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Unified Theory of Credit Spreads and Defaults

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School 1: Theory of Bonds and Credit Spreads

Fundamental identity of bond's risky return in response to rates:

$$R_{Bond} = EFFECTIVE\ DURATION_{Bond} * \Delta INTEREST\ RATES$$

Changes in interest rates can be expressed as:

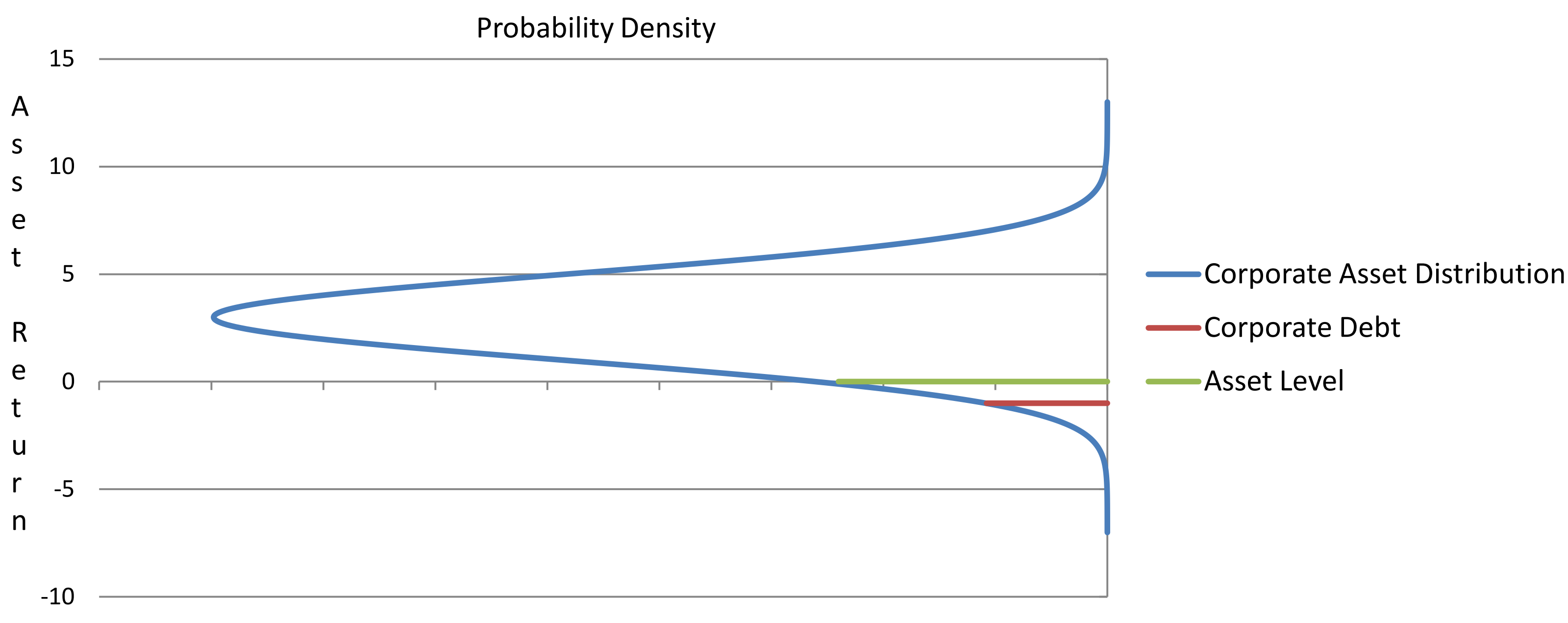
$$\Delta INTEREST\ RATE = \ \Delta RISK\ FREE\ RATE + \ \Delta CREDIT\ SPREAD$$

If the Risk Free Rate is held constant:

$$R_{Bond} = SPREAD\ DURATION_{Bond} * \Delta CREDIT\ SPREAD$$

Spread Duration is a version of Effective Duration that accounts for the shorter cash flow timeline under default scenarios in the risk neutral distribution (time related to Expected Life of Firms, diBartolomeo, 2010)

School 2: Theory of Contingent Claims Credit

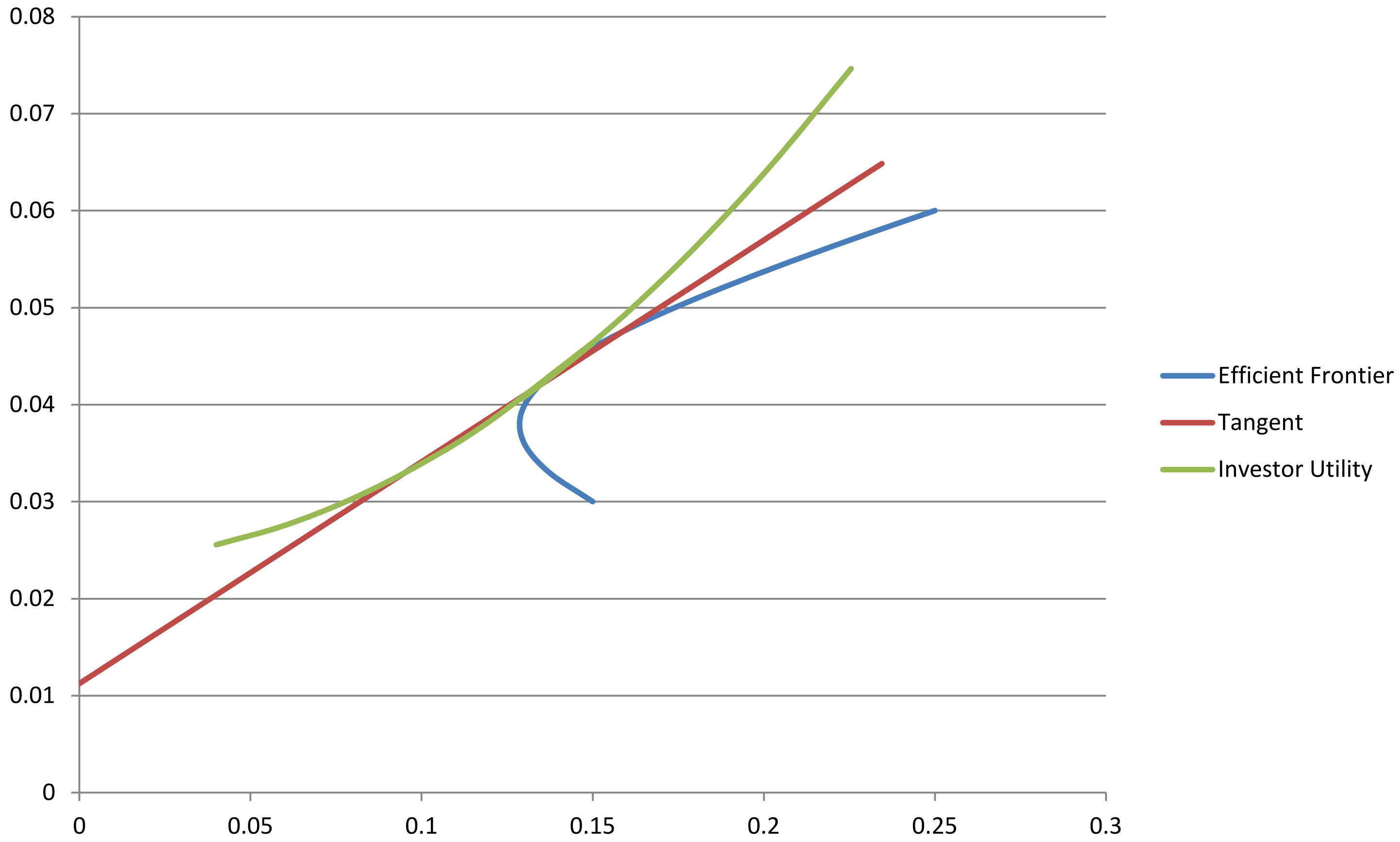


Inputs to Credit Model: **σ** **Debt** **Asset Level**

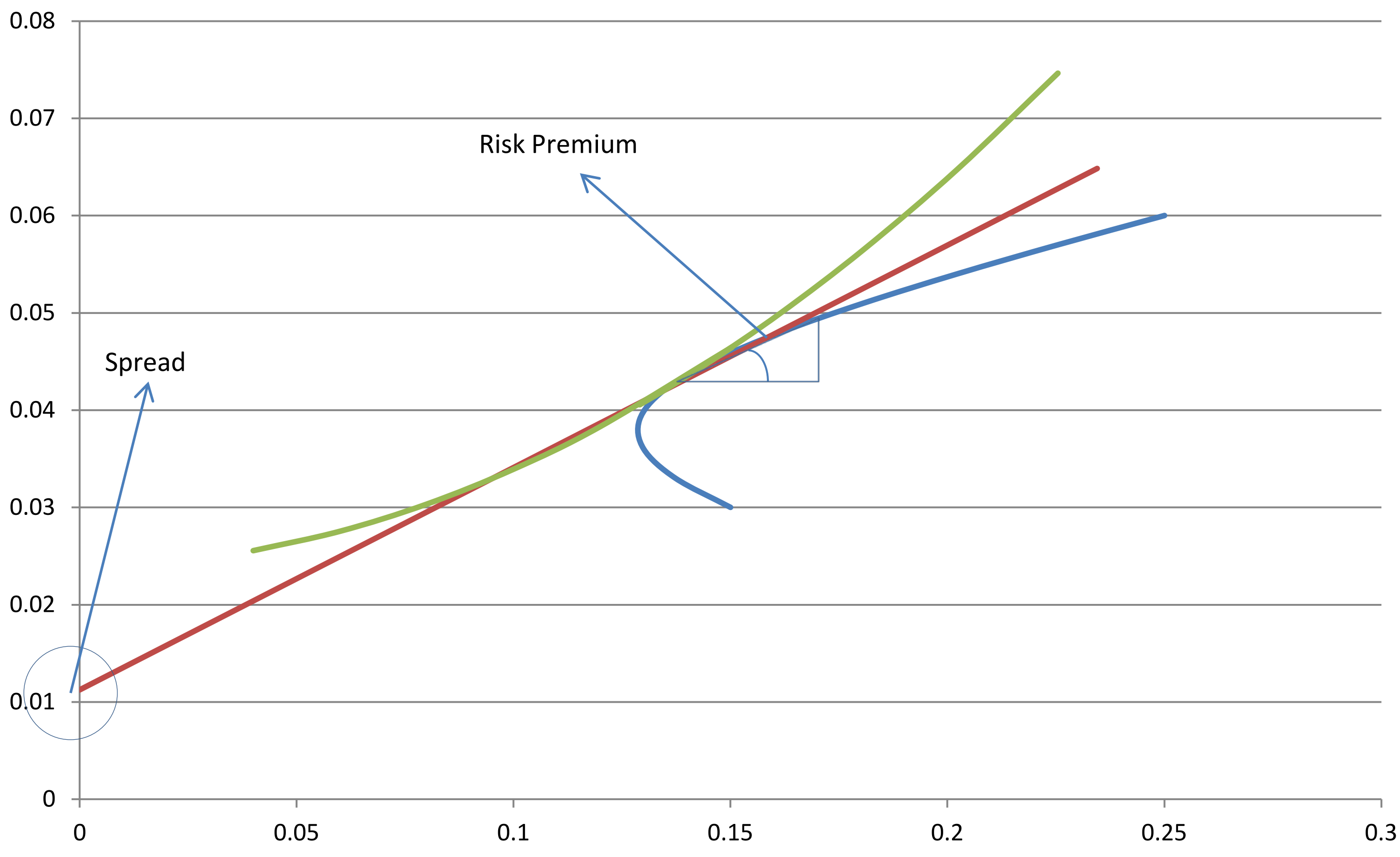
Model States: $\boldsymbol{\beta_{bond} = \beta_{stock} * -(P_{stock} / P_{bond}) * (\Delta_{put} / \Delta_{call})}$

Corollaries: ① LGD = P_{bond} * scalar; ② LGD & OAS ➔ Prob. Default

What is the Credit Spread to MVO Investor



OAS as an Intercept and Link to Risk Premium



An Apparent Paradox

- Assuming constant treasury rates, we can view OAS is the “Certainty Equivalent” from utility theory: “If I was offered an investment with no credit risk, or this risky investment, what is the deterministic rate of return that is going to make me indifferent ?”

- Then by exponential expected utility:

  OAS = Expected Return – Aversion Coefficient * Variance

- Paradox: for an risk-averse investor it appears that OAS < Expected Return

- But in practice OAS is always > Expected Return

- To resolve this we will start with a numerical example of two bonds – “investment grade” bond and a “junk” bond.

A Tale of Two Bonds

- An “investment grade” bond offering these payoffs (and probabilities):
    - $100 (75%)
    - $90 (20%)
    - $20 (5%)

- A “junk” bond offering these payoffs and probabilities:
    - $100 (10%)
    - $30 (50%)
    - $20 (40%)

- For both cases, at any entry price, the OAS will be higher than the expected return because the OAS will produce $100 for all outcomes.

A Tale of Two Bonds (cont'd)

- For high quality bonds an increase of the OAS is accompanied by a negative skew (the probability hump is towards high values) becoming more negative. Keeping the mean the same, negative skew means more probability in above average outcomes.

- For low of quality bonds an increase in OAS will correspond to a positive skew (the probability hump is towards low values) becoming more positive. Higher positive skew implies better, even if small chance of recouping the face amount.

- These imply two distinct relationships between OAS and skew

  OAS = Expected Return – Variance Aversion Coefficient * Variance **+/-** Skew Aversion * Skew

- How does this helps us make sense of the OAS > Expected return vs. the exponential utility expression paradox ?

A Small Detour: Entropy

- We can use an entropy argument to resolve the paradox

- From information theory Shannon Entropy is defined as:

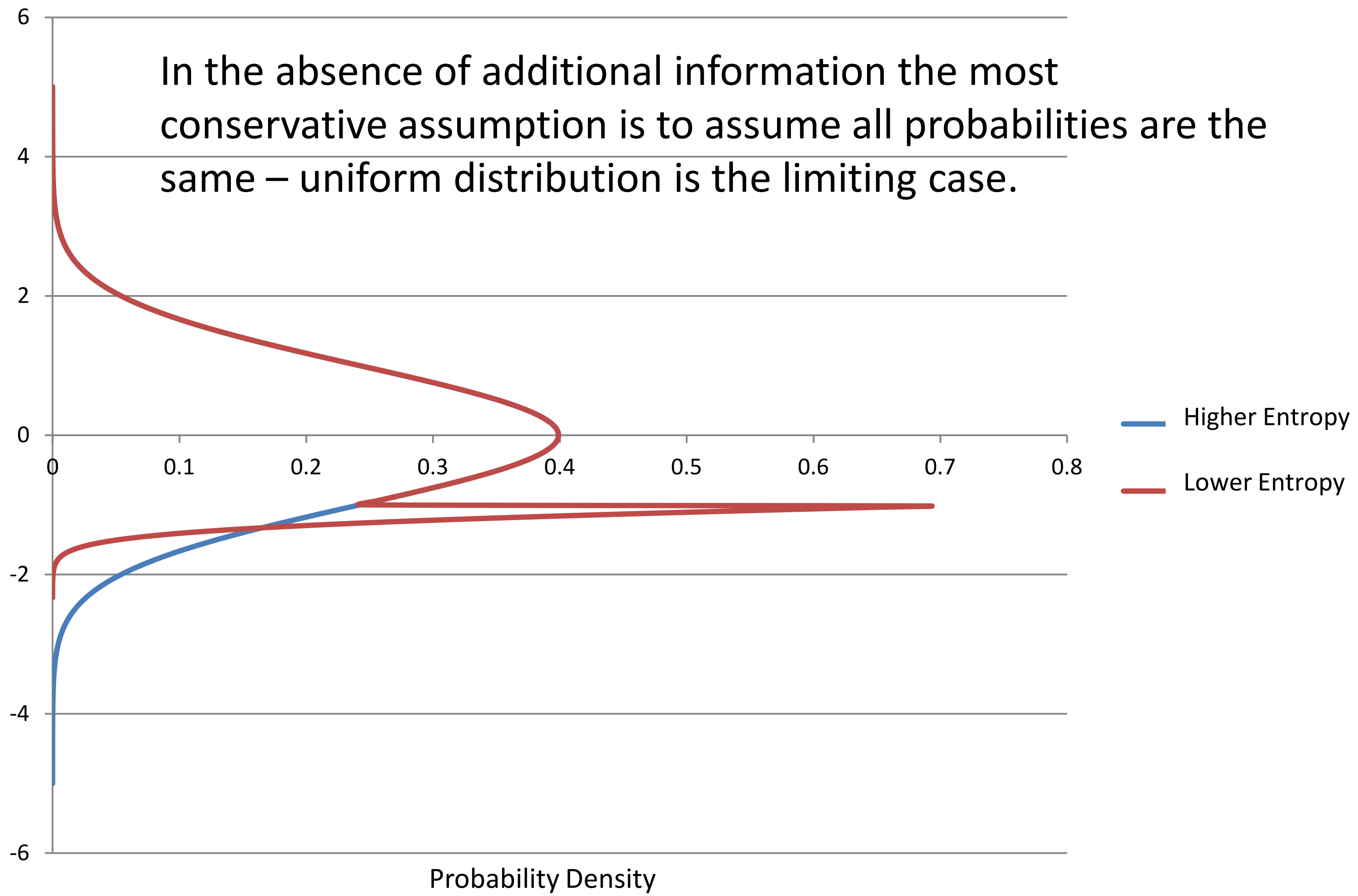
$$S = -\sum_{i=1}^{N} P_i * \ln(P_i)$$

- Entropy measures the level of order (i.e. distinctive information about mutually exclusive events) in a system (corresponding to the state of those events)

  - The more alike the probabilities of events "i" the higher the entropy
  - The more one or a couple of events have higher probabilities than the rest events the lower the entropy

Maximum Entropy Principle

In the absence of additional information the most conservative assumption is to assume all probabilities are the same – uniform distribution is the limiting case.



Implications of Maximum Entropy Principle

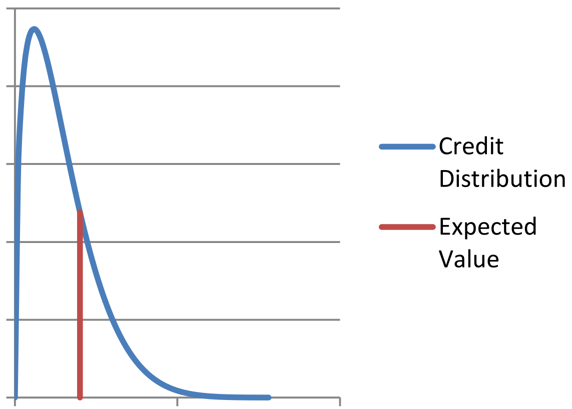
- Assume investors are aware of distributional higher moments of the distribution but they cannot reliably estimate them
  - By definition higher moment events occur in the tails and thus too rarely to be reliably estimated

- Based on the Maximum Entropy Principle, however, in utility terms, investors can consider the combined impact of variance with higher moments for truncated distributions like that of a bond.

- For bonds with positive skew (low credit quality) the principle will entail that in the absence of any other information an increase in variance will increase the positive skew

- For bonds with negative skew (high credit quality), in the absence of any other information an increase in variance will decrease in the negative skew

Variance Increase: Positive Skew Impact



$$M2 = \sum_{i=1}^{N} P_i\,(x_i - \mu)^2$$

$$A = \sum_{i=1}^{K \ni x<\mu} P_i\,(x_i - \mu)^2$$

$$B = \sum_{i=K+1}^{N} P_i\,(x_i - \mu)^2$$

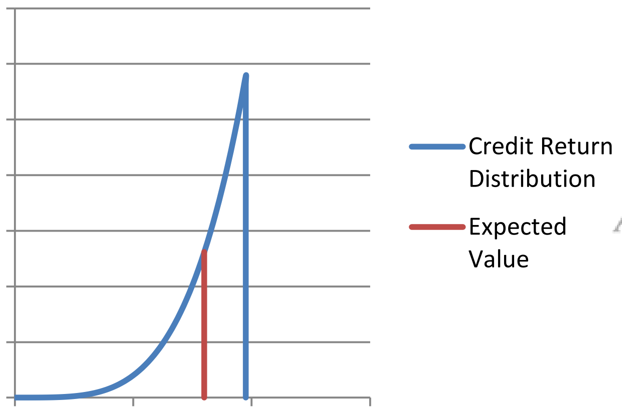
$$M2 = A + B$$

If we have a positive skew

$$\left|\frac{dB}{d[P(x_B)]}\right| > \left|\frac{dA}{d[P((x_A))]}\right|$$

So to increase M2 an increase in probabilities in B will accomplish more than an increase of probabilities in A. This will also minimize the decrease on entropy. *However, this pattern of probability change will also increase the third moment (skew)!*

Variance Increase: Negative Skew Impact



$$M2 = \sum_{i=1}^{N} P_i\,(x_i - \mu)^2$$

$$A = \sum_{i=1}^{K \ni x<\mu} P_i\,(x_i - \mu)^2$$

$$B = \sum_{i=K+1}^{N} P_i\,(x_i - \mu)^2$$

$$M2 = A + B$$

If we have a negative skew

$$\left|\frac{dB}{d[P(x_B)]}\right| < \left|\frac{dA}{d[P((x_A))]}\right|$$

So to increase M2 an increase in probabilities in A will accomplish more than an increase of probabilities in B. This will also minimize the decrease on entropy. *This pattern of probability change will also decrease the third moment (skew)!*

Example with a Beta Distribution

Beta Distribution, defined by parameters α and β :

$$Mean = \frac{\alpha}{\alpha + \beta}$$

$$Varaince = \frac{\alpha\beta}{(\alpha+\beta)^2 * (\alpha + \beta + 1)}$$

$$Skew = \frac{2\,(\beta - \alpha)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}}$$

To keep Mean the same:

$$\left[\frac{\Delta\alpha}{\Delta\beta} = \frac{\alpha}{\beta} => \Delta\beta = \Delta\alpha\frac{\beta}{\alpha}\,; Skew < 0 \rightarrow \beta < \alpha\right] => \boldsymbol{\alpha} \downarrow \cap\ \boldsymbol{\beta} \downarrow \rightarrow \boldsymbol{Skew} \downarrow$$

Summary of Utility Equality

- "Investment Grade" Bonds: investors like outcomes close to principal protection, which entails negative skew acceptance parameter:

*OAS = Expected Return – Variance Aversion Coefficient * Variance - Skew Coefficient * (negative Skew)*

By entropy, as variance increases Skew becomes more negative, which may overcompensate for positive variance aversion in terms of preference

- "Junk" Bonds: investors like speculative outcomes close to unlikely return of principal, which entails positive skew acceptance parameter:

*OAS = Expected Return – Variance Aversion Coefficient * Variance + Skew Coefficient * (positive) Skew*

We then can approximate via:

OAS = Expected Return + Adjusted Aversion Coefficient * Variance

Higher Moments of Log Return

- By *Fama and Booth* (1992):

$$E[\ln(1+r)] = \ln[1+E(r)] - \frac{VAR}{2[1+E(r)]^2} + \frac{M3}{3[1+E(r)]^3} - \frac{M4}{4[1+E(r)]^4}$$

- *Wilcox and Fabozzi* (2009) propose and *Belev and Gold* (2015) offer additional argumentation that the utility of a levered investor and hence their aversion will depend on leverage ratio L = Assets / Equity. By Wilcox and Fabozzi:

$$E[\ln(1+L*r)] = \ln[1+L*E(r)] - \frac{L^2VAR}{2[1+L*E(r)]^2} + \frac{L^3M3}{3[1+L*E(r)]^3} - \frac{L^4M4}{4[1+L*E(r)]^4}$$

- If we take LHS as the objective of optimization, dividing both side by L will not change the optimization result:

Higher Moments of Log Return (cont'd)

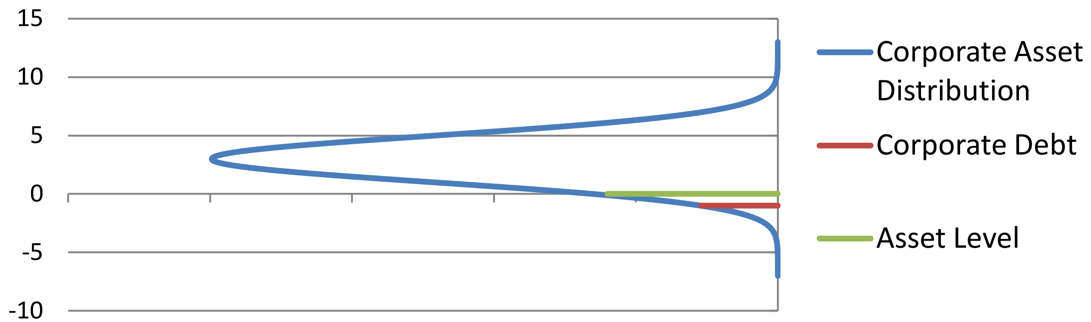
$$Utility = E - \frac{L * VAR}{2K^2} + \frac{L^2 M3}{3K^3} - \frac{L^3 M4}{4K^4}$$

- If leverage is two times then its impact to Variance is 4-fold and to Skew (M3) is 8-fold.
- For the sample "junk grade" bond (P=20), Variance Term = 0.077 and Skew Term = 0.17. Clearly, the 8-fold skew term will dominate with positive skew.
- Using probabilities as (a multitude of) free parameters, we can adapt the Wilcox approach to the "negative skew prone" bond investors, reversing the sign of the adjusted M3 and keeping E and VAR the same. The benefit of this derivation is that it will keep the "aversion" link to investor leverage.

$$Utility = E - \frac{L * VAR}{2K^2} + \frac{L^2 M3\_ADJUSTED}{3K^3} - \frac{L^3 M4}{4K^4}$$

Incorporating Higher Moments in MVO

- As shown previously, we project the truncated distribution of credit payoff – the segment under the *debt level*



- Then we can calculate higher moments and based on the leverage-based utility function (adjusted or unadjusted) we can estimate the equivalent increase in variance that will reflect the higher moments plus the aversion to them

- We ca also approach the incorporation of higher moments indirectly

Back to the Spread as a Certainty Equivalent

OAS = Expected Return + Adjusted Aversion Coefficient * Variance

Can be represented as:

OAS = E[Return Credit] + E[Other Factor] + Adjusted Aversion Coefficient * [Variance(Credit) + Variance(Other Factor)]

- "Other Factor" can be e.g. liquidity.
- If we want to capture the "Other Factor" for a group of bonds we can run a cross sectional regression of OAS per bucket against "Merton"-style volatilities of the constituents.
- Cont'd

Back to Spread as Certainty Equivalent (cont'd)

- We would expect from the slope of that regression to be positive based on previous arguments
- We would expect the intercept of that regression to consist of the average expected returns of the group of bonds plus the volatility aversion term to the "Other Factor."
- We would expect the residuals of that regression to consist of divergences in expected return and errors due to higher moment effects.
- In the absence of "Other Factor", if the credit distributions for all bonds in the bucket are stationary, then the regression intercept (the average spread) and slope will be identical through time.
- If the expectation and/or variance of "Other Factor" changes over time, then the intercept OAS will change.

The Importance of the Intercept

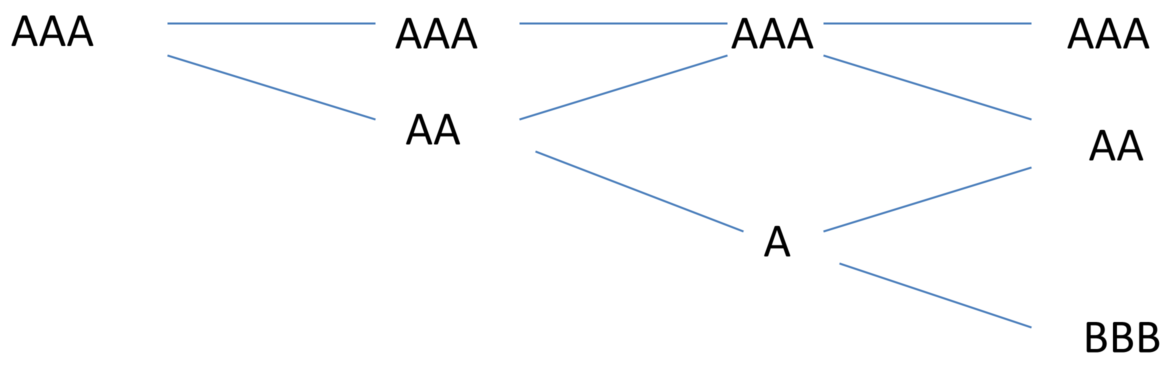
OAS = E[Return Credit] + E[Other Factor] + Adjusted Aversion Coefficient * [Variance(Credit) + Variance(Other Factor)]

- If we pick a bucket by a particular credit band (e.g. BB) and accept that E[Return Credit] and the variance terms are constant over time, the change of period to period of the intercept will be due to the changes in E[Other Factor]

- The volatility of this spread will be related to additional volatility of the bonds in that bucket beyond Merton-style volatility through the spread duration of the particular bond. We can use times series of this variable as the basis for (an) additional risk factor(s)

- Also the volatility of that component for the bucket can be found by using the average spread duration of the bucket. Having found this quantity we can subtract from the intercept to find the contemporaneous expected return. This is an important step in an alternate way to estimate higher moments of bond distribution.

Higher Moments 2



- Having estimated E[R] and VAR we can build a lattice of credit quality transition, trace the paths and estimate higher moments of the full resulting distribution.

- The benefit of this method is that assumes piece wise distribution but lets the full distribution of credit quality naturally arise rather than use a parameterized one. Also, this approach mitigates the "limited observation" aspect of higher moment estimation.

Higher Moments Yet Again

OAS = E[Return Credit] + E[Other Factor] + Adjusted Aversion Coefficient * [Variance(Credit) + Variance(Other Factor)]

- What if the E[R] and VAR are changing over time. Or what if Aversion is not constant over time.
- Isaac Newton vs. Christiaan Huygens argued about light being a wave or a particle. *Eventually we settled that it is both.*
- Didier Sornette and Nassim Taleb have argued about Dragon Kings vs. Black Swans – i.e. economic distributions are subject to changing regimes vs. they are subject to truly unknown higher moments. May be we should settle that these two views are two sides of the same phenomenon that plays out over time.
- If we keep our "intercept" factor observations relatively short we would be able to capture the changing nature of mean and variance and adapt the model to higher moments.

Empirical Results with Spreads and Volatilities

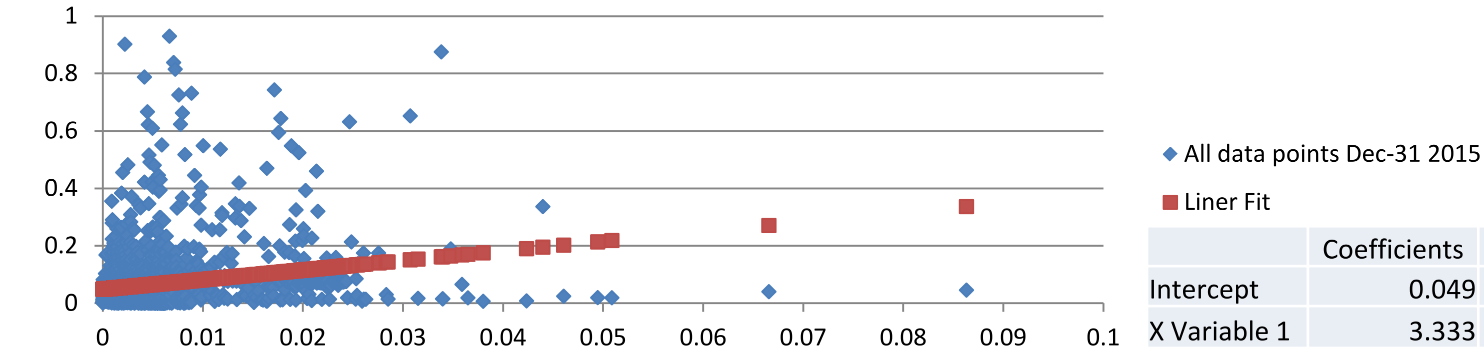
The data set consisted of credit spreads for 5 years (EOY) and their contemporaneous Merton-style volatilities. That entails 5 sets of approx. 200K bonds each with OAS calculated by Northfield per period.

The fundamental purpose of this analysis was to observe the cross-sectional relationship between spreads and Merton-style volatilities, and its evolution over time.

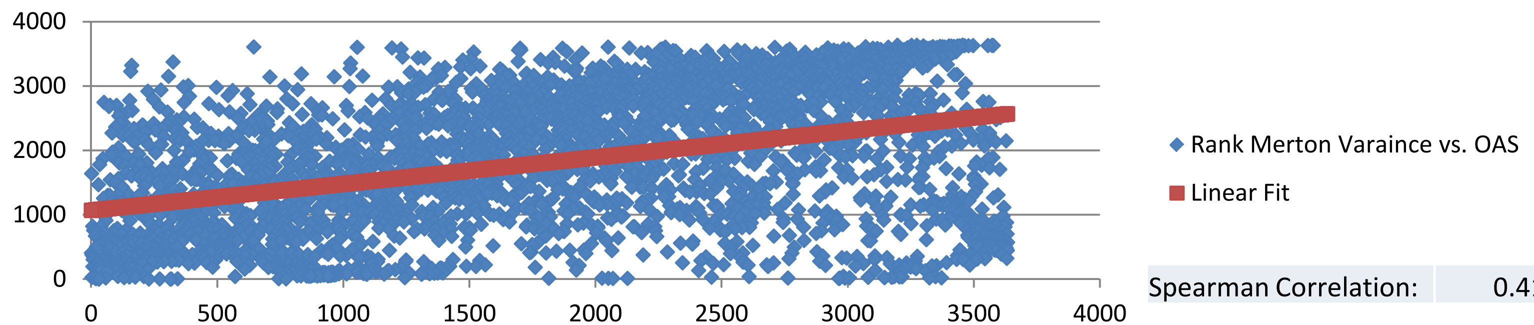
The variables to which we would pay special are:

- Positive or negative correlation (sign of the slope)
- Statistical significance of slope
- Statistical significance of intercept
- Relative size of the intercept across buckets
- Relative size of the intercept cross buckets

All Bonds Snapshot 2015-12-31

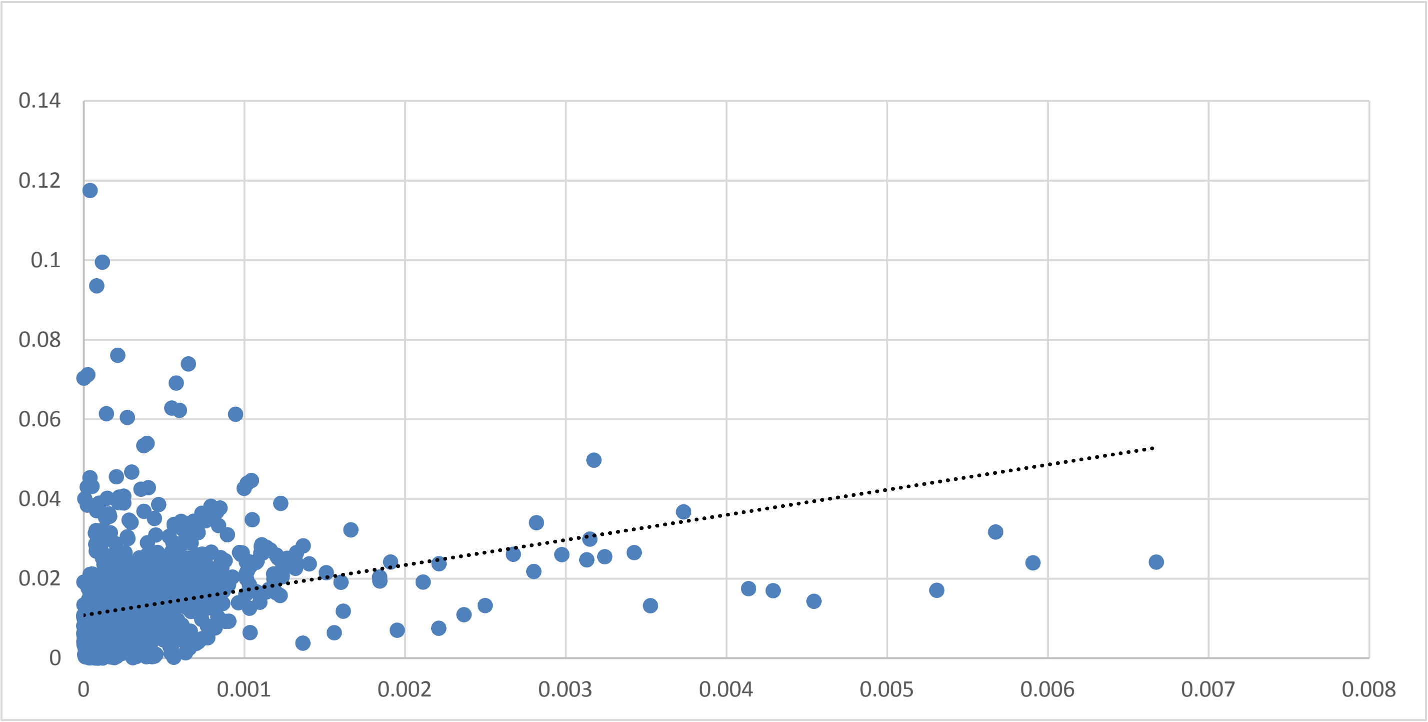


	Coefficients	t Stat
Intercept	0.049	33.965
X Variable 1	3.333	15.262



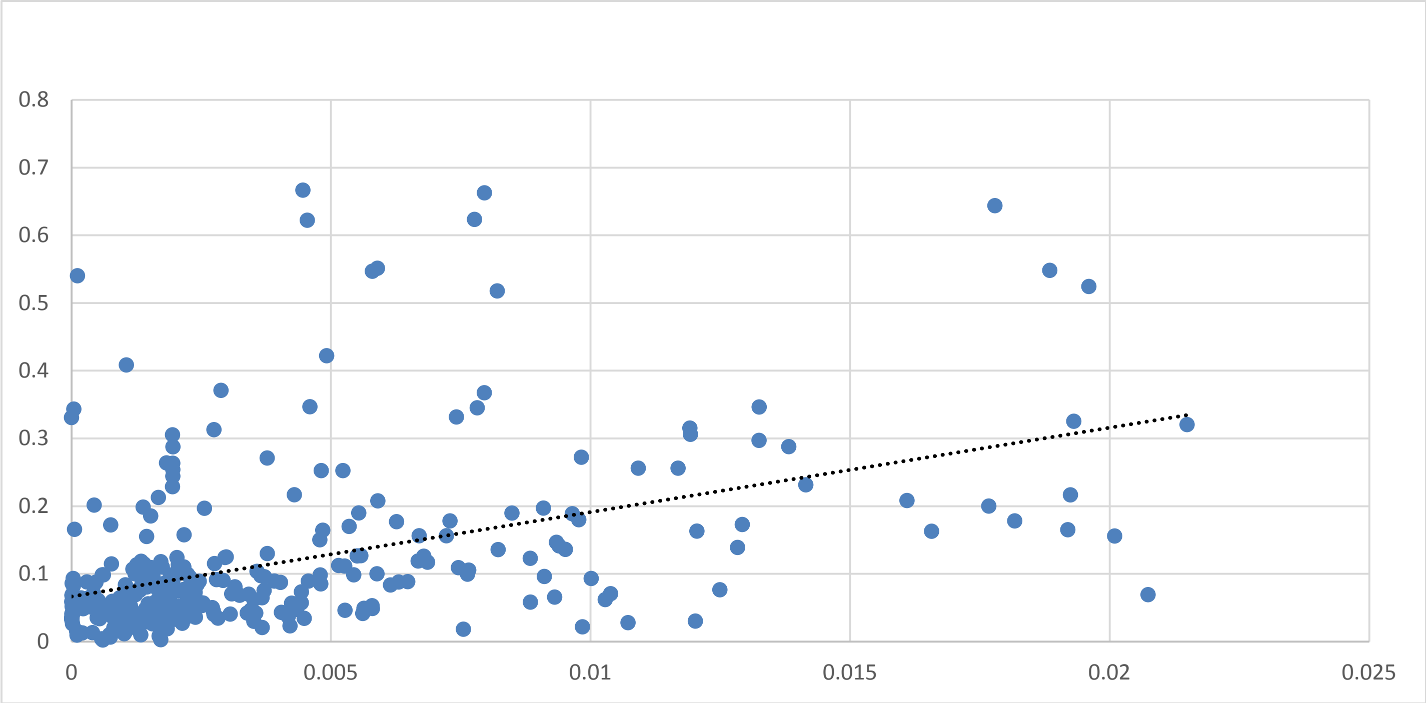
Spearman Correlation:	0.412

A Sample Plot for a Bucket: “A”-rated



	VAL	T-STAT
SLOPE	6.295828	12.20471
INTERCEPT	0.010829	33.35786

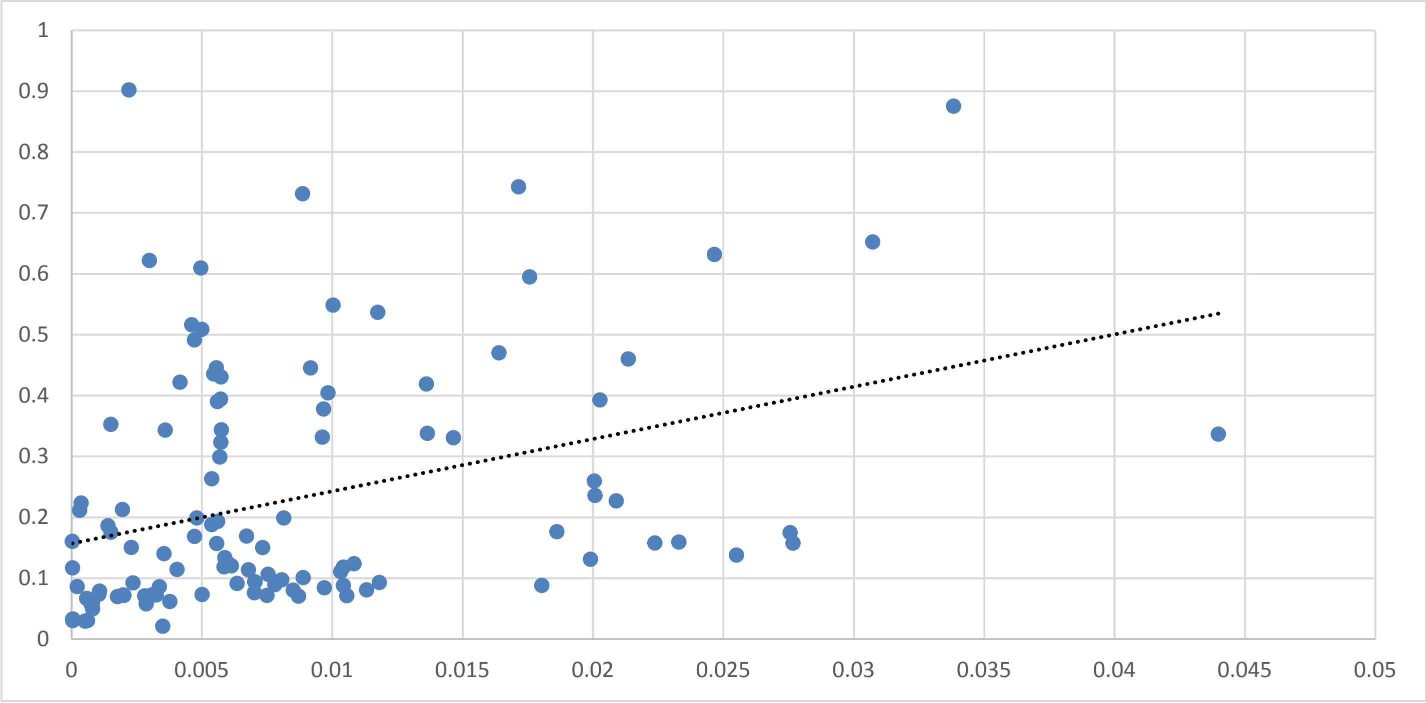
A Sample Plot for a Bucket: “B”-rated



	VAL	T-STAT
SLOPE	12.46111	9.256393
INTERCEPT	0.066457	8.593081

BofA ML US B spread: 7.15%

A Sample Plot for a Bucket: “CCC”-rated



	VAL	T-STAT
SLOPE	8.593996	4.066664
INTERCEPT	0.156904	6.384991

BofA ML US CCC spread: 16. 53 %

Summary Results for 2015-12-31

Year	Credit Band	Coefficient	Value	T-statistic
2015	AAA	SLOPE	1.408	2.391
		INTERCEPT	0.007	12.003
	AA	SLOPE	0.241	2.831
		INTERCEPT	0.010	28.852
	A	SLOPE	6.296	12.205
		INTERCEPT	0.011	33.358
	BBB	SLOPE	10.673	15.056
		INTERCEPT	0.019	26.531
	BB	SLOPE	6.712	7.709
		INTERCEPT	0.048	20.421
	B	SLOPE	12.461	9.256
		INTERCEPT	0.066	8.593
	CCC	SLOPE	8.594	4.067
		INTERCEPT	0.157	6.385

Summary Results for 2014-12-31

Year	Credit Band	Coefficient	Value	T-statistic
2014	AAA	SLOPE	0.197	3.727
		INTERCEPT	0.004	4.770
	AA	SLOPE	0.359	12.132
		INTERCEPT	0.002	4.083
	A	SLOPE	0.452	10.400
		INTERCEPT	0.004	6.592
	BBB	SLOPE	0.197	1.539
		INTERCEPT	0.013	8.878
	BB	SLOPE	0.172	0.863
		INTERCEPT	0.023	9.271
	B	SLOPE	0.121	1.323
		INTERCEPT	0.073	8.142

Summary Results for 2013-12-31

Year	Credit Band	Coefficient	Value	T-statistic
2013	AAA	SLOPE	0.200	2.858
		INTERCEPT	0.006	5.292
	AA	SLOPE	0.305	10.826
		INTERCEPT	0.005	10.643
	A	SLOPE	-0.021	-0.496
		INTERCEPT	0.013	20.551
	BBB	SLOPE	0.190	2.305
		INTERCEPT	0.013	11.627
	BB	SLOPE	1.147	6.449
		INTERCEPT	0.010	4.923
	B	SLOPE	0.163	0.158
		INTERCEPT	0.044	3.867

Summary Results for 2012-12-31

Year	Credit Band	Coefficient	Value	T-statistic
2012	AAA	SLOPE	0.155	7.836
		INTERCEPT	0.004	11.375
	AA	SLOPE	0.317	15.393
		INTERCEPT	0.002	7.418
	A	SLOPE	0.263	6.500
		INTERCEPT	0.009	13.545
	BBB	SLOPE	0.404	5.691
		INTERCEPT	0.016	13.259
	BB	SLOPE	0.968	5.562
		INTERCEPT	0.029	11.057
	B	SLOPE	0.466	0.259
		INTERCEPT	0.104	3.913

Summary Results for 2011-12-31

Year	Credit Band	Coefficient	Value	T-statistic
2011	AAA	SLOPE	-1.321	-3.093
		INTERCEPT	0.017	18.353
	AA	SLOPE	15.434	4.684
		INTERCEPT	0.012	13.867
	A	SLOPE	2.704	4.445
		INTERCEPT	0.020	24.444
	BBB	SLOPE	1.820	1.688
		INTERCEPT	0.024	14.120
	BB	SLOPE	7.625	3.743
		INTERCEPT	0.049	9.093
	B	SLOPE	5.242	2.267
		INTERCEPT	0.062	3.506

Summary

- Develops an explicit link between spreads and utility theory
- Provides the tools to estimate the impact of higher moments
- Supported by empirical evidence
- Reconciles the methodology of spread estimation and default analysis

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