

Reconciliation of Default Risk and Spread Risk

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Introduction

- There are two conflicting concepts of what credit risk actually is. The classic definition has to do with the likelihood that a given fixed income instrument will default (PD), and the expected severity of economic loss in the event of a default (LGD). In this view, the focus is on the “tail risk” (negative skew in the return distribution) associated with the default event
- Many fixed income market participants prefer to think of a given fixed income instrument as offering a yield spread above a comparable duration riskless instrument. These investors think of credit risk as the volatility of the credit related yield spread and impact on the market value of an instrument (conditional on the duration). If investors are not risk-neutral, the credit spread will compensate investors for their expected loss ($PD \cdot LGD$), plus provide a risk premium to induce risk averse investors to hold these instruments.

Presentation Objectives

- *These two concepts of credit risk are not equivalent because credit spreads can change over time both because of changes in expected loss, and separately because aggregate investor risk aversion can change, forcing a change in the risk premium (incremental yield) which fixed income borrowers must pay.*
- In this presentation we will provide review both approaches to credit risk, and illustrate a new analytical model which reconciles the two views. We satisfy the default risk concerns of “buy and hold” investors, while simultaneously explaining yield spread volatility for investors who are more concerned with controlling variation in period to period returns.
- Illustrate how a modified version of the Merton (1974) contingent claims model from diBartolomeo (2010,2012) is used to break each debt into two pieces, the first considered riskless debt and the second equity in the issuer.
- We will present a few empirical examples of how to parameterize the inputs to the model, at both the individual bond and market levels.

“Buy and Hold” Bond Investing

- A key to understanding the representation of credit risk is the concept of book value accounting
 - Widely used by long term investors who intend to hold bonds to maturity. They perceive interim fluctuations in bond prices as irrelevant.
 - Unless there is a substantial decline in credit quality, the value of the bond is *assumed* to be purchase cost plus amortization of premiums or discounts. *There is no “mark to market”.*
 - Return on investment in each period is the accrued income, plus price changes due to amortization, plus realized net capital gain or loss. Portfolios will almost always have positive returns as we can choose not to sell bonds that have fallen in market value.
 - Liquidity is irrelevant since all bonds it is assumed all bonds will be held to maturity.

Modern Performance Measurement

- The vast majority of investors do not use book value accounting for measuring investment performance.
 - The value of most bonds in a portfolio is *estimated* at the end of the measurement period, based on a “fair market yield” that includes the time value of money, and a “spread” that is a risk premium for both credit risk and illiquidity.
 - The spread is measured net of the impact of embedded options (e.g. callable bonds) and is often referred to as an “option adjusted spread” (OAS)
 - The total return on the portfolio for a period includes “mark to market” price change, income (including accrual) and the reinvestment of cash income that has been received.
 - This approach is consistent with FASB 157 requiring fair market valuation, even for illiquid assets.

Buy and Hold Credit Risk

- For investors that assume that instruments will be held to maturity (long term investors in bonds, bank loans), credit risk can be summarized as:
 - The probability of default (PD)
 - The “loss-given-default” (LGD).
 - The product of these two is the *expected loss* (EL)
- Most credit models assume PD and LGD are uncorrelated.
 - Frye (2013) proposes a functional form that implies a positive correlation
 - Credit defaults/foreclosures impact related entities causing a contagion effect of more defaults. Human and financial resources for “workouts” are both finite so LGD rises with PD

Credit Risk as “Spread Vol * Duration”

- For “marked to market” investors the modern view of credit risk is based on “spread duration”
 - OAS spreads are the market price of credit risk for a given security, conditional on stable liquidity
 - The volatility of OAS spread (yield units) times the effective duration of the bond (adjusted for embedded options) gives the credit related volatility of the bond price. There is wide empirical observation of OAS spreads.
 - To the extent that investors are risk averse, the OAS must cover the expected loss (EL) and also include a yield premium to compensate investors for bearing credit and illiquidity risk.
 - OAS is a function of not only PD and LGD, but also illiquidity and investor risk aversion. **Time series volatility of OAS must be a function of the variation of all these components**

Default and Equity Risks Linked: Basic Contingent Claims Literature

- Merton (1974) poses the equity of a firm as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt
 - Alternatively, lenders are short a put on the firm assets
 - Default can occur only at debt maturity
- Black and Cox (1976) provide a “first passage” model
 - Default can occur before debt maturity
 - Firm extinction is assumed if asset values hit a boundary value (i.e. specified by bond covenants)
- Leland (1994) and Leland and Toft (1996)
 - Account for the tax deductibility of interest payments and costs of bankruptcy. Estimate strike price “boundary” where firm equity value is maximized subject to bankruptcy

Estimating PD and LGD

- Underlying is the firm's assets with asset volatility determined from the equity factor model
 - How volatile would a firm's stock be if the firm had no debt?
 - This is the volatility of the assets
- Include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately
- You can solve numerically for the “implied expiration date” of the option that equates the option values to the stock price
 - *Market implied expected life of the firm, which is a transform of PD*
 - See Yaksick (1998) for numerical methods for evaluating a perpetual American option

Defining the Default Risk in Corporate Bonds

- **PD** is the “percent moneyness” of the put option
- One approach to approximate “Loss Given Default” is

$$\text{LGD} = -(T-B)/B * (\Delta_p / \Delta_c)$$

T is the value of the bond if it were riskless

B is the market value of the bond

Δ_p = delta of the put option

Δ_c = delta of the call option

Volatility in the values of PD and LGD at the individual security level is largely driven by the common change in the volatility of firm equity, resulting in positive correlation.

Extension to Sovereign Debt

- Bodie, Gray, Merton (2005, 2007)
 - The paper provides a complex system of theoretical balance sheet relationships among three types of entities: the Corporate Sector, the Financial Sector including Central Banks, and Sovereign Governments
- The interrelationships between sectors are modeled as a set of put and call options among the players
 - The government has a call on corporate assets (taxes)
 - The banks have a call on the government (bailouts)
 - *A key attribute (asset) of some but not all governments is a monopoly authority on the printing of money*
- Belev and diBartolomeo (2013) extends the model to include joint default risk of governments and banking systems
 - Winner of the PRMIA 2013 Award for “New Frontiers in Risk Management”

Basic Analytical Model of Spreads

- Our model assumes a mean/variance efficient investor

$OAS_t = E[\text{credit losses} + \text{trading costs}] + \text{risk premium}$

$OAS_t = (PD_t * LGD_t) + (C * T) + L_t(\text{var}((PD_t * LGD_t) + k_t))$

$OAS_t = (PD_t * LGD_t) + (C * T) + \text{var}(L_t * PD_t * LGD_t) + \text{var}(L_t * k_t)$

C = estimated cost to trade bond

T = decimal annual portfolio turnover

var = variance statistical operator

E [] = expectations operator

L = investor risk aversion

k = illiquidity risk measure (no counterparty available)

Variance of a Product

$$OAS_t = (PD_t * LGD_t) + (C * T) + \text{var}(L_t * PD_t * LGD_t) + \text{var}(L_t * k_t)$$

- Calculation of the term in green requires finding the variance of a product of two variables. Calculation of the term in red is the variance of three terms. If X and Y are independent, the variance of (XY) is

$$\text{Var}(XY) = \text{var}(X) * \text{var}(Y) + \text{var}(X) * E(Y)^2 + \text{var}(Y) * E(X)^2$$

This structure can be extended to multiple variables.

Incorporation of correlated variables is algebraically very messy but possible.

Basic Analytical Model of Spread Volatility

- If the OAS spread at time t is a function of four terms, the volatility of the OAS is square root of the summation of the variances and covariance of those terms.

$$\text{var}(\text{OAS}_t) = (\text{var}(EL_t) + \text{var}(\text{var}(L_t * EL_t)) + \text{var}(L_t * k_t) + \text{COV}()) \cdot 5$$

Note the higher order term in red. This will create a high degree of negative skew in distribution of returns at the individual security level. Bond investors must diversify widely to mitigate the impact of variation in expected loss, but cannot diversify the impact of variation in investor risk aversion over time.

Lets Get Some Data

- Our first intuition is that OAS spreads should be positively correlated with PD and LGD, and hence negatively with equity prices.
 - From the Merton model we know that PD and LGD should decline as the stock market rises (a firm can more easily sell shares to raise cash), so the correlation of OAS and equity market returns (S&P 500) should be negative
- For the first 11 months of 2015 using S&P ratings
 - AAA = -.36, AA = -.09*, A = -.39
 - BBB = -.46, BB = -.51, B = -.51

** Note that the AA category includes US Treasuries*

A Single Bond Example

- Let's consider a hypothetical single bond with a 5% yield, where the comparable riskless bond yields 3%
 - Assume the hypothetical bond is highly liquid
 - Nominal OAS is 2%
 - PD is 3% per year,
 - LGD is 30%
 - EL is .90%
 - Effective OAS is 1.94% = $(1 - \text{PD}) * 2\%$
 - Risk Premium is 1.04%
 - Volatility (EL) is 5.49%, **Variance (EL) = 30.1%²**
 - Skew(EL) = - 5.59,
 - Excess Kurtosis = 26.9
 - Adjusted Vol (EL) = 6.12%, **Adjusted Variance (EL) = 37.61%²**

Let's Estimate Risk Aversion and It's Variation

- We will estimate aggregate fixed income investor risk aversion from a comparison of returns to the Barclay's Treasury Index and the Barclays Aggregate Index (which involves bonds with credit risk).
- Sample period is from November 2004 to October 2015.
- Correlation of the two indices is .88
- We adjusted returns for differences in average bond duration. We calculate differences in realized return as a proxy for the expected value of OAS over 36 month rolling periods.
- Average realized return spread was .49% per year with a standard deviation of 1.57%
- Realized values of L had mean of .22 with a standard deviation of .45. About 20% of overlapping periods had negative realized L (i.e. the GFC period).

Illiquidity Risk

- Our model incorporates two aspects of liquidity risk.
 - The first is the expected value of trading costs during a bond's life. This is a function of the expected bid/asked spreads during a trade and the level of investor specific portfolio turnover.
 - Portfolio turnover goes to zero for “buy and hold” investors.
 - The second is a yield premium ($L_t^* k_t$) that must be paid to investors of illiquid assets as compensation for the fact that no buyers may be available when a holder wishes to liquidate.
 - We follow the method of Ang and Bollen (2010) modeling the random arrival of a viable buyer as Poisson distribution. They estimated a maximum annual return premium for a 10 year mean time between buyers of 6% or .05% per month.
 - This effect should be relatively small compared to the other terms in our model *in the absence of margin leverage*.

PD and LGD Variation for the Market

- The credit rating agencies routinely publish historical data on PD, LGD, and realized losses across time, sectors and rating levels. This data comes from *Moody's Corporate Bond Default and Recovery Rates 1920-2010* (published 2011). This information can be used to parameterize the variation of PD and LGD for diverse samples.
 - For all bonds from 1982 to 2010 the lowest recovery rate (1-LGD) was 2001 at 21.6% and with the highest in 2004 at 58.5%
 - From 1982 to 2010, the average realized loss was 2.78% for non-investment grade bonds. Worst year 8.26% and best .45%
 - From 1920 to 2010, the average realized PD for investment grade bonds was .15% with a standard deviation of .276 but with a maximum of 1.58
 - Closer examination of the data illustrates that the expected positive skew is observed in PD, LGD and particularly EL.

Conclusions

- While it has always been clear that fixed income credit risk defined as default risk, and credit risk defined as “spread duration” risk should be highly related, we believe that the model presented here provides an intuitive framework to link the two views.
- Our model highlights the importance of higher moments in the return distributions irrespective of which view of credit risk the investor adopts.
- While algebraically complex, we now have a mechanism to analytically estimate the expected levels of spread risk at the individual firm level from our Merton model and market level information on investor risk aversion.

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