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Risk Decomposition of Investment Portfolios

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Main Concepts for Today

- Investment practitioners rely on a decomposition of portfolio risk into factors to guide investment decisions
- While the total estimated risk of a given portfolio is usually unambiguous, the way in which risk is allocated to constituent factors can be radically changed by:
  - The estimation process for the model, even with the same factors
  - Reporting conventions vary widely across vendors and systems
  - How the covariance among any pair of factors is allocated to the members of the pair
  - Inclusion or exclusion of basic portfolio constraints (e.g. portfolio weights should sum to 100%)
  - Different metrics (volatility, tracking error, VaR) behave differently

Implicit Decomposition in Factor Specification

- All factor models rely on a simple linear representation of asset (or portfolio) returns

$R_t = \Sigma_{i=1 \text{ to } n} B_i F_{it} + \varepsilon_t$

R_t = the asset return in period t

B_i = the factor exposure to factor i

F_{it} = the return to factor i in period t

ε_t = the return to factor i in period t

In times series models we observe the F values and statistically estimate the B values. In "fundamental" models we observe the B values and statistically estimate the F values. In blind factor (PCA) models we jointly estimate both.

Implicit Decomposition in Factor Specifications

- Let's assume we have two categories of factors called "red factors" and "blue factors". We could write such a model as to have G red factors and H blue factors

$$R_t = \Sigma_{i=1 \text{ to } g} B_i F_{it} + \Sigma_{i=g+1 \text{ to } g+h} B_i F_{it} + \varepsilon_t$$

If we had a reason to do so, we could define the blue factors as being "net" of the influence of the red factors. For example, if inflation and interest rates were both factors in the model, we might choose to put inflation in the red group and redefine "nominal interest rates" as "interest rates net of the effect of inflation and other red factors (e.g. real interest rates).

Implicit Decomposition in Factor Specifications

- We accomplish this structuring of the decomposition by using a two step estimation procedure

$$R_t = \Sigma_{i=1 \text{ to } g} B_i F_{it} + \zeta_t$$

$$\zeta_t = \Sigma_{i=g+1 \text{ to } g+h} B_i F_{it} + \varepsilon_t$$

ζ_t = the residual return at time t net of red factors only

Since we have defined and estimated the blue factors net of the red factors the risk decomposition *will naturally allocate more risk to the red factors and less to the blue factors*

Why do "staged" Model Estimation?

- You have a lot more data on some factors than others.
  - You have a universe of 1000 stocks broken into 50 industries
  - You will have 1000 data points for estimating the return to a factor like P/E or size, but only an average of 20 data points to estimate the return to a particular industry group.
- You have two or more factors that are highly correlated
  - Statistical estimation procedures often produce unstable results when independent variables are correlated.
  - By defining one factor net of another correlated factor, we structurally remove their natural correlation
- You have particular strategies where it makes sense
  - There has been a long debate about whether countries or sectors are more important to global equity portfolios.
  - The answer depends on whether you see the world as cap weighted or equal weighted

Risk Decomposition Conventions - Northfield

- We decompose variance
  - Variances are *naturally additive*, while standard deviations (or VaR segments) are not
  - Consistent with a long-term view of risk as decreasing compounded returns relative to arithmetic returns
  - Decompose variance into factor and specific components
- For a model with N factors, we will have a factor covariance matrix of N*N elements
  - The matrix is square and symmetric about the diagonal
  - We use the conventional assumption that of the covariance between any two factors, we will credit half of the covariance to each factor. There is an algebraic (not necessarily economic) rationale for this.
  - We then create a **row subtotal for each factor and report it**

Risk Decomposition Conventions - Northfield

- Depending on which Northfield model is being used factor exposures may or may not be put on a common scale
  - For non-scaled exposures the sign of an active factor exposure is relevant for both benchmark relative and absolute risk
  - With scaled exposures (e.g. Z scores) are often difficult to interpret in terms of absolute risk or VaR
  - Magnitude of an active "bet" must be judged from factor contribution in **variance units.**
- Security specific risk is summed across positions and presented as a single value
  - For our multi-asset class "EE" model, the relationships of multiple securities from the same issuer (e.g. Bank of America stock and a Merrill Lynch bond) are accounted for properly

Risk Decomposition Conventions – Vendor 2

- Another popular risk vendor decomposes variance into both factor and specific.
  - Each factor variance contribution is reported separately
  - *All factor covariance terms (the off diagonal elements) are added up to a single sum* and are reported separately. This obfuscates the joint effect of two factors that are highly correlated
- Factor exposures are scaled (Z score or percentage).
  - Benchmark relative active factor exposures can be easily observed and are often incorrectly interpreted as being the relative measure of bet size in either **standard deviation** or variance units **as the factor variances are not uniform.**
  - Neither the sign nor magnitude of factor exposures are easily interpreted in absolute risk terms (i.e. cash doesn't have a market cap or PE).

Risk Decomposition Conventions – Vendor 3

- Decomposes variance into factor and specific risk
- Statistically defined factors are assumed orthogonal so no off-diagonal covariance terms exist
  - Statistical factors must be mapped onto real world factors for economic interpretation
- All factors have the same unit volatility
  - Factor exposures are rescaled to reflect the relative risk of each factor in **standard deviation units**
  - Signs on factor exposures are arbitrary [1*1=1= (-1*-1)].
  - We can define continuous factor exposures in an arbitrary fashion but it's very unintuitive to do this with something like industry weights. Your factor exposure to the "short oil industry factor" is the negative of your factor exposure to the "oil industry factor"

Allocation of Covariance: An Example

- As previously noted, we assume that across any pair of factors (or assets), half the covariance is allocated to one factor and half is allocated to the other. Let's do a simple two asset example in absolute variance units

Stocks 20% Volatility, 60% weight

Bonds 5% Volatility, 40% weight

Assume 20% correlation between Stocks and Bonds

The variance of the portfolio is

$V_p = 20^2 * .6^2 + 5^2 * .4^2 + 2 * 20 * 5 * .6 * .4 * .2 = 157.6$

$V_p = 400 * .36 + 25 * .16 + 2 * (20 * 5 * .6 * .4 * .2) = 157.6$

Allocation of Covariance

- The conventional thing to do is to allocate half the covariance of a pair to one asset (or factor) and the other half to the other asset (or factor). Rearranging can get:

V_p = [144 + (20 *5 * .6 *.4 *.2)] + [4 + (20 *5 * .6 *.4 *.2)]

V_p = 148.8 + 8.8 + 0

- Alternatively we can get

V_p = 144 + 4 + 9.6

Allocated to stocks, Allocated to bonds, Allocated to covariance

Proportional Allocation of Covariance?

- In the first decomposition, the total covariance is 9.6 of which 4.8 is added to the first term (stocks) and 4.8 has been added to the second term (bonds).
  - It's easy and algebraically simple but not necessarily indicative of any underlying economics.
  - Is it economically realistic to do this "half and half" split given that the variance contribution of the stocks alone (much more volatile and bigger weight) is 36 times as big as the variance contribution of the bonds alone (much less volatile and smaller weight)?
  - One could easily argue for something like:

$$V_p = [144 + (144/148 * 9.6)] + [4 + (4/148* 9.6)]$$

$$V_p = [144 + 9.34] + [4 + .26] = 153.34 + 4.26$$

Other Allocations of Covariance

- In the first formulation, the amount of variance allocated to bonds (e.g. factor 2) is more than double what it is in the second and third formulations
  - If I were the bond PM, I'd like the second formula a lot more than the first.
  - The conventional approach (half and half) is one end of a spectrum. The Vendor 2 approach of keeping covariance completely separate is the other end of that spectrum.
  - There is a lot of area in between where some kind of proportional allocation makes the most sense
  - We have proposed other proportionality procedures (e.g. allocating based on the relative absolute value of the product of weight and volatility) but *results are still somewhat arbitrary*

Decomposition of Risk by Positions

- Many systems (e.g. Vendor 4) try to decompose risk (either variance or standard deviation) by the position.
  - This is often perceived to be intuitive for VaR calculations at banks (e.g. how much risk comes from each loan) because the *alternative is to not make the loan*, and we are measuring risk of loss in dollar amounts.
  - For asset management, the amount of capital (AUM) to be put at risk is fixed (i.e. institutional clients don't pay asset managers to hide money under their mattress).
  - Basically, we are deciding whether we do or don't want to enforce the requirement that asset weights sum to 100%
  - As such, any algebraic decomposition of risk by position requires (either explicitly or implicitly) the definition of a "contra-asset" which defines where the proceeds of closing out a position will be deployed
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Defining the Contra-Asset

- For analysis of absolute risk or VaR, the usual algebra implicitly defines the contra-asset as riskless cash

- For benchmark relative decomposition of incremental tracking variance by position, you could define the contra-asset as:
  - Cash
  - An ETF for the benchmark
  - Void (reweight the remaining portfolio positions to again add to 100%)

Choosing Your Contra-Asset

- Choosing cash is often done
  - Causes confusion in asset management because it's easy to have positions that are big risk contributors in absolute terms (e.g. high beta) but diversifiers on a benchmark relative basis (high beta is diversifying if the rest of the portfolio is low beta relative to benchmark
  - Implicitly going to cash increases tracking error while reducing absolute risk
- Using a benchmark ETF is messy
  - If the position X you are selling out is a member of the benchmark, selling out position X and replacing it with the benchmark ETF implicitly buys back some of the stock X risk exposure hat you just thought you got rid of.
  - The problem becomes mathematically recursive.

More Decomposition by Position

- You can act like a bank calculating it's $ VaR
  - Reweighting the remaining positions for asset management creates confusion because the new portfolio weights you will end up with after selling out position X, or position Y will be different,
  - As such, you can't directly compare the incremental volatility or variance risk changes across positions, which defeats the purpose.
  - This approach often works well for VaR because removing the successive positions and reweighting reflects that economic value of your portfolio has declined by the correct increment
  - VaR is an *incoherent measure*. You show that it leads to clearly wrong conclusions about risk for some problems.
- Some choices of "contra-asset" allow closed form allocation of volatility (standard deviation) and VaR and others do not

Position Decomposition More Thoughts

- From an analytical perspective, the only risk quantities that are exactly known are the *marginal variances* (MV) of a factor or position.
  - The problem is that the marginal variance has to be obtained from basic calculus (which non-math people don't like)
  - The MV values are legitimate only for infinitesimally small changes in position size which non-quant people see as unintuitive and non-actionable
- However, the marginal variances, not the incremental contributions by position are what matter in optimality
  - When you consider trading a position you can close out any part of it, not just "all or none".
  - The exception to this assertion would be something very illiquid like real estate

Decomposing Volatility

- The algebraic issues get even worse when you try to decompose by position in standard deviation units
  - Usually done to make it easy to do parametric VaR by position
  - Since standard deviations are not naturally additive, you sometimes have to use some kind of algebraic trick to allocate SD risks by position to add to the SD total. Some of the proposed schemes distort the economics less than others but *there is no exact solution for some definitions of the contra asset.*
  - Many systems do the variance contributions and then divide everything by the standard deviation *as a scalar constant,* which creates percentage allocations of standard deviation that are identical to the percentage allocations by variance.
  - Some vendors try to decompose by position, and then by factor within position. This produces lots of numbers that add up to the volatility but are very hard to use to actually make portfolio decisions

Conclusions

- How much risk we allocate to a given factor is heavily influenced by the estimation process of the model.
  - The robustness of statistical estimates can often be improved by staged estimations, but at the cost of more complex interpretation
- Risk service vendors report the decomposition of risk differently
  - Many of the reporting procedures follow an algebraic rather than economic reasoning
  - Much of the ambiguity relates to how the covariance terms are allocated to the involved factors
- When dealing with "incremental risk contributions by position" we will be either implicitly or explicitly dealing with the existence of the contra-asset
  - Only some of the possible definitions of the contra-asset have simple algebraic structures