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An Optimized Approach to Scenario Driven Risk Simulations

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Introduction

- This presentation provides a new approach to risk assessment from numerical simulations. As risk-related regulation extends from commercial banking to other parts of the financial services industry, risk assessments arising from "stress tests" and "scenario analysis" have become more widely discussed and implemented.
- Unfortunately, traditional methods for this kind of risk assessment are often counter-productive for long term investors who are not levered.
- To resolve the shortcomings of numerical methods we have built a new process, extending the approach suggested in Meucci (2008) which combines Monte Carlos simulations with the flexibility to overlay complex explicit scenarios. The analytical output of the process is a robust representation of the distribution of possible outcomes, while being consistent with any mathematically feasible "stress scenario".

Is Risk Now?

- For financial intermediaries such as commercial banks that are generally highly levered, the conception of risk is about ***solvency.***
  - The liabilities of the entity are current and subject to immediate call.
  - The objective is to make as much money as you can each day while limiting the probability of going broke to some acceptable level so you likely to be in business tomorrow.
  - diBartolomeo (2010, JOI) found that the typical implied half-life of a financial firm is on the order of 20 years, but much shorter (e.g. 8 years) on a revenue weighted basis.
  - Risk is measured in value units (e.g. VaR, CVaR) and **material effort is spent to get accurate prices for assets**

- Geared hedge funds are in an even worse position as margin loans are at call, *and* prime brokers don't care about trading costs in a forced liquidation as long as they are whole.

Or Later?

- Sovereign wealth funds are the opposite end of the spectrum. You can't go broke if you don't owe anybody any money.
  - For long term unlevered investors, the key risk is ***the estimated variance of the future return stream*.**
  - $1 invested for 50 years at a fixed 8% annually produces $46.90
  - $1 invested for 50 years at an average 8% annually with a standard deviation of 20% produces only $17.42
  - If you want to get fancy you can adjust the volatility to account for skew and kurtosis. See Wilcox (2000).
  - *Precision in current asset pricing is largely irrelevant.*

- Pension funds and insurance companies have only actuarial liabilities, which are the present value of expected future liabilities.
  - The liabilities are not subject to immediate call.

Dealing with "Now" Risk

- Existing processes have worked in one of two ways. The first is Monte Carlo simulations of asset prices where there is random sampling from a parametric or empirical distribution to get a range of possible outcomes. Risk assessments are based on the lower tail of the portfolio value distribution.

- The second process is to forecast a single return value for a set or series of specific exogenous scenarios. For example "What will be the % change in the value of my portfolio *(notice it's a single point value)* if interest rates go up 2%? and oil prices go down 30%."
  - It is argued that if we look at enough different "stress scenarios" we can gain an intuition about "worst case" outcomes. Unfortunately, the way most stress scenarios are formulated, their actual probability of occurrence is very, very small. Investors predicating investment strategy on such low probability outcomes end up with portfolios that are materially sub-optimal in the vast preponderance of situations.

“We Told You So” from 2006

- In May and September 2006 we published articles describing the limitations of such analyses, and suggested fixes.
  - Scenario based stress tests produced only a single point estimate outcome for each scenario. Even with many scenarios you can think you know something only for a small portion of the probability density.
  - Strategies predicated on low probability events are sub-optimal the rest of the time. It’s hard to live your life never crossing a street.
  - Monte Carlo simulations just numerically get the distribution of portfolio values based on assumed underlying distributions. Not much different than a simple factor model unless complex derivatives are involved.

- During the Global Financial Crisis all sorts of financial institutions from global banks to hedge funds became insolvent.
  - All of these entities had elaborate risk systems, mostly based on numerical simulations as prescribed by banking and other regulators.

Other Issues with Stress Testing and Scenarios

- Any ***scenario should be mathematically coherent, which is often a non-trivial exercise in conditional probability.***
  - A partial equilibrium solution to a full equilibrium world
  - Alternatively, the expected outcome for each factor must be coherent in terms of the expected outcome of every other factor, not just the factor or factors for which we intend to explicitly forecast outcomes.
  - Let's assume that we have a 50 factor risk model, of which oil prices are listed as factor #1. If we hypothesize a 45% to 55% rise in oil prices, we must ensure that our expected range outcome for factor #2 is consistent with the correlation between factor #1 and factor #2.
  - For our 50 factor model, there will be 1225 relationships
- It's hard to simulate events that have never *yet* happened like the 1987 crash, "Flash Crash," or the August 2007 liquidity problem.
  - Chebyshev's inequality comes in very handy
  - 1, 3, 600,19 and the CLT

A Numerical Method We Like

- Since the future is unlikely to be exactly like the past, we should be interested in whether the sequence of past events we have lived through is typical or unusual, given available history.
- As described in our June 2013 newsletter, our preferred numerical simulation method for exploring the distribution of a set of outcomes is "bootstrap" resampling.
  - We can use bootstrap methods to answer the broader question of "what if things had been different" but drawn from a similar distribution. set of factor return experiences.
  - However, rather than using the actual sequence of events (e.g. factor returns) we will be using many sequences of randomized events drawn from an historic set of experiences.
  - In essence, we will assume that the future may follow *any one of an infinite number of paths that we might have experienced in the past.*

Basic Bootstrapping in Brief

- Mechanically, the process is easy and very, very fast.
  - We use any of our risk models to get the factor profile of the portfolio
  - Let's assume we want to make a period by period forecast of the return distribution for the next 12 months and that we have a 240 month history of factor returns.
  - To create our first sequence of synthetic history as our forecast, we draw random number N between 1 and 240. The factor returns for month N are now the first month of our first sequence of our forecast factor events. If we repeat the process 12 times, we will have one full sequence of potential future events.
  - Note that since the choice is random each time, not only is the order of events randomized but some observations may be omitted and some observations may be repeated more than once.
  - The probability of choosing each observation is 1/N at each moment
  - For each path we estimate the return on the portfolio for each month, assuming a random draw from the distribution of idiosyncratic risk.

Let’s Add a Little More Realism

- Given the simple computational process, we can repeat this entire procedure many thousands of times in a few minutes to *produce a very robust estimate of the future distribution.*
  - At each point in each path, we can estimate calculate the estimated mean, volatility, cumulative returns, maximum drawdown, etc.
  - We can also analyze the cross-section of paths at each moment in time to describe the period by period distribution of the statistics.
- We can also account for serial properties
  - If we believe that asset returns are serially correlated randomizing the sequences will fail to represent this aspect of the data.
  - To address this we can follow the procedure above, but build our sequences of future events from blocks of multi-month periods so as to capture most of the dependence from one month to the next.
  - The length of the blocks would relate to the number of lags in an autoregressive process.

“Even God Cannot Change the Past”: Agathon

- So far, we are just sampling from an empirical distribution.
  - Any of the paths we generate are plausible
  - All of the statistical relationships between factors would hold together
  - We can see how typical or atypical the actual sequence of history within the range of the paths we generate.

- The results are not a lot different than if we did Monte Carlo simulations that incorporated the higher moments and serial properties of the expected distribution.
  - But the use of an empirical distribution at least ensures that effective distribution is realistic (it did actually happen).

- But a lot of things have changed since 450 BC. Even if we can’t change the past we can pretend that we can.

Let's Try Playing God

- In terms of our risk simulations what we really want is to combine the rich distributional information of a numerical simulation with the "intuitive" nature of a set of explicit scenarios. Such a combined process is described in Meucci (2008)
  - Attilio and I were part of a session on this at the Society of Actuaries conference last March.
- It is possible to "stress test" the projections by filtering the set of past observations from which our projected sequences are built.
  - We could include only months from periods of economic recession, or had rising interest rates or include only months that were perceived as particularly volatile. *Meucci refers to this as "crisp conditioning."*
  - If we have a "seed sample" of N observations and we filter out P observations, the probability of any observation being drawn to fill a position in a particular path is *1/(N-P) or zero.*
  - Now we have dense simulated data in both time series and cross-section, *conditional on the stressful or benign filtering.*

Now Let's Get Flexible

"Who is to say that truth is in the crystal and not in the mist?"
Kahlil Gibran

- We can set up a more flexible process where the probability of any particular observation being drawn for inclusion in a bootstrap path is explicitly defined by the user.
  - Instead of the probability of inclusion being 1/(N-P) we can choose a vector of explicit values for each observation.
  - Each probability p_t must be between zero and one
  - The sum of all values of p_t must equal one
  - Meucci refers to this as *flexible conditioning*
- While obviously feasible, it is not immediately obvious how an investor would decide what values should populate the probability vector.

Scenario Based Flexible Conditioning

- We would like to combine bootstrap simulations with explicit scenarios.
  - We can build the probability vector for inclusion of observations so as to fulfill the some explicit scenario within a confidence interval.
  - For example, we could say "Do a bootstrap simulation where ***on every path,*** the 10 year interest rate rises between 297 and 303 basis points, and oil prices decline 11 to 13% over a 12 month interval."
  - Observations with increasing interest rates and declining oil prices get more weight and vice-versa.
  - We can specify any variable for which data exists for the seed sample. We are not limited to the factors of an underlying model.
  - We can generate several different scenarios and select the number of paths to be run for each to represent weights. We just do our cross-sectional statistics on the aggregated paths.
  - *The cross-sectional variation in the paths is an implicit measure of the likelihood of the scenario.* If all paths are similar we know that the only a small fraction of all feasible paths fulfill the scenario.

Calculating the Probability Vector

- Figuring out what probability vector best expresses a given scenario is an optimization problem. We want to find the vector of probabilities such that:
  - All values of p_t are equal to or greater than zero
  - All values of p_t are less than one
  - All values of p_t sum to one
  - The attributes described in the scenarios are fulfilled within the prescribed ranges.
  - We preserve maximum randomness by minimizing the sum of the differences (absolute or squared) between each value of p_t and 1/N.

- If you use the Northfield Optimizer to solve the problem, it will also come up with the closest possible probability vector if the scenario is infeasible within the range of outcomes of the seed sample.

A Quick Review

- In our process, we use our regular risk models to get a time invariant representation of the portfolio and/or liabilities.
  - Our SIENS process can be used incorporate complex derivatives.
  - Unlike normal risk model usage, we represent not only variation around the mean, but uncertainty of the mean return.
- We use bootstrap resampling to compile a wide range of alternative simulations of history drawn from a seed sample of historical data.
  - The probability of any observation being included in a simulated path can be conditioned by filtering (crisp) or by a probability vector (flexible).
  - The path driven simulations provide a rich set of statistical metrics in both time series and cross-section.
  - For any feasible explicit scenario there exists a corresponding best probability vector. Multiple scenarios may be easily combined.
  - Finding the best probability vector is a tractable optimization problem.

Conclusions

- We have long held reservations as to the usefulness of “stress tests” and “scenario analysis” for financial institutions where day to day solvency is not the primary goal of risk management.
  - Strategies based on low probability scenarios are sub-optimal for the vast preponderance of circumstances.
- Irrespective of our view, regulatory reforms in many countries are forcing more financial organizations to at least consider these concepts in their risk management process.
- Combining our normal factor risk models, bootstrap resampling and scenario driven conditioning can provide a rich set of information about the potential distribution of future periodic or cumulative return outcomes over a short or long time horizon, in a way that can be more intuitive for fundamental investors. The process is also very computationally efficient