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A Cointegration Approach to Portfolio Allocation

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Motivation

- Many problems in finance are naturally structured as hedging problems, wherein the objective to find some set of assets that will offset (hedge) the changes in value of a particular set of liabilities.

- Integration of assets and liabilities for institutional investors is a long-horizon example of such a problem. At the short-term end of the time horizon scale would be the hedging transactions entered into by a trading desk in order to offset the risks of security positions held.

- The hedging paradigm can also be applied to an actively managed portfolio process where the objective is to outperform some specified market index. Such framing of the active management is not only possible, but has many desirable aspects.

Outline for the Presentation

- This presentation explores the use of cointegration methods to determine advantageous asset class weights within an institutional investment portfolio.

- We begin by framing the issue as a hedging problem, wherein the liability grows over time at some rate which may be fixed, fixed in real terms, or equal to the return on a selected policy benchmark index

- The statistical property of cointegration is then employed to find portfolios that are suitable long-term hedges for the liability.

- We will also review prior work on utilizing cointegration as a mechanism to pursue active management. An empirical example involving country weights within the MSCI EAFE countries will be used to illustrate mechanics

Basic Conception

- Assume that we are asset owners seeking to find the portfolio that funds a particular set of liabilities.
  - Our goal is to find the portfolio that would be a "successful hedge" against the liabilities where the present value of the liability grows at some fixed rate (the actuarial return assumption) or a stochastic discount rate (e.g. FASB 87)
  - Assuming we are currently fully funded (i.e. zero surplus) our goal will be to hedge fluctuations in the present value of the liability
  - As our portfolio is held through time, our hedge is successful as long as the value of surplus never gets too large in magnitude, either positive or negative. The time series of the value of surplus would then be called *stationary. As long as the value* of surplus has this property, we know that the changes in asset values are adequately keeping up with changes in the value of the liability.
  - If we can find a set of portfolio assets that result in a stationary surplus, we can say that the portfolio members are *cointegrated with the liability.*

Not a Panacea but Maybe a Useful Direction

- There has been widespread discussion regarding the significant degree of underfunding of public pension plans in the United States
  - In September 2012, the GASB changed some of the rules about how local governments compute and report pension liabilities
  - Even before the change, the aggregate funding gap nationally was estimated to be at least $1.5 Trillion

- Rather than raise contributions most local plans are retaining expected return assumptions that many people believe are very optimistic
  - The national average is 7.7% (4.4% real, 3.3% inflation)
  - If you lower the return assumption to 3% which is what you need for a 99% probability that no future contributions would be needed, about $5 Trillion in additional funding is needed

- Managing funding risk gets a lot easier if you have strong mean reversion in the values of surplus, *which is what cointegration is all about.*

Active Returns from Passive Management

- Assume we are active managers seeking to outperform the Standard & Poors 500 index by 300 basis points per year.

- This problem could easily be viewed in a hedging framework.
  – Our goal is to find the portfolio that would be a "successful hedge" against a liability whose value grows at a rate equal to the return on the Standard & Poors 500 index plus 300 basis points per year.
  – Within the hedging construct for active management, we can arbitrarily assign the initial value of the liability to be equal to the initial value of our investment assets. Hence we have an initial surplus (assets minus liabilities) of zero.

- As long as we can find a set of assets that "keeps up" with the growth rate of the pseudo-liability, the value of surplus will be stationary and we will have found the cointegrating portfolio.

Methodology

- By stationary we mean that the time series has consistent mean, standard deviation and autocorrelation properties. We can formally define stationarity with equation (1)

$$X_t = aX_{t-1} + b_t + c + e_t \qquad (1)$$

Where

X_t = the value of series X at time t

t = the increment count of time

e_t = error term at time t

Cointegration Defined

- Equation (1) can be estimated using typical regression analysis methods. Series X will be considered stationary if the absolute value of coefficient *a is statistically significantly less* than one.

- Since we must consider both the possibility that *a is equal to or greater than one* and the possibility that *a is less than or equal to negative one, a regular t-statistic test for* statistical significance is not appropriate.
  - The most popular alternative test is called the Dickey-Fuller statistic.

- *Cointegration is the situation where we are able to form a stationary time series from a linear combination of series that are not individually stationary.*
  - For example, stock price time series are generally not stationary.

Cointegrating Portfolios

- If we could form a **fixed weight** portfolio of stocks such that the portfolio had a value time series that was stationary, we would say that those stocks formed a cointegrated set.
  - The list of the stock weights would be called the cointegrating vector.
- In our process portfolio cointegration is mathematically evaluated in a two step process. First, we define the surplus in equation (2).

$$Si = S_t = {}_{1 \text{ to } n}[w_i A_{it}] - L_t \qquad (2)$$

St = the value of surplus at time t

wi = the weight of asset i in the portfolio

Ait = the value of asset i at time t

Lt = the value of the liability at time t

Second Step for Cointegrating Portfolios

- Next, we use equation (1) to determine if the time series of S is stationary. If it is, then we have a portfolio that is cointegrated with the liability.

- The set of portfolio weights is called a cointegrating vector.

- For a detailed discussion of the mathematics of cointegration see Hamilton (1994).

- As previously noted L the time series liability, could be defined in many ways. For example, it could have a value that grows at a fixed rate like the actuarially assumed return of a pension fund, or it could grow at a rate equal to return on some market index.
  - We can also define L in an active management framework by defining it as a time series that grows at a rate equal to some benchmark index plus a desired incremental return.

Contrast with MPT

- Let's contrast this approach with the determination of finding optimal portfolio weights in mean-variance analysis as pioneered by Markowitz (1959).

- When using mean-variance methods, we are analyzing return time series that we must assume have the property of stationarity. In addition, we must assume that asset returns are normally distributed and that correlations between assets are stable.

- Cointegration methods work directly on portfolio values (not returns) and make no assumptions about the distributional properties or stationarity of the asset value time series.

- To the extent that investors pay their liabilities with wealth (not returns which are changes in wealth), using cointegration fits very well with the balance sheet concepts of traditional accounting

Finding Cointegrating Portfolios

- There are a number of ways that we can estimate the portfolio weights that will form the cointegrating vector, if such a vector exists. One way is to use multiple regression analysis to find the set of asset weights such that the time series of portfolio values best fit the time series of liability values

$$L_t = S_i = {}_{1 \text{ to } n}[w_i A_{it}] + e_t \qquad (3)$$

w_i = the weight of asset i in the portfolio

A_{it} = the value of asset i at time t

L_t = the value of the liability at time t

e_t = error term at time t

- The vector w would be the resultant regression coefficients. The error time series (e_t) would represent the surplus (S_t) from equation (2). Equation (1) would now be applied to the error time series to test for stationarity.

The "Shortcomings" of Regressions

- Our goal in using a regression is to find mean reverting error terms
  - This is inconsistent with the usual assumptions of random error terms

- A regression could also lead to negative values for some of the portfolio weights. If we choose not to take short positions in the portfolio, we must have values for the weights that are subject to two constraints.
  - All of the weights must be non-negative and sum to one

- These constraints are also employed in returns-based style analysis, as developed by Sharpe (1992).
  - To estimate equation (3) under such constraints, we must use methods of quadratic programming.
  - Unfortunately, there is no analytical solution for the standard errors of the regression coefficients (the portfolio weights) in such a case. An analytical approximation to the standard error is presented in diBartolomeo and Lobosco (1997).

Back to the Future

- Methodology and an empirical case were developed in diBartolomeo(1999)
- An alternative for finding the cointegrating vector would be to use Monte-Carlo simulation to create a large number of portfolios of randomly arranged weights, subject to whatever constraints we care to apply
  - We could then apply equation (2) directly and test each one with equation (1) to determine if it was cointegrating
- In the 1999 study, we tried to define if a set of country index weights would outperform the MSCI EAFE index in the future using cointegration methods
  - Our primary data consisted of actual index values for seventeen of the countries contained in the MSCI EAFE as well as the EAFE index itself
  - All index data ran from the beginning of 1970 through the end of 1998, a total of twenty-nine years (348 monthly index values). The seventeen countries are the subset of the twenty-one EAFE countries with complete data for the period.
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Experimental Design

- The experimental design was a simple one. We used the first fifteen years of data to find the vector of weights of the seventeen countries that cointegrated with our hypothetical liability stream
  - We tested five different liability streams that corresponded with increasing levels of active return over the basic EAFE return
- Our asset portfolios were constructed randomly with the constraint that all weights had to be positive. All seventeen asset indices had to have at least some weight in the portfolio.
- Once the cointegrating vectors had been found, we assumed that the portfolios were held at fixed weights for the remaining fourteen years of the data
  - The "out of sample" returns on our portfolios were then computed and compared to the return on EAFE.
  - We also compared our returns to those on the hypothetical liabilities, whose growth rate corresponded to EAFE plus the desired active return.

A Robust Solution

- To find the cointegrating vectors, we chose to use Monte Carlo methods. We simply create random portfolios subject to our desired constraints and capture those that cointegrated during the sample period
- Our approach was to let our simulation system create and test as many portfolios as needed until we had a sample of forty that had cointegrated
- Having a distribution of cointegrating portfolios allows us to directly test robustness of our result
  - Contrast this is to MPT where traditional mean-variance optimization has various “error maximizing” properties
  - Broadie (1993)
- We can also form combinations of the cointegrating portfolios (e.g. the average or median weight vector) and test it for cointegration as the “least likely to fail”

1999 Results - Discussion

- The results of our tests were encouraging.
- We tested five different levels of active return premium and found that we could obtain forty cointegrating vectors for each within a few minutes of simulation time.
- The key finding was that as we increased the desired active return, *a smaller fraction of the cointegrating portfolios beat the target in the out of sample period, but an increasing fraction of the cointegrating*
- *portfolios outperform EAFE itself.*
- The average weighting of the forty cointegrating portfolios for the EAFE plus 5 basis points per month was found to also be cointegrating

Still Partying in 1999

- Most of the random portfolios that were not cointegrating also outperformed EAFE in the out of sample period. This is due to our constraint that all seventeen indices participate in the asset portfolio.
  - As such, the resultant random weightings often were close to equal weighted, placing relatively greater emphasis on small countries relative to the composition of EAFE which is weighted by market capitalization.
- The fact that an equal-weighted portfolio of EAFE countries has outperformed EAFE itself has been well explored in the financial literature by Wilcox (1994).
  - However, in each case the fraction of cointegrating portfolios outperforming EAFE was greater than the fraction of non-cointegrating portfolios outperforming EAFE.
  - This difference was statistically significant at a better than 1% level, according to the Kolmogorov-Smirnov Type II test.
  - This test determines whether two samples have been drawn from the population.

Improving the 1999 Result

- The 1999 example was merely illustrative and is not meant to suggest actual investment practices.
- In our example, we held the cointegrating portfolios for a test period of
- fourteen years without ever trying to determine whether the property of cointegration was still present at each point in time during the test period.
  - In an actual investment strategy, we would certainly test periodically to determine if our cointegrating portfolios were still cointegrating, or whether some new mix of the assets now had the property of cointegration.
  - Monte Carlo methods, while simple to implement, did not provide us with any information that our cointegrating portfolios are the “best” cointegrating portfolios available.
  - In addition, this example also ignored transaction costs, although these should be quite small given that index futures contracts provide an inexpensive and very liquid way to rebalance the country index portfolio as needed.

Extending to the Current Day

- We have reproduced the country index example problem through 2012 and undertaken similar experiments with both asset classes and with sectors within a few selected equity markets.
  - We will be presenting these empirical results in detail at our annual conference (Montreal in October)

- All of the results are qualitatively the same. The key finding remains:
  - As we increased the desired active return, *a smaller fraction of the cointegrating portfolios beat the target in the out of sample period, but an increasing fraction of the cointegrating portfolios outperform the "base" liability representation*

- This result seems intuitive:
  - If you can find a way to jump over a three foot hurdle to the point that you succeed a high percentage of the time, jumping over a one foot hurdle seems **really easy**

Conclusions

- Cointegration analysis is a powerful statistical technique for finding asset allocations that may be able to act as a hedge against a variety of liabilities
- The relative merits of many forms of dynamic portfolio allocation such as portfolio insurance (pro-cyclical) and frequent rebalancing (counter-cyclical) become clearer in a cointegration framework.
- We believe the technique has particular merit as an implementation of "liability driven investing" where asset owners seek to minimize the volatility of the their funding ratios.
- The technique can also be used in a hedging framework to pursue active management returns. Empirical tests of cointegration on country index and asset class data provide encouragement that such techniques may provide avenues to earn returns in excess of market indices.

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