aperiogroup

What Would Yale Do If It Were Taxable?

Patrick Geddes
Lisa Goldberg
Stephen Bianchi

Aperio Group, LLC
Three Harbor Drive, Suite 315
Sausalito, CA 94965
415.339.4300
www.aperiogroup.com

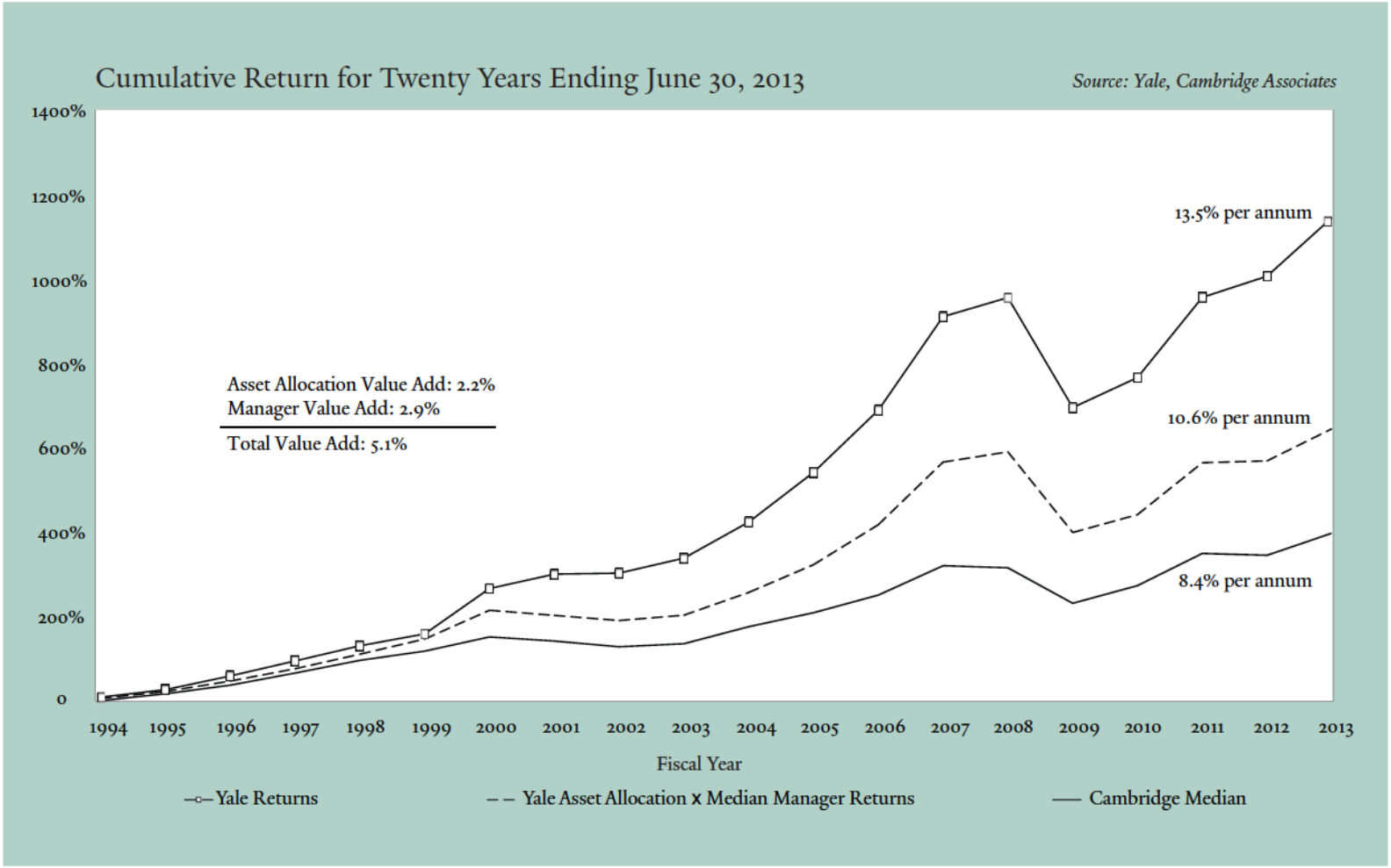
Aperio *v.* [Latin] to make clear, to reveal the truth

A Much-Admired Asset Allocation

The Yale Endowment Model

- Developed and popularized by maverick investor David Swensen, CIO of the Yale endowment
- Widely envied, widely emulated by other endowments, pensions, and individual investors
- Predicated on:
  - Diversification
  - Equity bias
  - Alternative (illiquid) investments

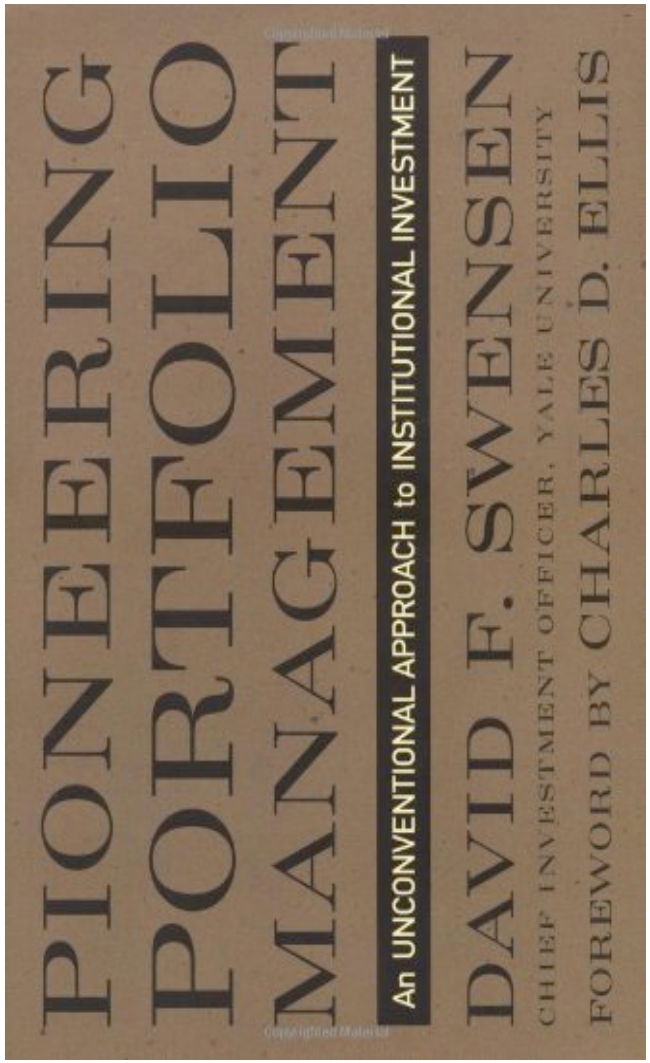
The Yale Endowment Model



Source: 2013 Yale Endowment (http://investments.yale.edu/images/documents/Yale_Endowment_13.pdf)

aperiogroup

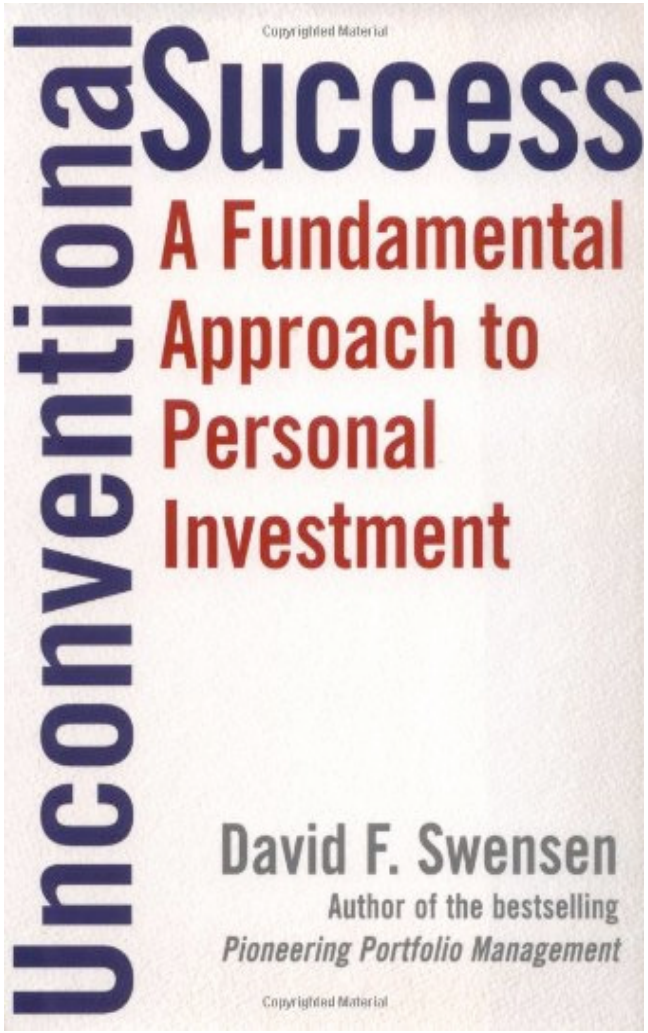
Swensen Speaks to Investors



In *Pioneering Portfolio Management: An Unconventional Approach to Institutional Investing,* "Swensen provides a brief history of endowment funds and explains the purpose of endowment accumulation and the goals for institutional portfolios. He distinguishes between traditional and alternative asset classes, looks at performance evaluation issues and tools, and considers the investment decision-making process."

Source: Amazon.com

Swensen Speaks to Investors



In *Unconventional Success: A Fundamental Approach to Personal Investment*, "Swensen reveals why the mutual fund industry as a whole does a disservice to the individual investor. Soft money, 12b-1 fees, overtrading, market timing, and other management practices lower performance and virtually guarantee that most mutual fund returns will fall short of their benchmark."

Source: Amazon.com

Don't Try This at Home

- Swensen counsels individual investors to avoid the alternative asset classes for which he is so famous

  Trendy investors often pursue the cocktail-party-chatter benefits of commitments to the promise, seldom filled, of actively managed alternatives. Sensible investors avoid the non-core asset classes.

  David Swensen, *Unconventional Success: A Fundamental Approach to Personal Investing* (New York, Simon & Schuster, 2005), 93.

- Cites inability of individual investors to access top managers

What Would Yale Do If It Were Taxable?

- Sea change: emergence of the "New Institutionals," who are ultra high net worth investors with access to Yale-level managers
- Problem: adapt the Yale model for a taxable investor
- Challenges:
  - Tax treatment varies across taxable investors
  - Identification of reasonable proxies for asset classes
  - Lack of reporting of after-tax returns

Swenson on Taxes

In an industry guilty of many crimes against investors, ignoring the tax consequences of portfolio transactions ranks among the most grievous.

David Swensen, *Unconventional Success: A Fundamental Approach to Personal Investing* (New York, Simon & Schuster, 2005), 93.

Another Great Thinker on Taxes

Although it is clear to everyone in the industry that after-tax returns are what economically matter to taxable investors, many firms continue to take the view that "we can ignore taxes entirely as long as our competitor managers do the same." In our view, this position of intentionally providing less than the best available services to separate account clients is unsustainable in the long run, and borders on a breach of fiduciary responsibility.

Dan diBartolomeo, *Northfield News,* September 2010, p. 1.
http://www.northinfo.com/Documents/390.pdf

Adapting the Yale Asset Allocation for a Taxable Investor

Adjusting Expected Returns for Taxes

- Focus on ultra high net worth investors
- Assume the highest tax bracket
- Identify the component of return that is realized
  - Decompose into short- and long-term gains, dividends, ordinary income
  - Apply the relevant tax haircuts
  - Calculate after-tax returns

Adjusting Expected Returns for Taxes

- Pre-Tax Weights + Pre-Tax Covariance Matrix ➔ Implied Pre-Tax Returns
- Apply Tax Haircut to Implied Pre-Tax Returns ➔ Implied After-Tax Returns
- Implied After-Tax Returns + Pre-Tax Covariance Matrix ➔ After-Tax Weights

Deriving Implied Expected Returns

- Based on William F. Sharpe's "Imputing Expected Security Returns from Portfolio Composition," *Journal of Financial and Quantitative Analysis*, June 1974
- Aperio study
  1. Use pre-tax weights to calculate pre-tax returns (reverse optimization)
  2. Apply "tax haircut" to derive after-tax returns
  3. With new after-tax returns, use optimizer to calculate new after-tax weights, using standard mean-variance optimization
- Purpose of reverse optimization: avoid controversy over return expectations

Asset-Class Weights and Implied Returns for Yale

Asset Class	Benchmark	Yale Pre-Tax Weight	Implied Pre-Tax Return
Absolute Return	DJ Credit Suisse Hedge Fund	17.8%	2.2%
World Public Equity	MSCI All Country World	15.7%	9.7%
Bonds	Barclays Aggregate	4.9%	1.5%
Natural Resources	Goldman Sachs Natural Resources	7.9%	10.7%
Real Estate	DJ US Real Estate	20.2%	12.3%
Private Equity	Russell 2000	32.0%	11.7%
Cash	US Treasury Bills 0-3 Month	1.5%	1.5%

Sources: Weights from annual report of the Yale Endowment as of December 31, 2013; returns calculated by Aperio Group using reverse optimization. Past performance is not a guarantee of future returns. The performance above is hypothetical and does not represent actual performance of the Endowment or any portfolio. Please refer to important disclosures at the end of this presentation.

Asset-Class Weights and Implied Returns for Yale

What's going on here?

Asset Class	Benchmark	Yale Pre-Tax Weight	Implied Pre-Tax Return
Absolute Return	DJ Credit Suisse Hedge Fund	17.8%	2.2%
World Public Equity	MSCI All Country World	15.7%	9.7%
Bonds	Barclays Aggregate	4.9%	1.5%
Natural Resources	Goldman Sachs Natural Resources	7.9%	10.7%
Real Estate	DJ US Real Estate	20.2%	12.3%
Private Equity	Russell 2000	32.0%	11.7%
Cash	US Treasury Bills 0-3 Month	1.5%	1.5%

Sources: Weights from annual report of the Yale Endowment as of December 31, 2013; returns calculated by Aperio Group using reverse optimization. Past performance is not a guarantee of future returns. The performance above is hypothetical and does not represent actual performance of the Endowment or any portfolio. Please refer to important disclosures at the end of this presentation.

Estimation of Tax Penalty Across Asset Classes

	% of Total Return from					
Asset Class	**Ord. Income**	**Realized Short Gains**	**Realized Long Gains**	**Unrealized Gains**	**Dividend Return**	**Tax Benefit**
Absolute Return	0%	23%	47%	30%	0.0%	0.0%
Equity, Active*	0%	23%	47%	30%	2.0%	0.0%
Equity, Indexed*	0%	0%	0%	100%	2.0%	0.0%
Equity, Tax-Adv.*	0%	0%	0%	100%	2.0%	1.9%
Taxable Bonds	100%	0%	0%	0%	0.0%	0.0%
Municipal Bonds	0%	0%	0%	0%	0.0%	0.0%
Nat. Resources	0%	10%	20%	70%	0.0%	0.0%
Real Estate	30%	0%	0%	70%	0.0%	0.0%
Private Equity*	0%	0%	30%	70%	2.0%	0.0%
Cash	100%	0%	0%	0%	0.0%	0.0%

*For equity strategies, % of Total Return applies to only the non-dividend portion of total return, i.e., gains. For all public and private equity strategies, dividends are assumed to be taxed at qualified dividend rates applied to a constant 2.0% dividend yield.
The above estimates are hypothetical and are not based on specific individual investments. Please refer to important disclosures at the end of this presentation.

Estimation of Tax Penalty Across Asset Classes

	% of Total Return from					
Asset Class	Ord. Income	Realized Short Gains	Realized Long Gains	Unrealized Gains	Dividend Return	Tax Benefit
Absolute Return	0%	23%	47%	30%	0.0%	0.0%
Equity, Active*	0%	23%	47%	30%	2.0%	0.0%
Equity, Indexed*	0%	0%	0%	100%	2.0%	0.0%
Equity, Tax-Adv.*	0%	0%	0%	100%	2.0%	1.9%
Taxable Bonds	100%	0%	0%	0%	0.0%	0.0%
Municipal Bonds	0%	0%	0%	0%	0.0%	0.0%
Nat. Resources	0%	10%	20%	70%	0.0%	0.0%
Real Estate	30%	0%	0%	70%	0.0%	0.0%
Private Equity*	0%	0%	30%	70%	2.0%	0.0%
Cash	100%	0%	0%	0%	0.0%	0.0%

*For equity strategies, % of Total Return applies to only the non-dividend portion of total return, i.e., gains. For all public and private equity strategies, dividends are assumed to be taxed at qualified dividend rates applied to a constant 2.0% dividend yield.
The above estimates are hypothetical and are not based on specific individual investments. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation hedge fund index)

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation HF index)

Assets with the worst tax haircut drop to nothing

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation HF index)

Looks like a 1960s pension allocation

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation HF index)

Tax losses allow for more alternatives

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

The alts can really prove their value as sources of diversification once the tax problem is solved.

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation HF index)

Bonds drop since alts now can reduce risk

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

Correlations Matter

The Only Free Lunch of Investing

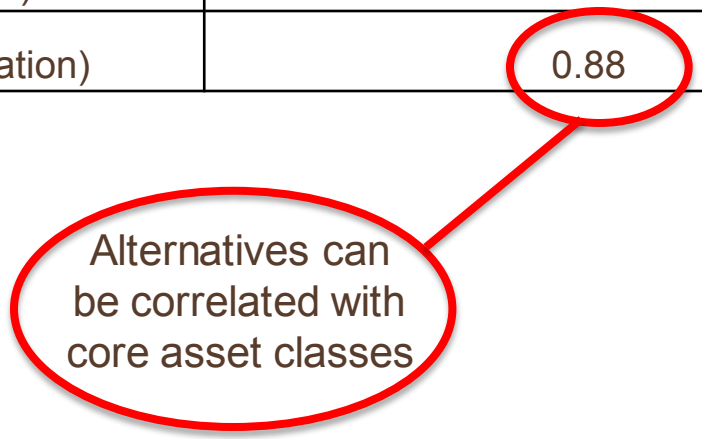
- **Diversification** is one of the main attractions of hedge funds and other alternatives
- The **presumption** is that alternatives are uncorrelated with core asset classes
- Is that true?

Equity/Hedge Fund Correlations

Index Benchmark for Absolute Return (hedge funds)	Correlation with Equities (MSCI ACWI)
DJ Credit Suisse Hedge Fund (low correlation)	0.14
HFRI Fund Weighted Composite (high correlation)	0.88

Equity/Hedge Fund Correlations

Index Benchmark for Absolute Return (hedge funds)	Correlation with Equities (MSCI ACWI)
DJ Credit Suisse Hedge Fund (low correlation)	0.14
HFRI Fund Weighted Composite (high correlation)	0.88



After-Tax Weights for Yale (high correlation HF index)

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	4.2%	3.3%	-0.9%	17.8%	0.0%	0.0%	0.0%
Equity, Active	9.2%	7.1%	-2.1%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.2%	8.7%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.2%	10.6%	+1.4%	0.0%	0.0%	0.0%	25.0%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	25.6%
Nat. Resources	10.2%	9.2%	-1.0%	7.9%	12.2%	0.0%	5.8%
Real Estate	11.4%	9.9%	-1.5%	20.2%	13.9%	9.0%	2.3%
Private Equity	11.0%	9.8%	-1.2%	32.0%	38.9%	19.6%	41.3%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (high correlation HF index)

Correlated hedge funds don't add value after tax

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	4.2%	3.3%	-0.9%	17.8%	0.0%	0.0%	0.0%
Equity, Active	9.2%	7.1%	-2.1%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.2%	8.7%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.2%	10.6%	+1.4%	0.0%	0.0%	0.0%	25.0%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	25.6%
Nat. Resources	10.2%	9.2%	-1.0%	7.9%	12.2%	0.0%	5.8%
Real Estate	11.4%	9.9%	-1.5%	20.2%	13.9%	9.0%	2.3%
Private Equity	11.0%	9.8%	-1.2%	32.0%	38.9%	19.6%	41.3%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low correlation HF index)

This is what we had before

Asset Class	Returns			Weights			
	Yale P/T Implied Return	Yale A/T Implied Return	Yale Tax Haircut	Yale P/T Weight	Yale A/T: Active Equity	Yale A/T: Indexed Equity	Yale A/T: Tax-Adv. Equity
Absolute Return	2.2%	1.7%	-0.5%	17.8%	0.0%	0.0%	11.9%
Equity, Active	9.7%	7.5%	-2.2%	15.7%	0.0%	0.0%	0.0%
Equity, Indexed	9.7%	9.2%	-0.5%	0.0%	0.0%	45.6%	0.0%
Equity, Tax-Adv.	9.7%	11.1%	+1.4%	0.0%	0.0%	0.0%	26.4%
Bonds*	1.5%	0.9%	-0.7%	4.9%	35.0%	25.8%	14.5%
Nat. Resources	10.7%	9.7%	-1.0%	7.9%	12.2%	0.0%	5.2%
Real Estate	12.3%	10.7%	-1.6%	20.2%	13.9%	9.0%	2.8%
Private Equity	11.7%	10.4%	-1.3%	32.0%	38.9%	19.6%	39.1%
Cash	1.5%	0.8%	-0.7%	1.5%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

Volatilities Matter Too: Adjusting a Covariance Matrix for Taxes

Risk and Expected Return

- Taxes generate a return penalty
- But taxes also lead to a smoother (less volatile) return series
- In the previous examples, we penalized tax-inefficient asset classes with lower return …
- … but we did not reward them with lower risk
- In the examples that follow, we tax-adjust the covariance matrix

Adjusting a Covariance Matrix for Taxes

- Pre-Tax Weights + Pre-Tax Covariance Matrix ➔ Implied Pre-Tax Returns
- Apply Tax Haircut to Implied Pre-Tax Returns ➔ Implied After-Tax Returns
- Implied After-Tax Returns + ***After-Tax*** Covariance Matrix ➔ After-Tax Weights

Adjusting a Covariance Matrix for Taxes

- Assume that pre-tax returns of tax-inefficient asset classes and tax haircuts are normally distributed with positive correlation ρ
- Higher correlation means smoother after-tax returns
- Sample from the distribution of tax haircuts (conditional on pre-tax returns)
- Simulate the after-tax return series
- Estimate the after-tax covariance matrix based on the simulated after-tax return series

After-Tax Volatilities (low correlation HF index)

Asset Class	Change in Annualized Volatilities using A/T Covariance Matrix				
	With P/T Covariance Matrix	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$
Absolute Return	6.0%	-0.3%	-0.7%	-1.0%	-1.3%
Equity, Active	17.2%	-1.0%	-1.9%	-2.8%	-3.8%
Equity, Indexed	17.2%	0.0%	0.0%	0.0%	0.0%
Equity, Tax-Adv.	17.2%	0.0%	0.0%	0.0%	0.0%
Bonds*	4.6%	0.0%	0.0%	0.0%	0.0%
Nat. Resources	23.5%	-0.6%	-1.1%	-1.7%	-2.2%
Real Estate	23.9%	-0.8%	-1.6%	-2.4%	-3.2%
Private Equity	20.6%	-0.4%	-0.8%	-1.2%	-1.5%
Cash	0.1%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Volatilities (low correlation HF index)

Greater risk reduction in active equity and real estate

Asset Class	Change in Annualized Volatilities using A/T Covariance Matrix				
	With P/T Covariance Matrix	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$
Absolute Return	6.0%	-0.3%	-0.7%	-1.0%	-1.3%
Equity, Active	17.2%	-1.0%	-1.9%	-2.8%	-3.8%
Equity, Indexed	17.2%	0.0%	0.0%	0.0%	0.0%
Equity, Tax-Adv.	17.2%	0.0%	0.0%	0.0%	0.0%
Bonds*	4.6%	0.0%	0.0%	0.0%	0.0%
Nat. Resources	23.5%	-0.6%	-1.1%	-1.7%	-2.2%
Real Estate	23.9%	-0.8%	-1.6%	-2.4%	-3.2%
Private Equity	20.6%	-0.4%	-0.8%	-1.2%	-1.5%
Cash	0.1%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low volatility)

Weight shifts to active equity and real estate

Asset Class	Change in Weights using A/T Covariance Matrix: Tax-Advantaged Equity				
	With P/T Covariance Matrix	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$
Absolute Return	11.9%	-0.2%	-0.6%	-3.2%	-6.2%
Equity, Active	0.0%	0.0%	+3.6%	+11.3%	+20.9%
Equity, Indexed	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Tax-Adv.	26.4%	-0.2%	1.0%	+3.9%	+7.6%
Bonds*	14.5%	-2.5%	-5.4%	-8.4%	-12.2%
Nat. Resources	5.2%	+0.9%	+0.5%	-1.3%	-3.6%
Real Estate	2.8%	+3.0%	+6.1%	+9.7%	+13.8%
Private Equity	39.1%	-1.3%	-5.3%	-12.2%	-20.4%
Cash	0.0%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Yale (low volatility)

Tax-advantaged equity is more important than ever

Asset Class	Change in Weights using A/T Covariance Matrix: Tax-Advantaged Equity				
	With P/T Covariance Matrix	**ρ = 0.25**	**ρ = 0.50**	**ρ = 0.75**	**ρ = 1.00**
Absolute Return	11.9%	-0.2%	-0.6%	-3.2%	-6.2%
Equity, Active	0.0%	0.0%	+3.6%	+11.3%	+20.9%
Equity, Indexed	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Tax-Adv.	26.4%	-0.2%	1.0%	+3.9%	+7.6%
Bonds*	14.5%	-2.5%	-5.4%	-8.4%	-12.2%
Nat. Resources	5.2%	+0.9%	+0.5%	-1.3%	-3.6%
Real Estate	2.8%	+3.0%	+6.1%	+9.7%	+13.8%
Private Equity	39.1%	-1.3%	-5.3%	-12.2%	-20.4%
Cash	0.0%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

Conclusions

Conclusions

- Since 2005, when David Swensen counseled individual investors to avoid alternatives, a class of ultra high net worth "New Institutional" investors has emerged
  - Perhaps the New Institutional investors have access to top managers …
  - … but unlike the Yale endowment, their gains are still taxable
- The impact of taxes on asset allocation is dramatic

Conclusions

- It may be possible to design an after-tax asset allocation that incorporates the diversification benefits and liquidity premiums of the Yale endowment. However:
  - After-tax investing principles must be incorporated
  - Diversification is essential and cannot be assumed
- Tax-efficient equity can facilitate the inclusion of diversifying tax-inefficient alternatives …
  - … regardless of whether after-tax returns are smoother than pre-tax returns

Thank You

Appendix

Asset-Class Correlations

Proxy #	Asset Class	Index Benchmark
1	Low Corr. Absolute Return	DJ Credit Suisse Hedge Fund USD
2	High Corr. Absolute Return	HFRI Fund Weighted Composite Index
3	World Equity - Active	MSCI ACWI NR USD
4	World Equity - Tax Neutral	MSCI ACWI NR USD
5	World Equity – Tax-Advantaged	MSCI ACWI NR USD
6	Fixed Income	Barclays Aggregate Bond TR
7	Municipal Bonds	Barclays Municipal
8	Natural Resources	Goldman Sachs Natural Resources
9	Private Equity	Russell 2000 TR
10	Real Estate	DJ US Real Estate TR USD
11	Cash	ML US Treasury Bills 0-3 Mon TR USD

	1	2	3	4	5	6	7	8	9	10	11
1	1.00	0.27	0.14	0.14	0.14	-0.15	-0.01	0.10	0.14	0.14	-0.08
2	0.27	1.00	0.88	0.88	0.88	-0.02	0.10	0.80	0.80	0.56	-0.32
3	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
4	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
5	0.14	0.88	1.00	1.00	1.00	0.02	0.07	0.80	0.87	0.72	-0.21
6	-0.15	-0.02	0.02	0.02	0.02	1.00	0.63	-0.04	-0.10	0.17	0.13
7	-0.01	0.10	0.07	0.07	0.07	0.63	1.00	0.00	0.00	0.19	-0.02
8	0.10	0.80	0.80	0.80	0.80	-0.04	0.00	1.00	0.69	0.50	-0.25
9	0.14	0.80	0.87	0.87	0.87	-0.10	0.00	0.69	1.00	0.76	-0.14
10	0.14	0.56	0.72	0.72	0.72	0.17	0.19	0.50	0.76	1.00	-0.05
11	-0.08	-0.32	-0.21	-0.21	-0.21	0.13	-0.02	-0.25	-0.14	-0.05	1.00

Source: Return data from Morningstar Principia for all indices except for proxy #2, from HFRI. For more information on HFRI indices, see https://www.hedgefundresearch.com/mon_register/index.php?fuse=login_bd&1397845036.
Correlations were calculated from monthly pre-tax total return data 12-31-98 through 06-30-13.

Impact of Tax-Adjusted Covariance Matrix on Asset Allocation

After-Tax Covariance Matrix for Yale (low correlation HF index)

Asset Class	Change in Weights using A/T Covariance Matrix: Active Equity				
	With P/T Covariance Matrix	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$
Absolute Return	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Active	0.0%	0.0%	0.0%	0.0%	+3.2%
Equity, Indexed	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Tax-Adv.	0.0%	0.0%	0.0%	0.0%	0.0%
Bonds*	35.0%	-3.1%	-6.6%	-10.4%	-15.6%
Nat. Resources	12.2%	+1.0%	+2.2%	+3.4%	+4.0%
Real Estate	13.9%	+2.8%	+6.1%	+9.9%	+14.0%
Private Equity	38.9%	-1.0%	-1.7%	-3.0%	-5.6%
Cash	0.0%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Covariance Matrix for Yale (low correlation HF index)

Asset Class	Change in Weights using A/T Covariance Matrix: Indexed Equity				
	With P/T Covariance Matrix	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.75$	$\rho = 1.00$
Absolute Return	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Active	0.0%	0.0%	0.0%	0.0%	0.0%
Equity, Indexed	45.6%	-8.8%	-19.5%	-30.6%	-42.4%
Equity, Tax-Adv.	0.0%	0.0%	0.0%	0.0%	0.0%
Bonds*	25.8%	-0.6%	-1.6%	-3.3%	-5.7%
Nat. Resources	0.0%	+2.6%	+6.6%	+11.0%	+15.9%
Real Estate	9.0%	+3.6%	+7.9%	+12.9%	+18.7%
Private Equity	19.6%	+3.2%	+6.6%	+10.0%	+13.5%
Cash	0.0%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

Advisor Allocation

After-Tax Weights for Advisor (low correlation HF index)

Asset Class	Returns			Weights			
	Sample Advisor P/T Implied Return	Sample Advisor A/T Implied Return	Sample Advisor Tax Haircut	Sample Advisor P/T Weight	Sample Advisor A/T: Active Equity	Sample Advisor A/T: Indexed Equity	Sample Advisor A/T: Tax-Adv. Equity
Absolute Return	2.5%	1.9%	-0.5%	25.0%	13.8%	11.3%	24.5%
Equity, Active	10.9%	8.4%	-2.5%	30.0%	0.0%	0.0%	4.4%
Equity, Indexed	10.9%	10.4%	-0.5%	0.0%	0.0%	51.0%	0.0%
Equity, Tax-Adv.	10.9%	12.3%	+1.4%	0.0%	0.0%	0.0%	24.6%
Bonds*	1.7%	0.9%	-0.7%	20.0%	42.3%	34.1%	23.2%
Nat. Resources	12.0%	10.8%	-1.1%	5.0%	13.1%	0.0%	5.7%
Real Estate	12.9%	11.2%	-1.7%	10.0%	8.0%	2.5%	0.0%
Private Equity	12.1%	10.8%	-1.3%	10.0%	22.9%	1.1%	17.5%
Cash	1.5%	0.8%	-0.7%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

After-Tax Weights for Advisor (low correlation HF index)

Asset Class	Returns			Weights			
	Sample Advisor P/T Implied Return	Sample Advisor A/T Implied Return	Sample Advisor Tax Haircut	Sample Advisor P/T Weight	Sample Advisor A/T: Active Equity	Sample Advisor A/T: Indexed Equity	Sample Advisor A/T: Tax-Adv. Equity
Absolute Return	2.5%	1.9%	-0.5%	25.0%	13.8%	11.3%	24.5%
Equity, Active	10.9%	8.4%	-2.5%	30.0%	0.0%	0.0%	4.4%
Equity, Indexed	10.9%	10.4%	-0.5%	0.0%	0.0%	51.0%	0.0%
Equity, Tax-Adv.	10.9%	12.3%	+1.4%	0.0%	0.0%	0.0%	24.6%
Bonds*	1.7%	0.9%	-0.7%	20.0%	42.3%	34.1%	23.2%
Nat. Resources	12.0%	10.8%	-1.1%	5.0%	13.1%	0.0%	5.7%
Real Estate	12.9%	11.2%	-1.7%	10.0%	8.0%	2.5%	0.0%
Private Equity	12.1%	10.8%	-1.3%	10.0%	22.9%	1.1%	17.5%
Cash	1.5%	0.8%	-0.7%	0.0%	0.0%	0.0%	0.0%

*Bonds are assumed to be taxable for the pre-tax allocation and tax-exempt municipals for the after-tax portfolios. All distributions from tax-exempt bonds are assumed to be tax-exempt, i.e., possible capital gains are ignored.
Note: P/T = Pre-Tax, A/T = After-Tax. The above weights and returns are hypothetical and are not based on actual investments. Cash has been constrained in optimization to show a weight of zero. Please refer to important disclosures at the end of this presentation.

Disclosure

The performance reflected in the tables and charts in this report are hypothetical, shown for illustrative purposes only, and not based on actual investments. Furthermore, they do not reflect the deduction of any management fees or transaction costs, which would lower performance returns. Hypothetical performance has inherent limitations, and investors may experience investment results materially different from those portrayed.

Any forward-looking statements or estimates are based on assumptions, and actual returns are expected to vary from any such statements or estimates. No reliance should be placed on any such statements or forecasts when making any investment decision. The assumptions and projections displayed are estimates, hypothetical in nature, and meant to serve solely as a guideline. The results and analysis are not guarantees of future results because they are derived from mathematical modeling techniques of the economic and financial markets that may or may not reflect actual conditions and events.

The information contained within this presentation was carefully compiled from sources Aperio believes to be reliable, but we cannot guarantee accuracy. We provide this information with the understanding that we are not engaged in rendering legal, accounting, or tax services. In particular, none of the examples should be considered advice tailored to the needs of any specific investor, and nothing herein is intended to be investment advice. There is risk of loss from an investment in securities, including the risk of loss of principal. Different types of investments involve varying degrees of risk, and there can be no assurance that any specific investment will be profitable or suitable for a particular investor's financial situation or risk tolerance. Asset allocation and portfolio diversification cannot assure or guarantee better performance and cannot eliminate the risk of investment losses. Accordingly, you should not rely solely on the information contained in these materials in making any investment decision, and we recommend that all investors seek out the services of competent professionals in any of the aforementioned areas

Aperio's investment advisory fees are described in Part 2A of its Form ADV. Aperio's investment advisory fees and any other expenses that may be incurred in the management of any account will reduce performance results of Aperio clients. The Monte Carlo results do not include management fees. A difference in fees between a separate account and an ETF or other strategy could lead to a return benefit lower than those shown. With respect to the description of any investment strategies, simulations, or investment recommendations, we cannot provide any assurances that they will perform as expected and as described in our materials. Past performance is not indicative of future results. Every investment program has the potential for loss as well as gain.

The value added from the tax-advantaged equity strategy reflect Monte Carlo simulations based on the following assumptions:

- Time Horizon—10 years
- Individual Stock Volatility—41%
- Dividend Yield—2.0%
- Bid/Ask Spread (round-trip)—0.08%
- Annual Delisting from Index—4.0%
- Per-share Commissions—$0.01
- Expected Market Return—7.0%

Due to the complexity of tax law, not every single taxpayer will face the situations described herein exactly as calculated or stated, i.e., the examples and calculations are intended to be representative of some but not all taxpayers. Since each investor's situation may be different in terms of income tax, estate tax and asset allocation, there may be situations in which the recommendations would not apply. Please discuss any individual situation with tax and investment advisors first before proceeding. Taxpayers paying lower tax rates than those assumed or without taxable income would earn smaller tax benefits from tax-advantaged indexing or even none at all compared to those described.