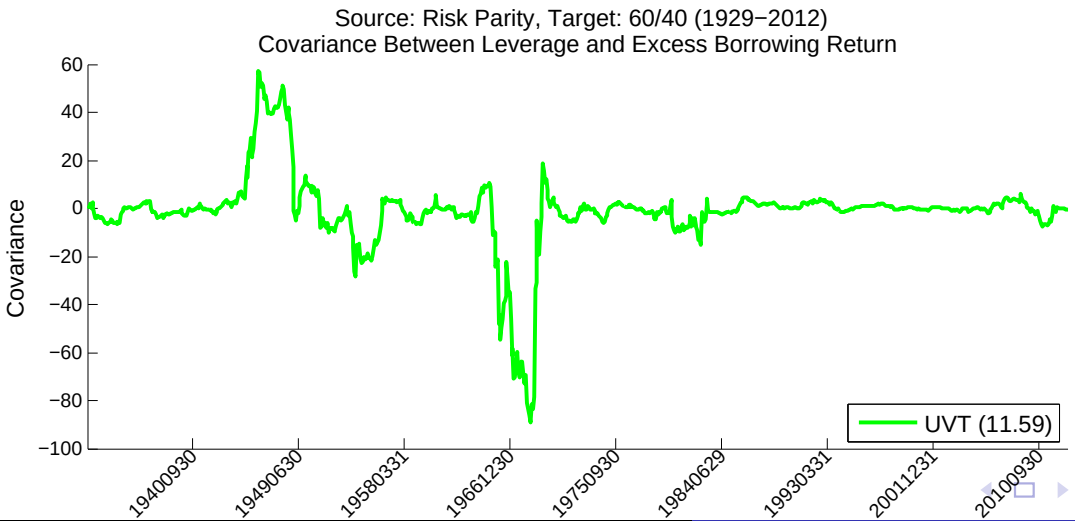
(1929–2012) The covariance term is unstable...





- rolling 36-month estimates
- covariance is in units of percent annualized return

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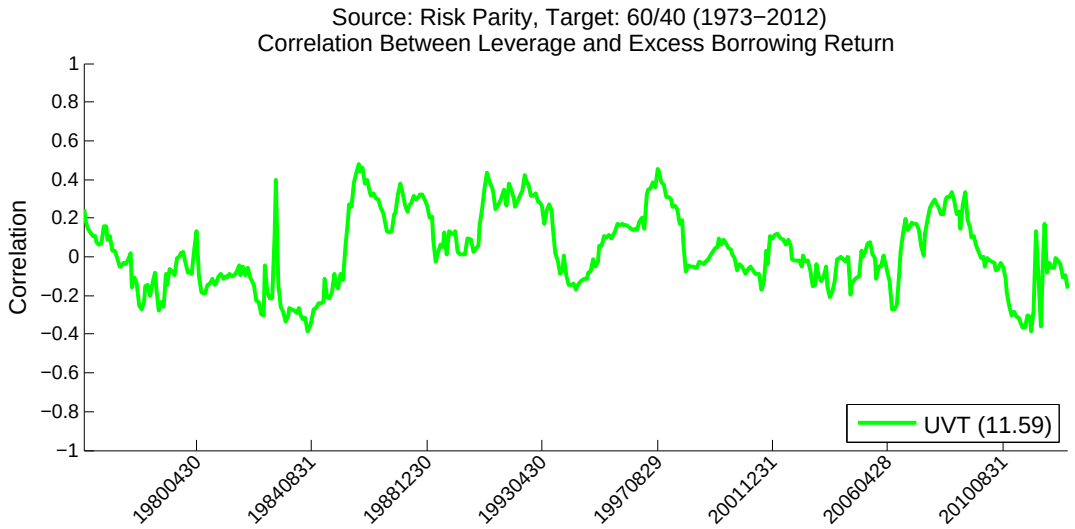
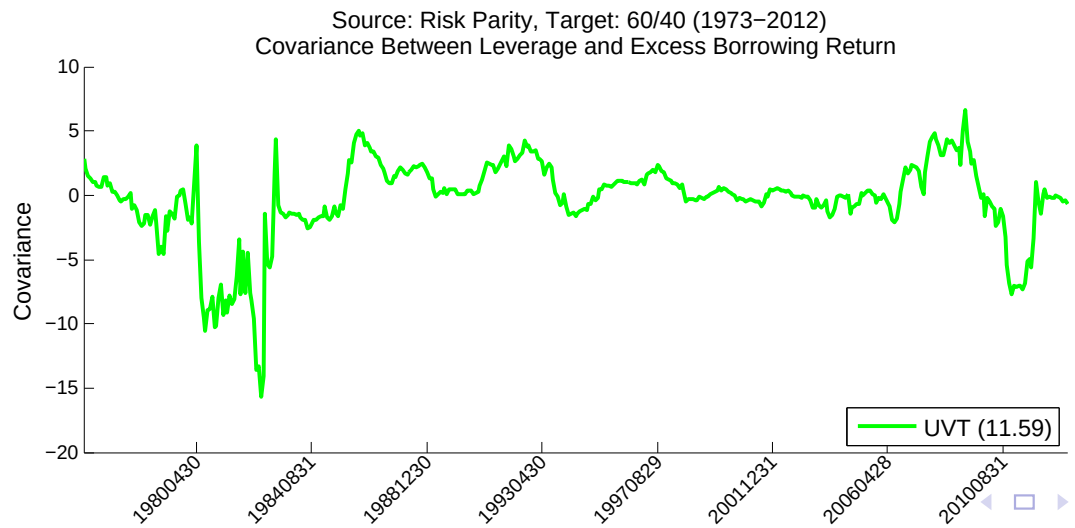
...Even in recent decades (1973–2012)





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Performance (after transaction costs, geometric return)

	1929–2012	1973–2012
Source (Risk Parity)	6	6
Target (60/40)	2	5
UVT ($\sigma = 11.59$)	5	4
FLT ($\lambda = 3.69$)	1	1
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CVT (60/40)	4	2

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Performance (after transaction costs, geometric return, 1929–2012)

	60/40	Market
Source (Risk Parity)	6	6
Target (60/40)	2	3
UVT ($\sigma = 11.59$)	5	4
FLT ($\lambda = 3.69$)	1	1
FLT ($\lambda = 2.75$)	3	2
CVT (60/40)	4	5

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Linear trading cost model

Trading is penalized by deducting from equity κ times the dollar amount required to rebalance

$$\kappa = \begin{cases} 1\% & \text{1929–1955} \\ .5\% & \text{1956–1970} \\ .1\% & \text{1971–2012} \end{cases}$$

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