

# Risk Tolerance in Optimizations

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User Seminar

February 9, 2017

## Goals today:

- Drill fundamentals for users!
- What is RAP (risk acceptance parameter)?
- What numbers do I put in the box?
- How can it break things?
- Have some fun on a snow day (time permitting)

# Optimization Objective Function

- Investor objective is to maximize risk- adjusted returns net of costs:

$$\text{Utility} = \text{Return} - \text{Risk} - \text{Costs}$$

- Most commercial portfolio optimizers uses the objective function described in Levy and Markowitz (1979)
- Portfolio return variance is the proper measure of risk because the difference between the arithmetic average rate of return and the geometric average rate of return is proportional to the variance (see Messmore, 1995)

# Objective Function Review

$$U = \underbrace{\alpha}_{\text{Return}} - \underbrace{\left( \frac{\sigma_s^2}{RAP_s} + \frac{\sigma_u^2}{RAP_u} \right)}_{\text{Risk Component}} - \underbrace{((C + T) * A)}_{\text{Implementation Cost}}$$

Factor Risk
Stock Specific Risk

- **$\alpha$  = the “certainty equivalent” expected portfolio return**
- $\sigma_s^2$  = portfolio variance risk due to common factors (correlation across securities)
- $\sigma_u^2$  = portfolio variance risk due to stock specific risks
- **RAP = risk tolerance**
- C = transaction costs for the optimization
- T = capital gain taxes for the optimization
- **A = amortization constant**

Note that this is a convex function and the various scalars transform all terms into certainty equivalent expected return units

# Objective Function Review

$$\bullet U = \alpha - \underbrace{\left( \frac{\sigma_s^2}{RAP_s} \right)}_{\text{Factor Risk}} - \underbrace{\left( \frac{\sigma_u^2}{RAP_u} \right)}_{\text{Stock Specific Risk}} - ((C + T) * A)$$

Risk Component

Factor risk is (Active exposure to factors)<sup>2</sup> x (Factor covariances)

Stock specific risk is (Active weight of asset)<sup>2</sup> x (Residual risk of asset)

RAP (Risk Acceptance Parameter) scales relative importance of each type of risk. Very large numbers make the risk terms drop out of the utility function.

# Objective Function Review

$$\bullet U = \alpha - (\sigma_s^2 / RAP_s) - (\sigma_u^2 / RAP_u) - \underbrace{((C + T) * A)}_{\text{Implementation Cost}}$$

- C = transaction costs for the optimization
  - Can be dollars per share or %
  - Includes brokerage commissions, half of bid/ask spread, liquidity costs
- T = capital gain taxes for the trade, as % of the trade
  - Can be positive (sell a gain) or negative (sell a loss)
- A = amortization constant

# Optimization Review

- Our goal is to adjust the security weights to maximize the objective function, given your input settings
- Buy the best asset, sell the worst asset:

===== DETAIL OPTIMIZATION REPORT =====

----- Main Cycle -----

1>> Assets To BUY: XOM=20.000%, Swap = 10.0121557 (sorted by MuBuy)


N	Ticker	MUBUY	MA	MF/RAP	MR/RAP	MTRANS	MIND	MSECT	MQPEN	TRNSELL+	TRNBUY+
1	*\$\$\$	0	0	0	-0	-0	-0	-99	-0	0	0
2	D	-0.22	0	-0.089	0.012	-0.15	-0	-99	-0	0	0
3	AEE	-0.33	0	-0.075	0.0044	-0.26	-0	-99	-0	0	0
...											
492	*XOM	-5.5	0	-3.2	-2.2	-0.11	-0	-99	-0	0	0

1>> Assets To SELL: AAPL = 19.999, Swap = 10.0121557 (sorted by MuSell)

N	Ticker	MUSELL	MA	MF/RAP	MR/RAP	MTRANS	MIND	MSECT	MQPEN	TRNSELL+	TRNBUY+
1	AAPL	-13	0	-4.5	-8.1	0.02	-0	-99	-0	0	0
2	BAC	-10	0	-5.6	-12	6.7	-0	-99	-0	0	0
3	IBM	-10	-2	-4.3	-3.8	0.057	-0	-99	-0	0	0
26	XOM	-3.9	0	-3.2	-2.2	1.4	-0	-99	-0	0	0
479	SBUX	-0.37	3	-3	-5.5	5.1	-0	-99	-0	0	0

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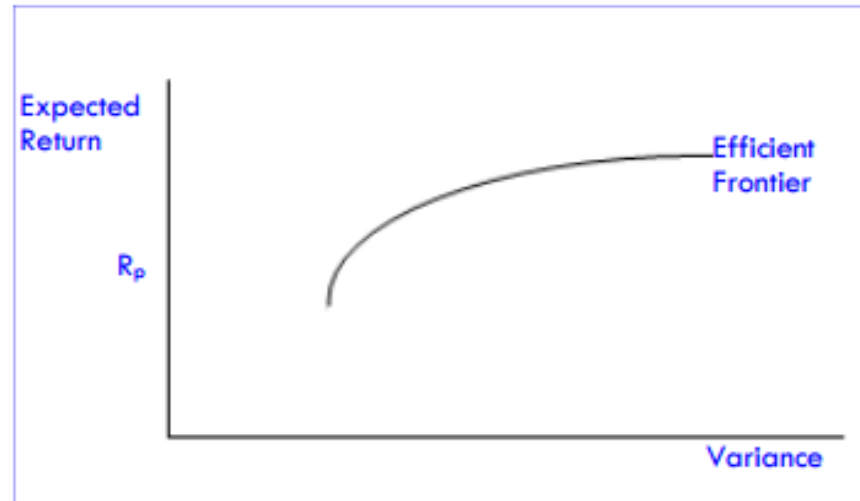
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# What is RAP?

The efficient frontier is a set of portfolios that offers:

- The highest level of return for a given amount of risk
- The lowest level of risk for a given amount of return

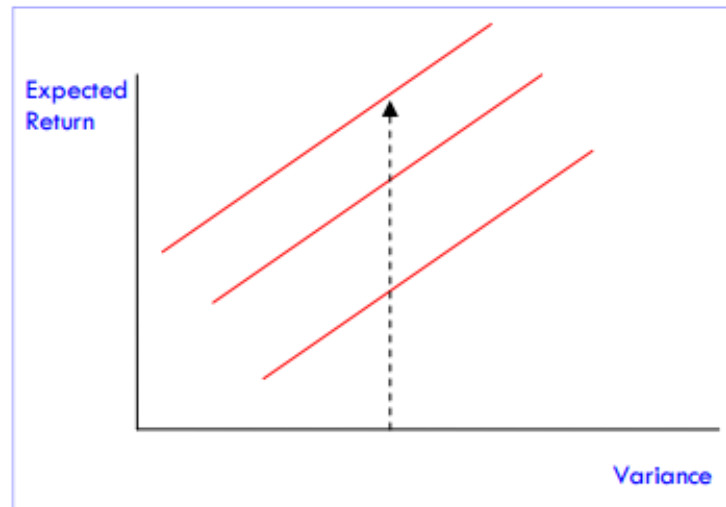


<http://www.northinfo.com/docs/tech032011.pdf>

# What is RAP?

An indifference curve is:

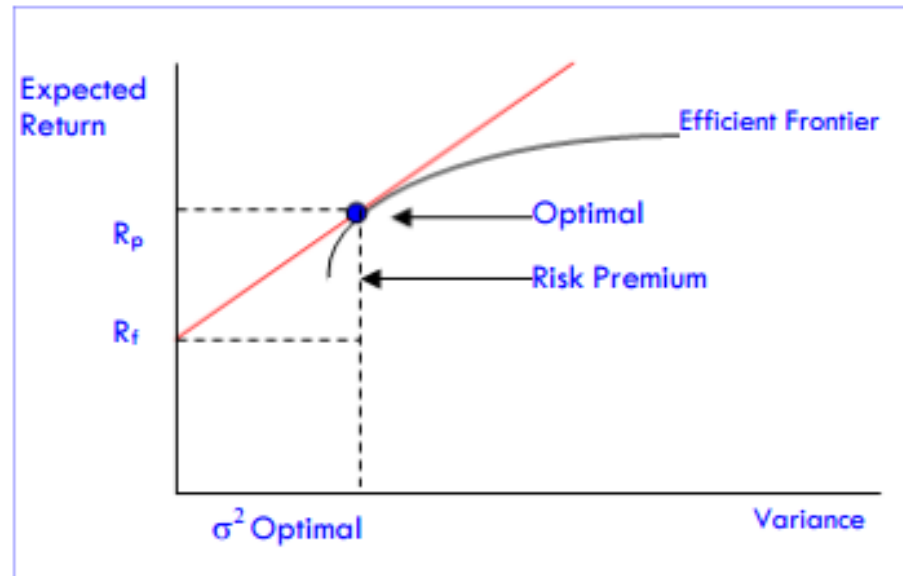
- An upward sloping line that represents an investor's preferences – a rational investor will choose any point along a given line as their willingness to take incremental risk in exchange for additional return
- Higher curves are preferred over lower curves – more return for the same amount of risk



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# What is RAP?

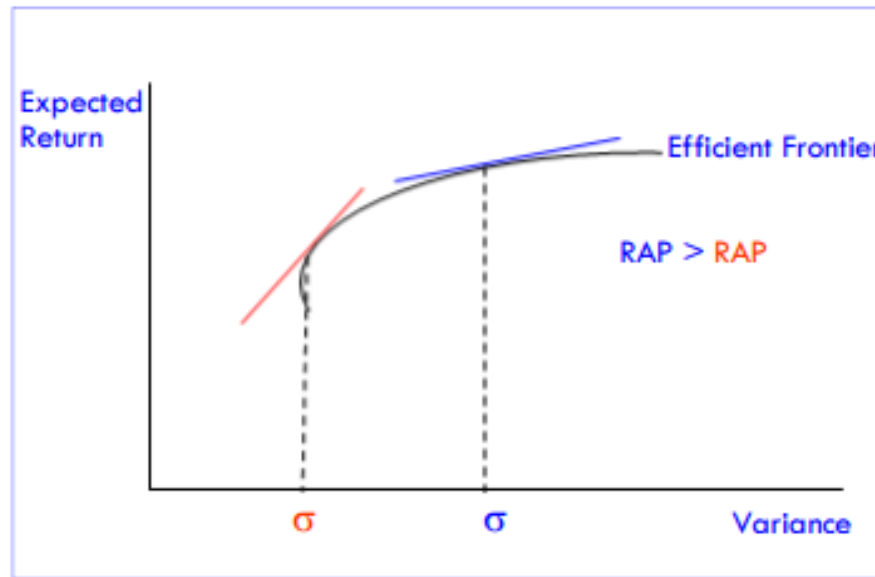
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# What do I type in the box?

- How do I select my RAP?
- The lower bound is zero (the RAP term is in the denominator)
- The upper bound is that of a “growth optimal” investor (infinite time horizon) is 200
  - $C = A - (\sigma^* \sigma) / 2$ 
    - $C$  = compound annual rate of return in decimals
    - $A$  = the arithmetic mean rate of return in decimals
    - $\sigma^* \sigma = V$  = variance of the annual return, in decimal  $\%^2$
- ...convert from decimal to % as used in the optimizer...
  - $C = A - (\sigma^* \sigma) / 200$
- Messmore (Messmore, Tom "Variance Drain, "Journal of Portfolio Management, 1995, v21 (4), 104-110.)

<http://www.northinfo.com/documents/16.pdf>

# What do I type in the box?

- Rule of thumb for investors without an infinite time horizon (hopefully you)
- In absolute terms, think of your risk tolerance as the amount of your initial wealth you are willing to lose; e.g. "Maximize return, but never losing more than 20% of my initial amount"
- Divide your wealth into two portions – the amount that will take no risk (RAP = 0) and the portion on which you are maximizing return (RAP = 200)

$$\text{RAP} = (D/100)*200 + (1-D/100)*0$$

$$\text{If } D = 20, \text{ then RAP} = 40$$

- Jarrod Wilcox, "Better Risk Management," Journal of Portfolio Management, 2000, v26 (4, Summer), 53-64]

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- Rule of thumb for investors without an infinite time horizon (hopefully you)
- In benchmark relative terms, think of a loss of D% of wealth as a number of standard deviations, assuming normally distributed returns
- A “worst case scenario” is a three standard deviation event, or 99% of events
- We can then say  $D = 3 * \sigma$
- Substitute this into the previous slide and we get

$$RAP = 6 * \sigma$$

Or **six times tracking error**

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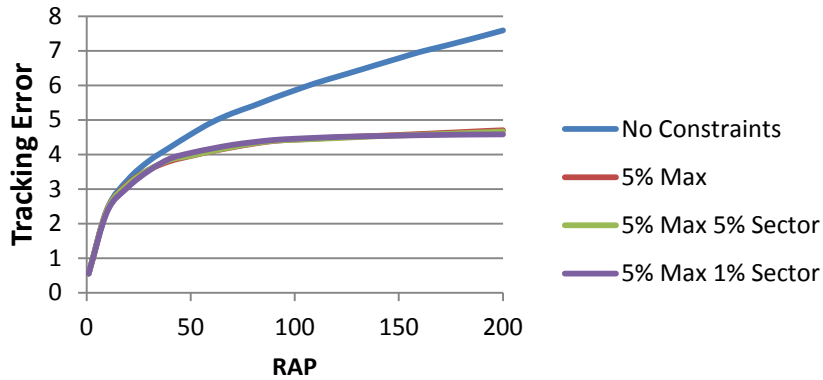
# What do I type in the box?

- Real results from the optimizer:
  - Fundamental model, 1/31/2017
  - Cash Portfolio
  - S&P 500 Benchmark
  - Alphas ~ -3% to +3%
  - Adding in some realistic constraints
    - 5% max position size
    - +/- 5% sector collar
    - +/- 1% sector collar
  - Optimize with RAPs from 1-200

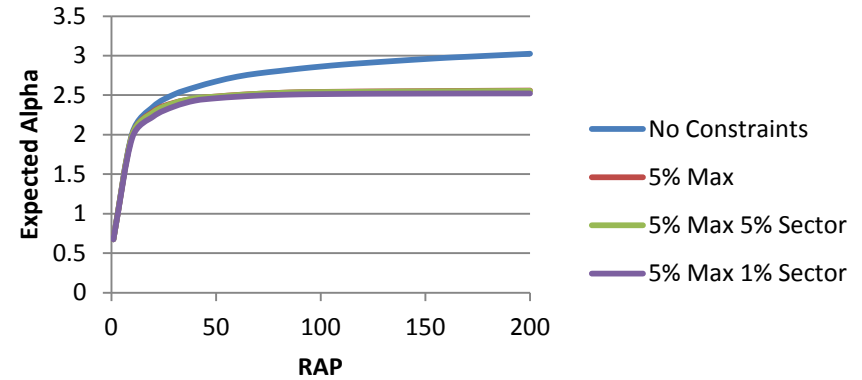


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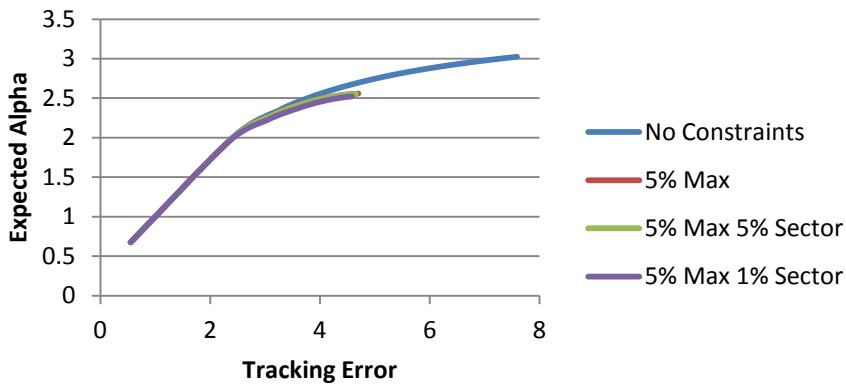
## RAP vs TE



## RAP vs Alpha



## TE vs Alpha



## RAP vs Names

