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Risk Model Testing, or Horses for Courses
Part 2 - Minimum Variance Portfolios

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A Brief Reprise

- In November 2015, at the Northfield Annual Conference in St Petersburg, Florida, we gave a talk called ‘Risk Model Testing, or Horses for Courses’, which compared three different types of US risk model
- After the presentation, one of the questions asked was whether we had tried running Minimum Risk Portfolios using the different risk models, and if so, what the results were like
- As it happened, we had not done so, but we thought it was a good idea
- In addition to simply running absolute Minimum Risk Portfolios, we also thought it would be interesting to run Minimum Tracking Error Portfolios against the S&P 500 benchmark
- In this presentation, we show the results of one such exercise, which has been run over 11 years, from December 2004 to November 2015

Standard Risk Models - Three Types

- There are three basic types of multi-factor risk models :
  - Statistical (using Principal Component analysis)
  - Cross-sectional (usually with dummy variables for betas)
  - Time Series (estimating stock betas against factor returns)

- Northfield pioneered the idea of Hybrid risk models, which capture the main advantages of each method
  - Statistical (ensures no systematic risk left in residuals)
  - Cross-sectional (necessary when stock betas change quickly)
  - Time Series (to distinguish between weight & exposure)

Horses for Courses

- All models work better for some portfolios than for others
- All models work better in some time periods than others
- Stock R-Squareds will vary considerably (e.g. XRD US)
  - Average 36% for top 3,000 by MktCap (max = 84% min = 0%)
  - Average 40% for top 1,000, and 42% for top 500 largest stocks
  - Average 20% overall for model universe of around 9,000 stocks
- As a simple but obvious example, models in which Market and Industry factors are market-capitalisation–weighted tend to work best on capitalisation-weighted portfolios of large stocks

The Risk Models to be Tested

- For this exercise, we used four different US models
  - The US Fundamental risk model
  - The US Macro-Economic hybrid risk model (APT-based)
  - The US XRD hybrid risk model
  - A Statistical factor model we built for this research project
- The first two are based on a history of 60 calendar month returns (a 5 year look-back)
- The last two are XRD (Cross Reference Day) models, derived from four SRD (Single Reference Day) models, each based on a set of 80 4-weekly returns (just over 6 years), and staggered one week apart

Methodology – Minimum Risk Portfolio case

- We began by optimising the top 1,000 stocks by capitalisation, excluding foreign stocks (such as ADRs) to create a Minimum Risk Portfolio on 31st December 2004
- We invested in a notional $10,000,000 portfolio at that date
- The portfolios are rebalanced quarterly throughout the back test period, up to the last rebalance on September 30th 2015
- We set a minimum initial holding size for a new stock holding of 0.5%, and a maximum initial holding size of 5.0%
- However, if a current holding was below 0.5% or above 5.0% at a subsequent rebalancing, it was allowed to stay as it was, unless the optimiser wanted to change its holding size or sell it completely

Methodology – Minimum Tracking Error case

- In this case, we use the same initial universe, but now create Minimum Tracking Error Portfolios against the S&P 500
- We also used the same initial holding size constraints as in the Minimum Risk Portfolio case
- In both cases, we measured the quarterly turnover, and deducted transaction costs from the portfolio valuations based on 10 bps for both Buys and Sells
- Note that changing the assumed trading cost does not affect the portfolio risks very much, but mainly affects the realised returns

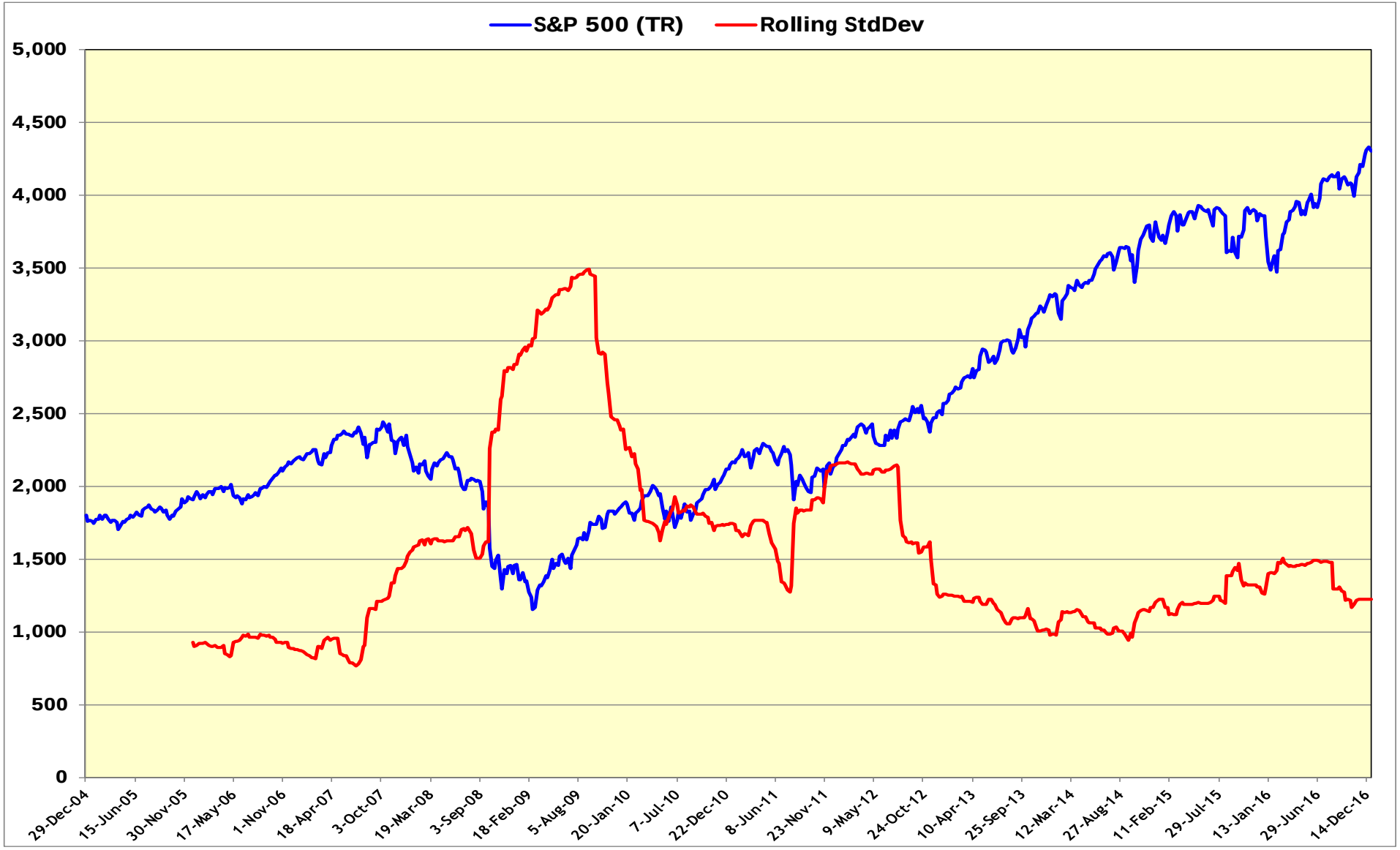
Expectation of Known Bias - 1

- All risk models are based on statistical estimation techniques: some use cross-sectional regressions to estimate factor returns, while others use time series regressions to estimate stock betas
- All risk models will therefore have estimation errors in them somewhere
- There two kinds of possible estimation error:
  - The risk of individual stocks may be too high or too low
  - The diversification properties of individual stocks in a portfolio may be better or worse than the model estimates

Expectation of Known Bias - 2

- Over the whole universe, these errors are likely to distributed randomly
- However, if we are focusing on low risk stocks to build low risk portfolios, we are likely to have more under-estimates of risk then over-estimates
- So before we even begin, we must recognise that our *ex ante* portfolio risk forecasts are likely to be biased downwards, and that the portfolios' actual *ex post* risks will probably be higher than the models predict
- This bias will be exacerbated if we have only relatively small numbers of holdings in the portfolios

Interesting Sub-Periods - 1



Interesting Sub-Periods - 2

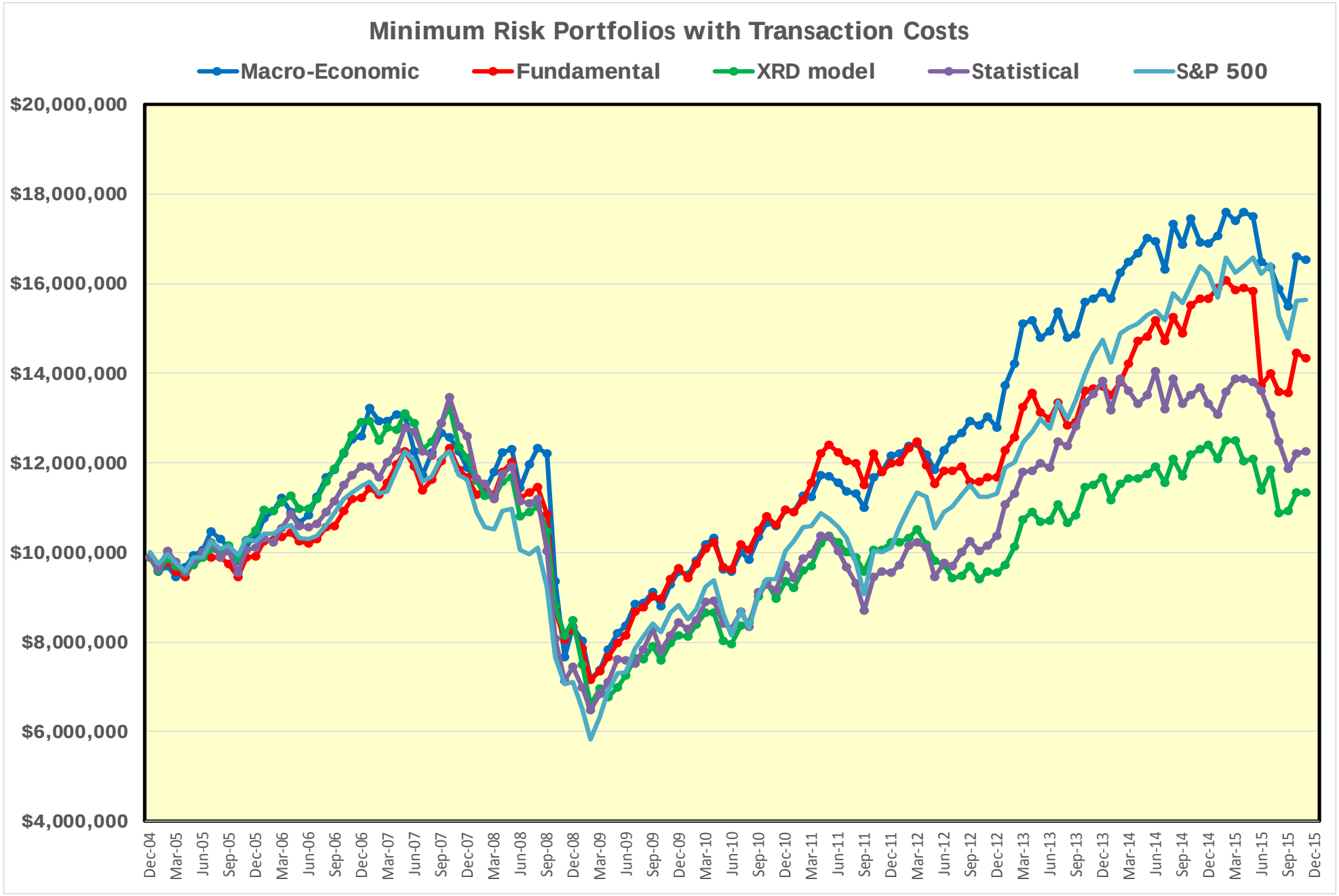
- We divided the 11 year period into 4 Sub-Periods:
  - Low Risk Bull Market Jan 2005 to July 2007 (31 months)
  - The GFC Crash Aug 2007 to March 2009 (20 months)
  - The Recovery April 2009 to Dec 2012 (45 months)
  - The New Normal Jan 2013 to Nov 2015 (35 months)

- Unsurprisingly, we get different results from each of the different sub-periods

Portfolio Characteristics from Different Models

Averages:	MVP case # Holdings	Minimum T.E. case # Holdings	Minimum T.E. case Beta
• Macro-Economic	50	78	0.911
• Fundamental	49	73	0.941
• XRD model	49	80	0.935
• Statistical	70	82	0.968

- Biggest factor bet in MVP cases was negative beta on Price Volatility
- Biggest factor bet in MTE cases was negative beta on Log of MktCap


Minimum Risk Portfolios with Transaction Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
$20,000,000
$18,000,000
$16,000,000
$14,000,000
$12,000,000
$10,000,000
$8,000,000
$6,000,000
$4,000,000
Dec-04
Mar-05
Jun-05
Sep-05
Dec-05
Mar-06
Jun-06
Sep-06
Dec-06
Mar-07
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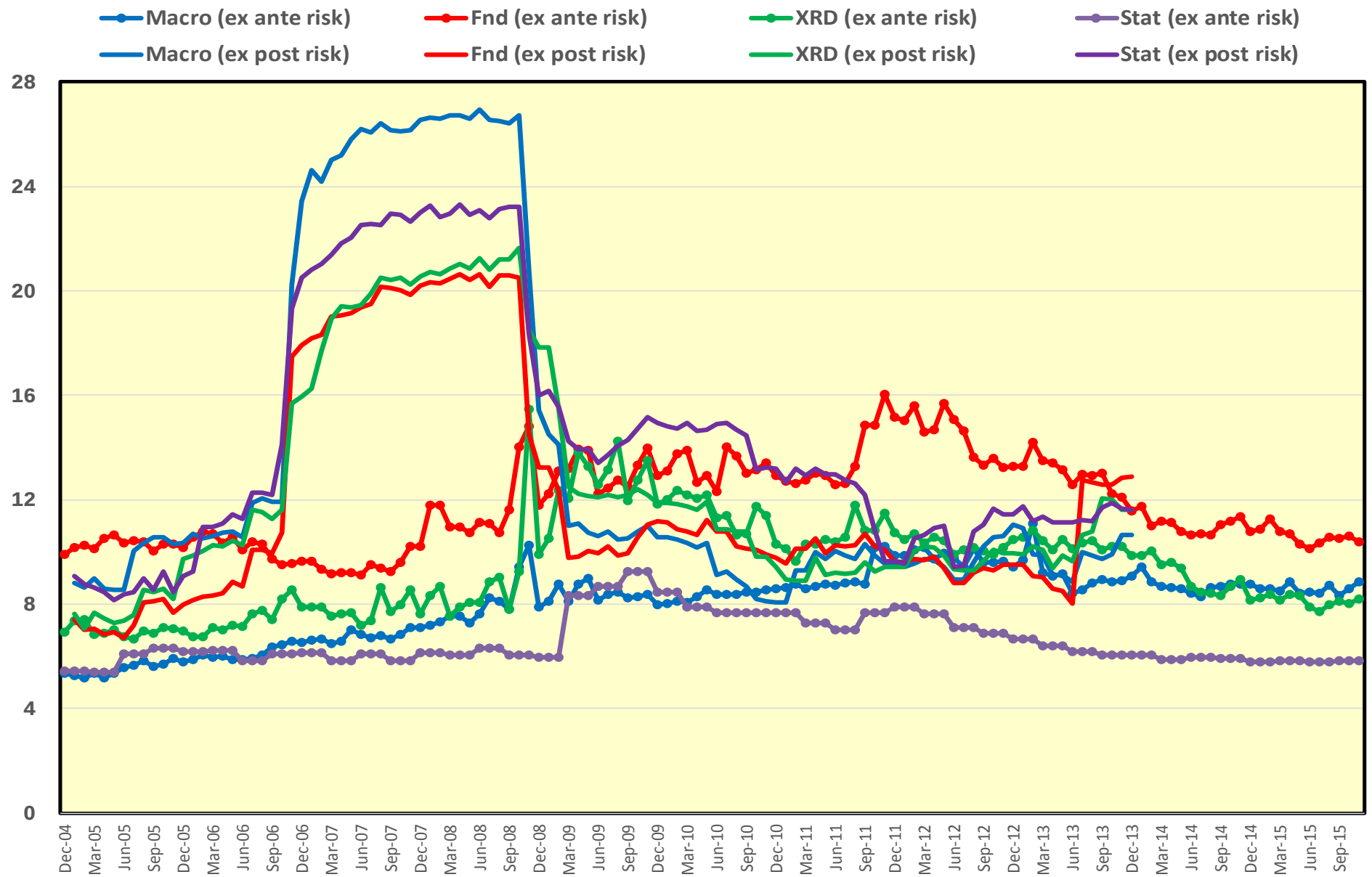
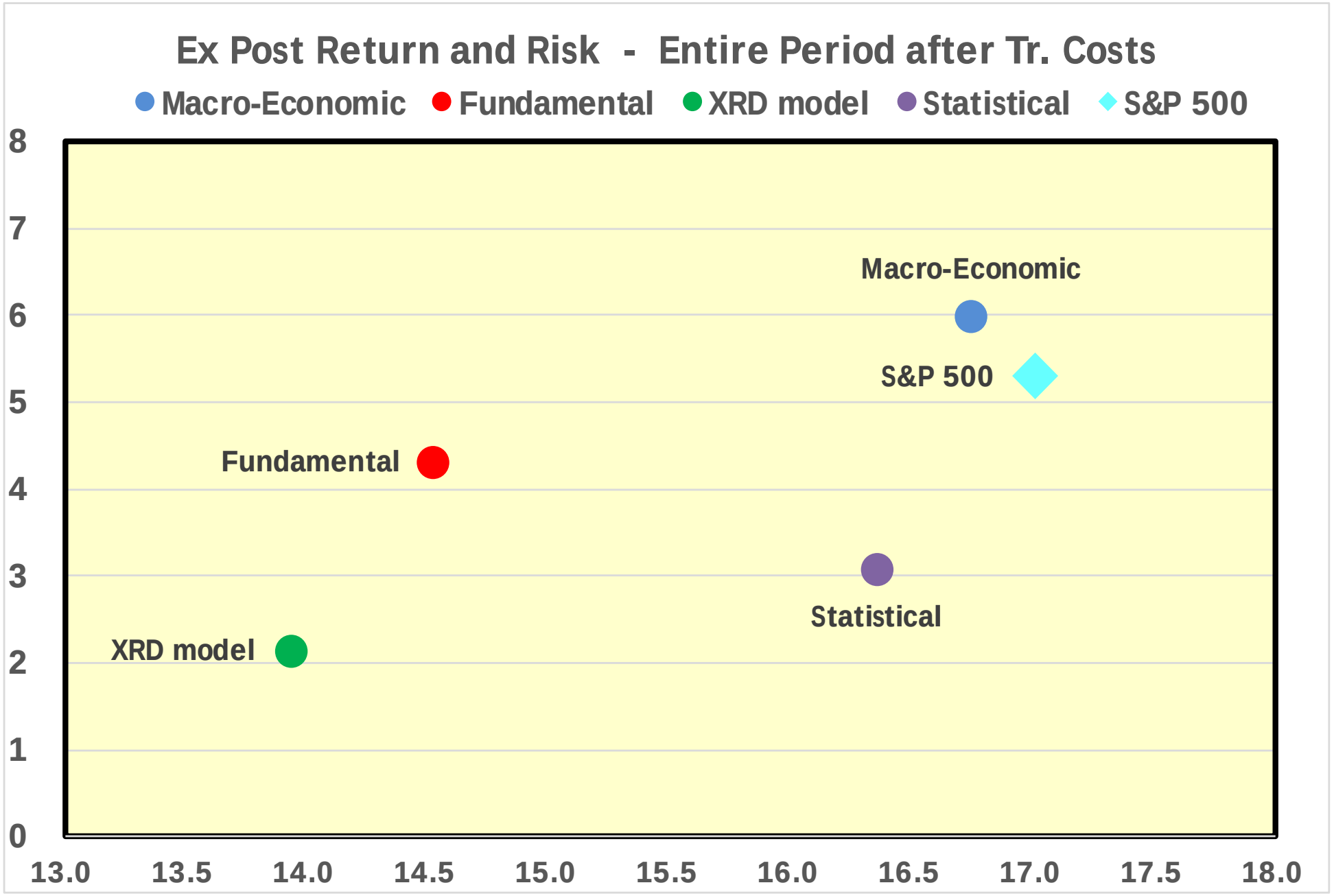
Jun-15
Sep-15
Dec-15

Ex Ante vs Ex Post (24m) Risks - Minimum Risk Portfolios



Summary of Minimum Risk Portfolio results - 1

- We obviously cannot draw definitive conclusions from a single test, especially when we are comparing the *ex ante* risk forecasts with only a 24-month *ex post* outcome
- The *ex post* risk calculation multiplies the monthly standard deviation over 24 months by the square root of 12, which assumes that the monthly returns have no serial correlation
  - In fact, they had an average positive correlation of 0.12, which itself varies from sub-period to sub-period
- In the next five charts, we have therefore adjusted the risks of the observed monthly returns in each period for their serial correlation


Ex Post Return and Risk - Entire Period after Tr. Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
8
7
6
5
4
3
2
1
0
Macro-Economic
S&P 500
Fundamental
Statistical
XRD model
13.0
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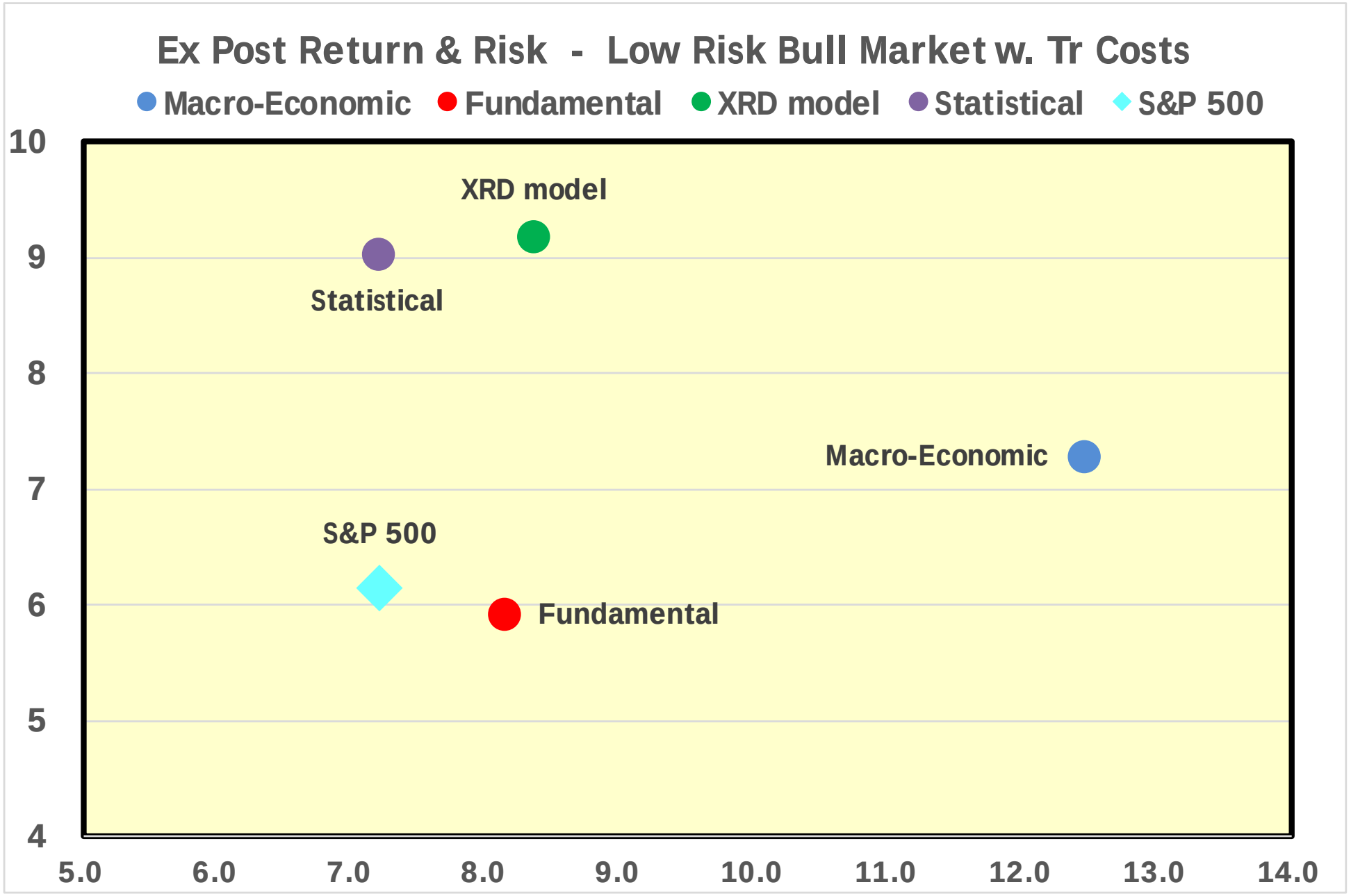
17.5
18.0


Ex Post Return & Risk - Low Risk Bull Market w. Tr Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
10
9
8
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4
5.0
6.0
7.0
8.0
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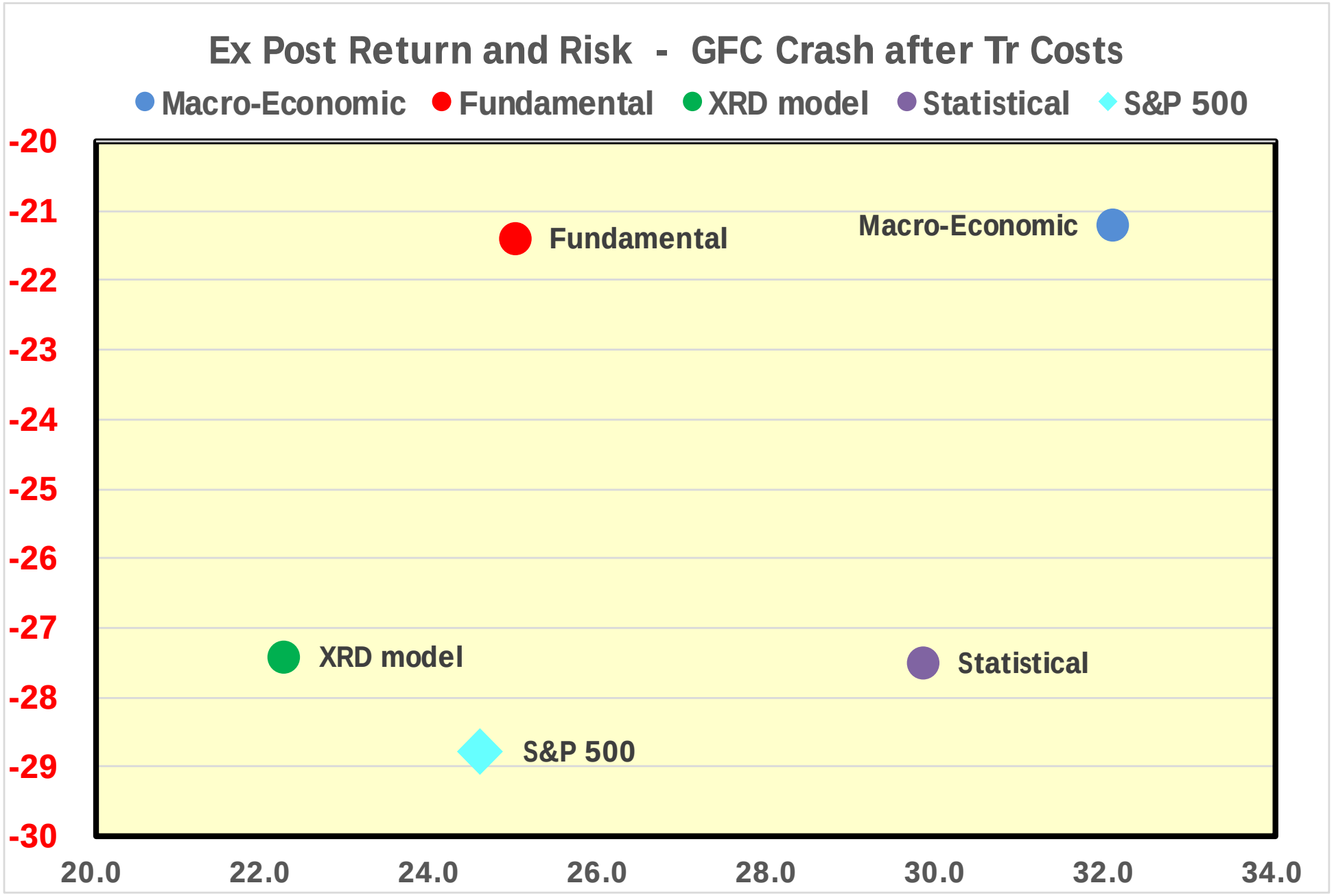
12.0
13.0
14.0
XRD model
Statistical
Macro-Economic
S&P 500
Fundamental


Ex Post Return and Risk - GFC Crash after Tr Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
-20
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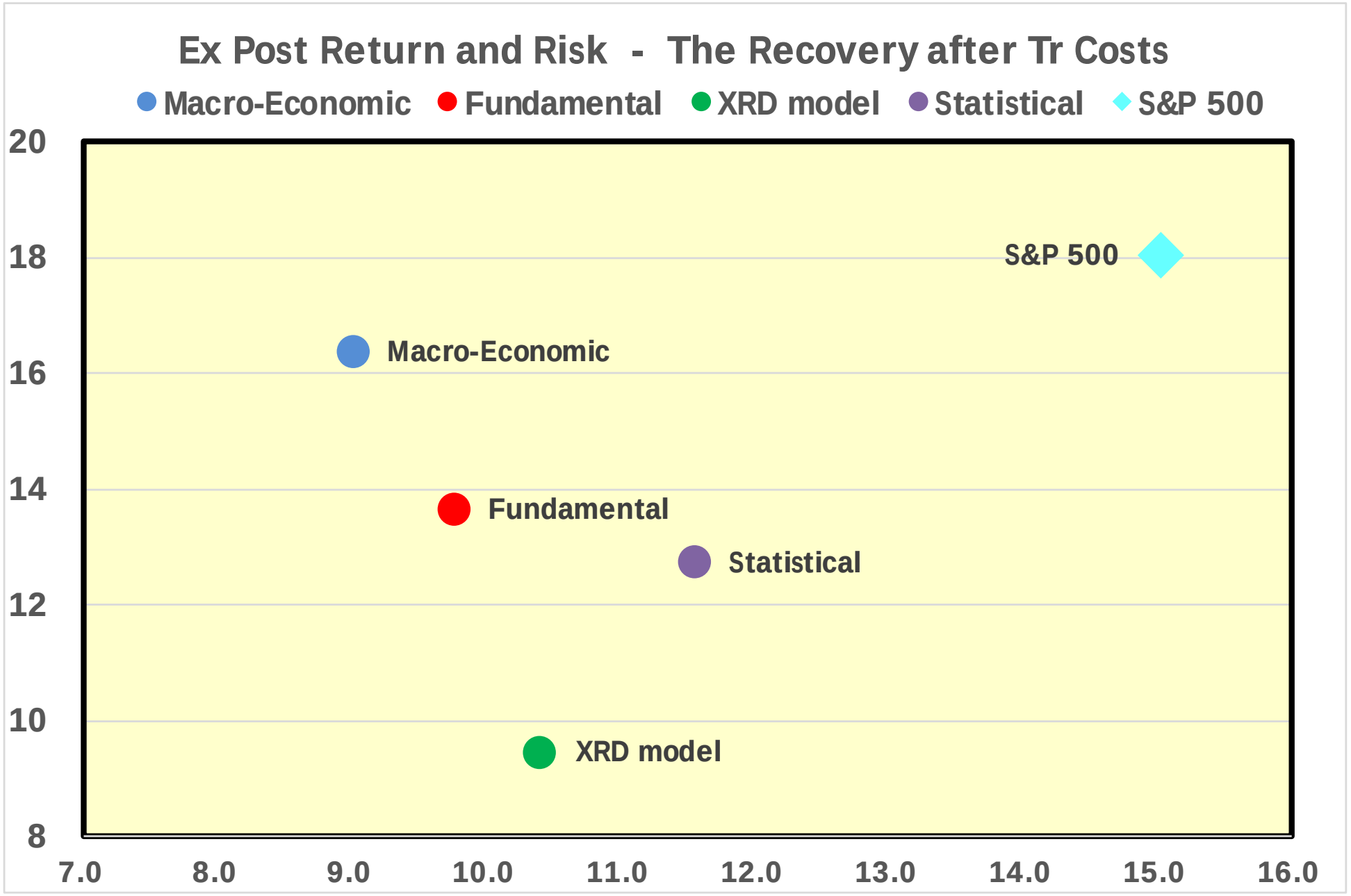
32.0
34.0
Fundamental
Macro-Economic
XRD model
Statistical
S&P 500


Ex Post Return and Risk - The Recovery after Tr Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
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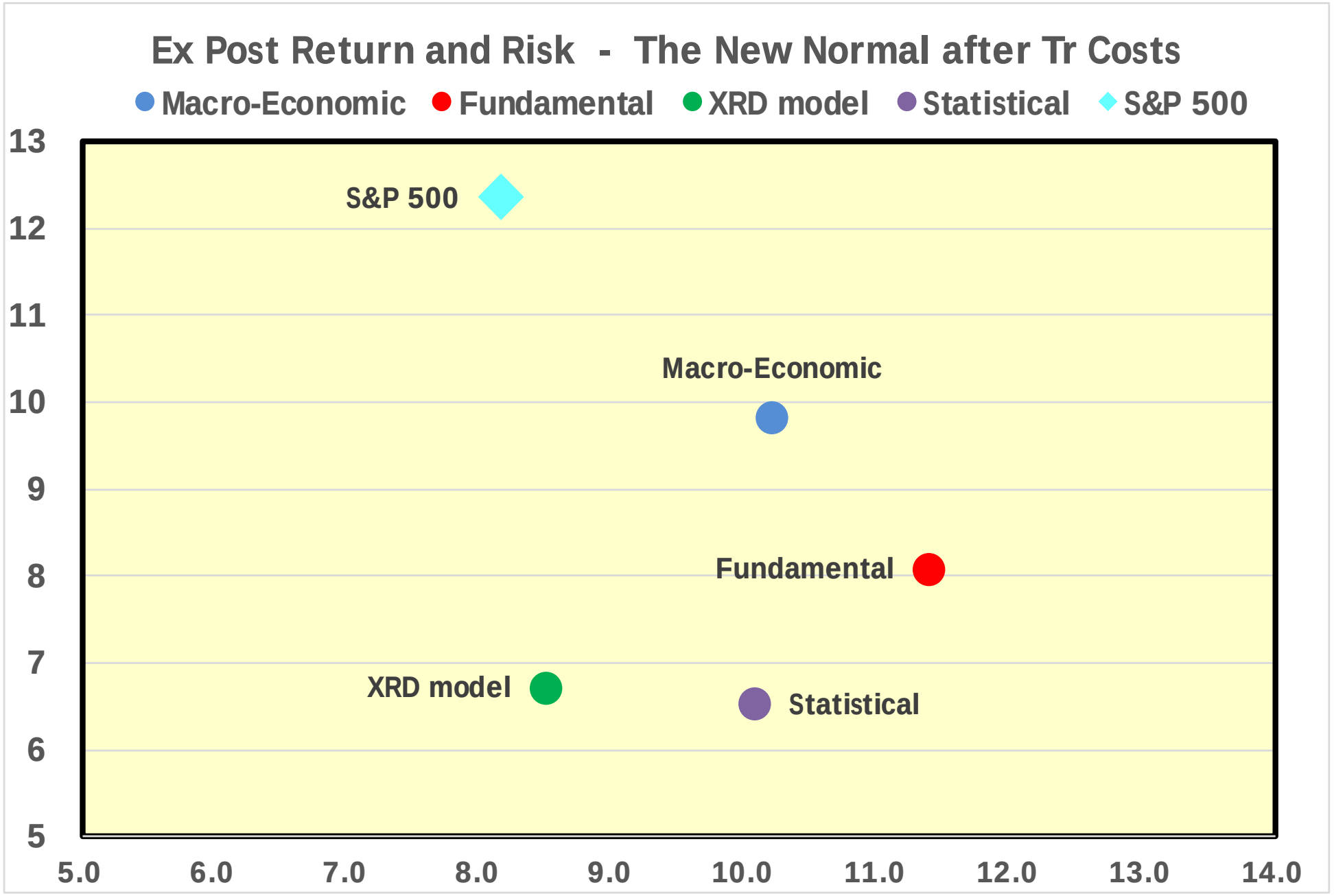
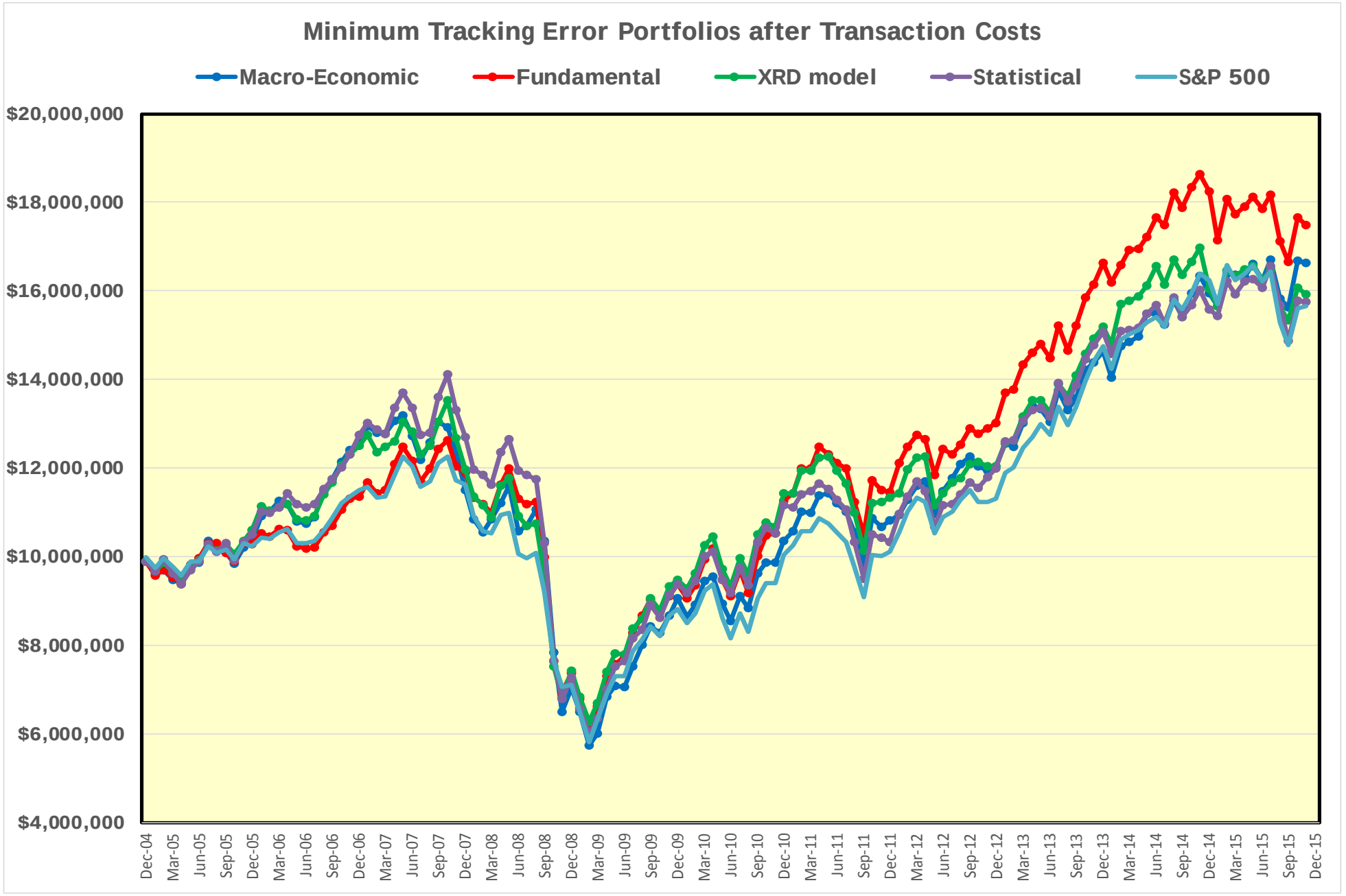
S&P 500
Macro-Economic
Fundamental
Statistical
XRD model

Ex Post Return and Risk - The New Normal after Tr Costs



Summary of Minimum Risk Portfolio results - 2

- The MVP portfolio based on the Macro-Economic model had the highest risk over the whole period at 16.7, but the lowest in the Recovery period at 9.0
- The MVP portfolio based on the Statistical model had almost the same risk overall at 16.4, but the lowest in the Low Risk Bull Market period at 7.2
- The MVP portfolio based on the Fundamental model had an overall risk of 14.5
- The MVP portfolio based on the XRD model had an overall risk of 13.9, and the lowest risk (22.2!) during the GFC period
- All four of the MVP portfolios had a lower overall risk than the S&P 500, whose risk was 17.0


Minimum Tracking Error Portfolios after Transaction Costs
Macro-Economic
Fundamental
XRD model
Statistical
S&P 500
$20,000,000
$18,000,000
$16,000,000
$14,000,000
$12,000,000
$10,000,000
$8,000,000
$6,000,000
$4,000,000
Dec-04
Mar-05
Jun-05
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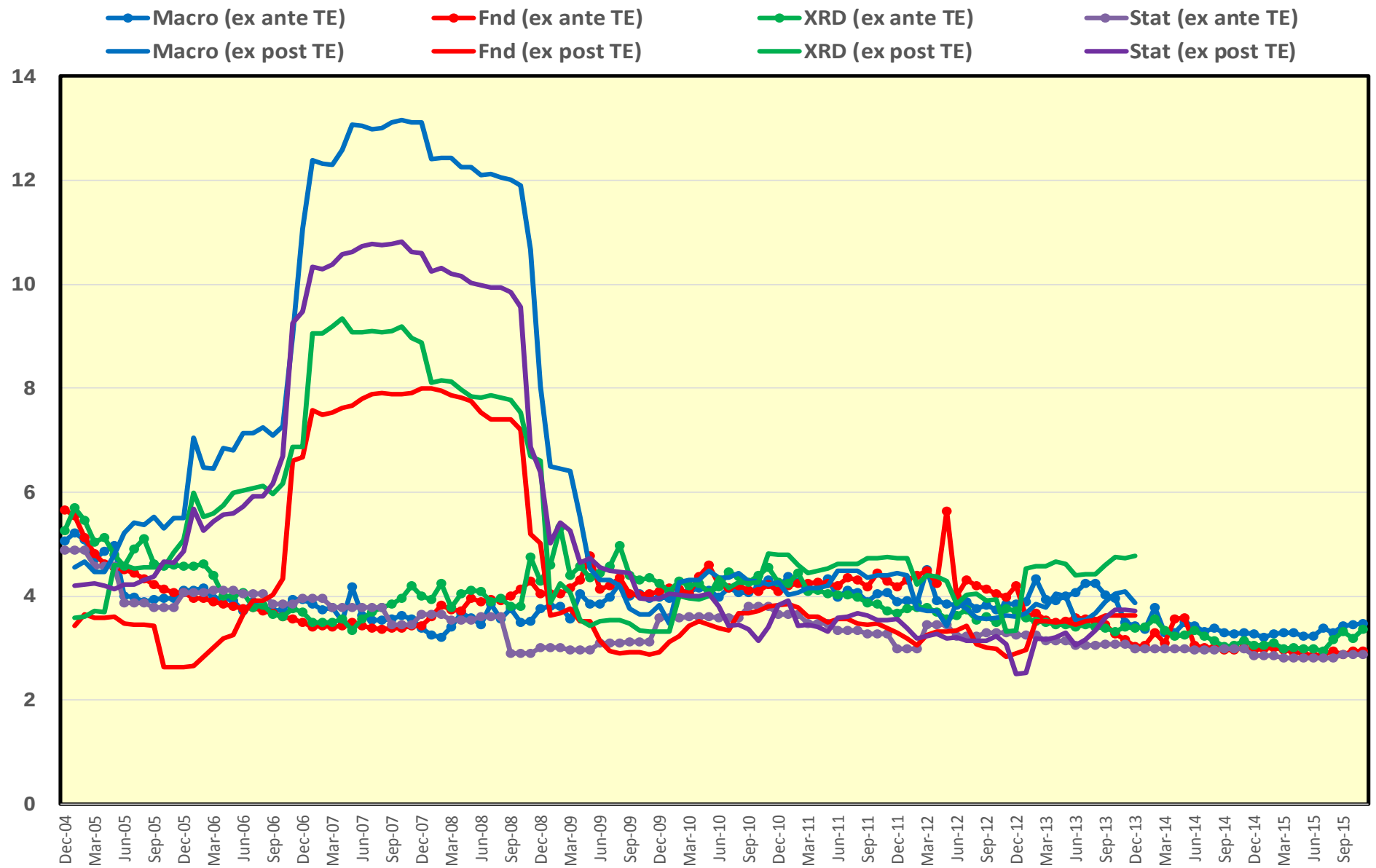
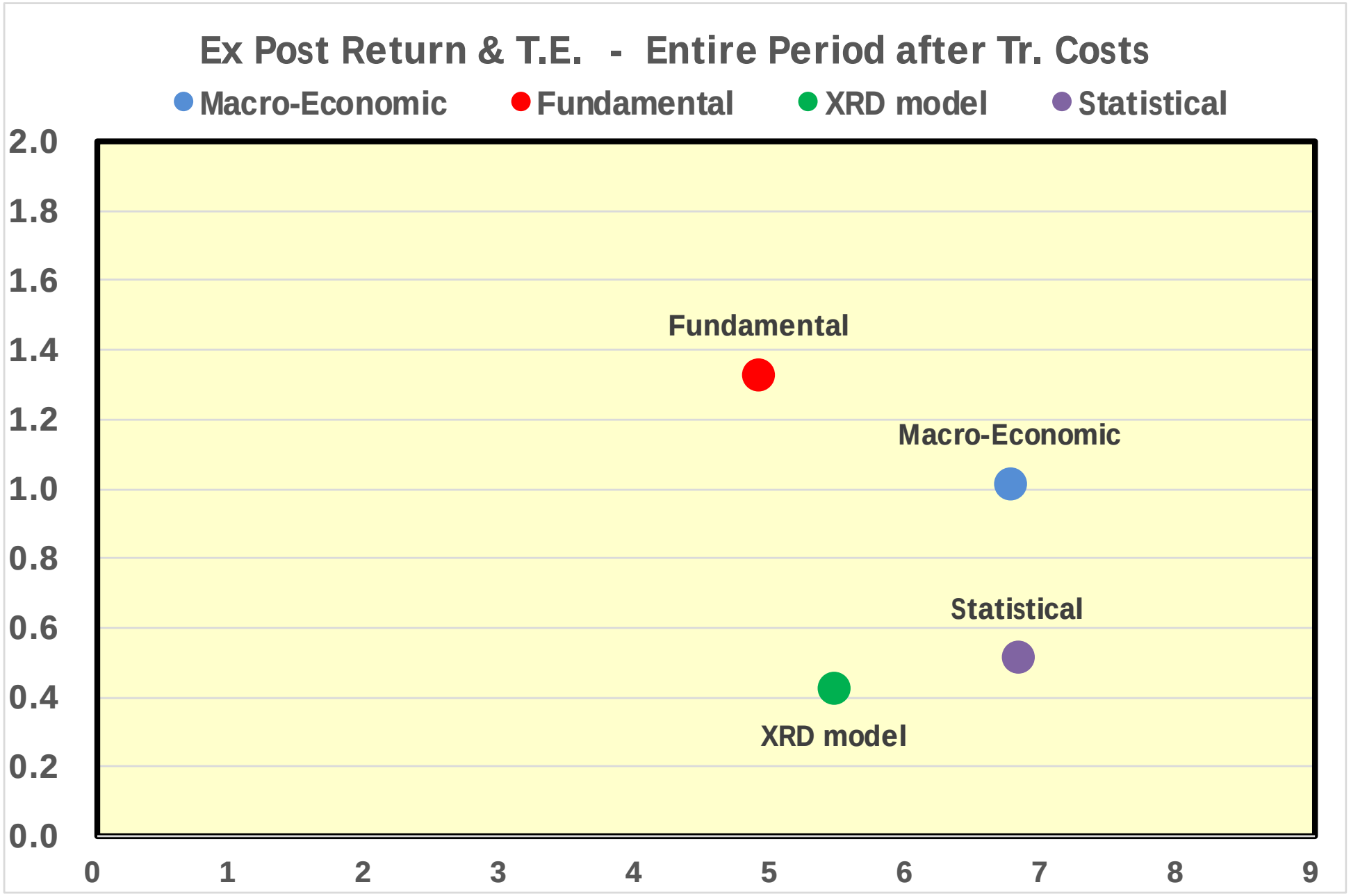
Dec-15

Ex Ante vs Ex Post (24m) T.E. - Minimum T.E. Portfolios



Summary of Minimum Tracking Error results - 1

- Here we are trying to minimise risk relative to a benchmark, in this case the S&P 500 index
- Since there are no expected returns involved, it is again a simple test of the risk models, although as before, we should remember that it is only one test over one particular historic period, and we cannot, therefore, draw very general conclusions from the results
- In the next five charts, as in the MVP case, we have adjusted the tracking errors of the observed monthly relative returns in each period for their serial correlation, which averaged 0.06 over the whole period, but with considerable variation in the four sub-periods


Ex Post Return & T.E. - Entire Period after Tr. Costs
Macro-Economic
Fundamental
XRD model
Statistical
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
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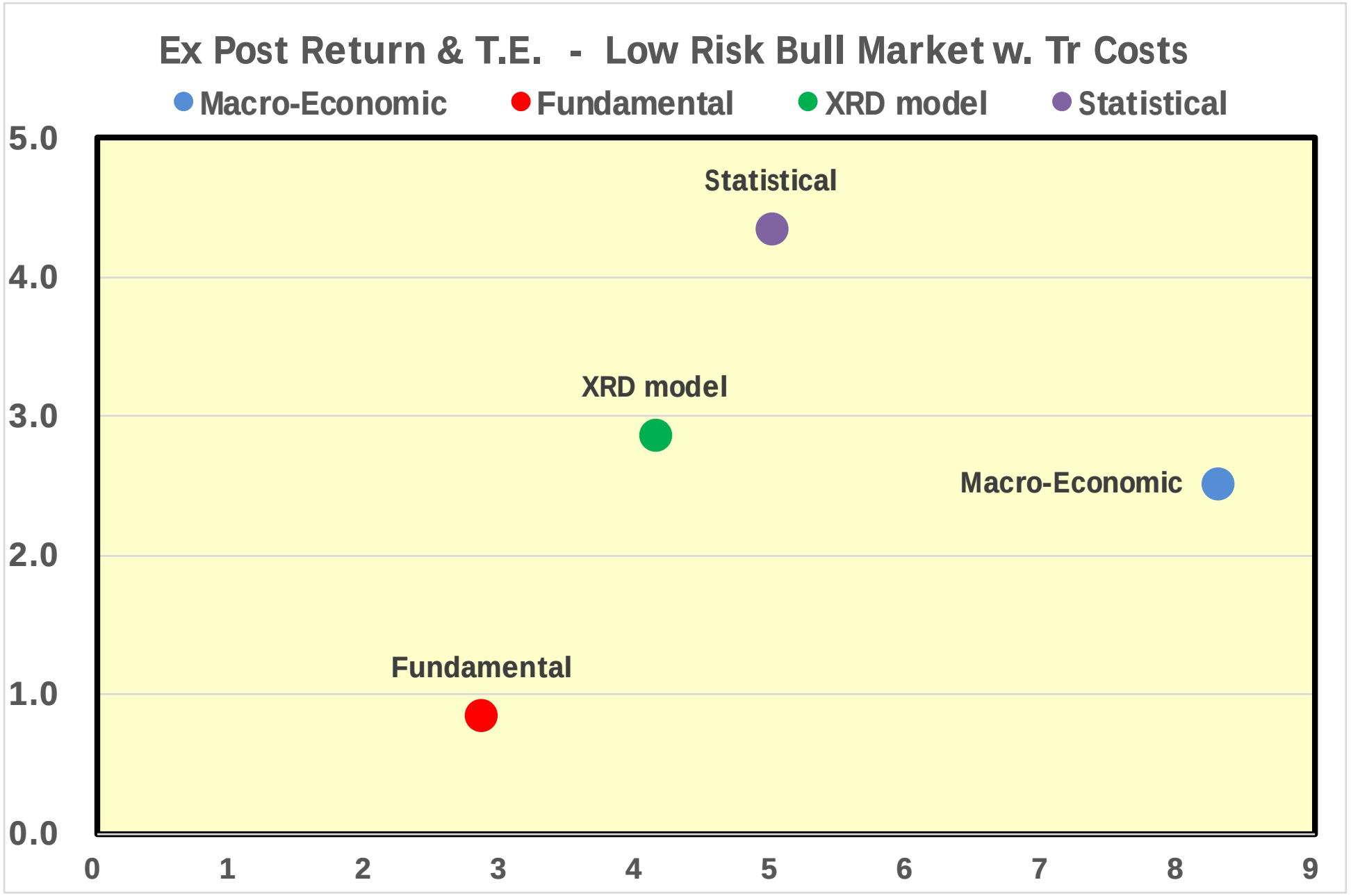
Fundamental
Macro-Economic
Statistical
XRD model


Ex Post Return & T.E. - Low Risk Bull Market w. Tr Costs
Macro-Economic
Fundamental
XRD model
Statistical
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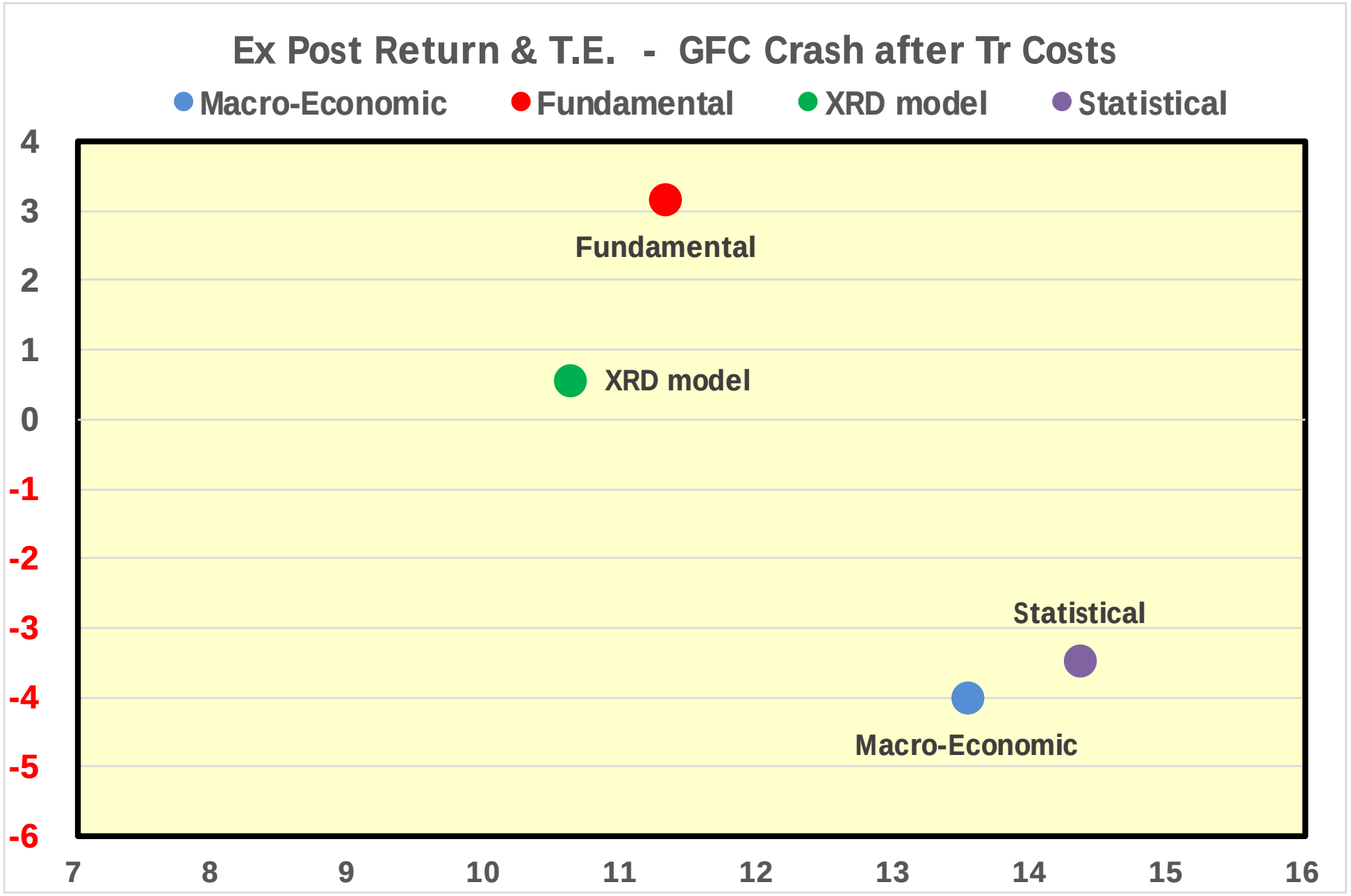
Statistical
XRD model
Macro-Economic
Fundamental


Ex Post Return & T.E. - GFC Crash after Tr Costs
Macro-Economic
Fundamental
XRD model
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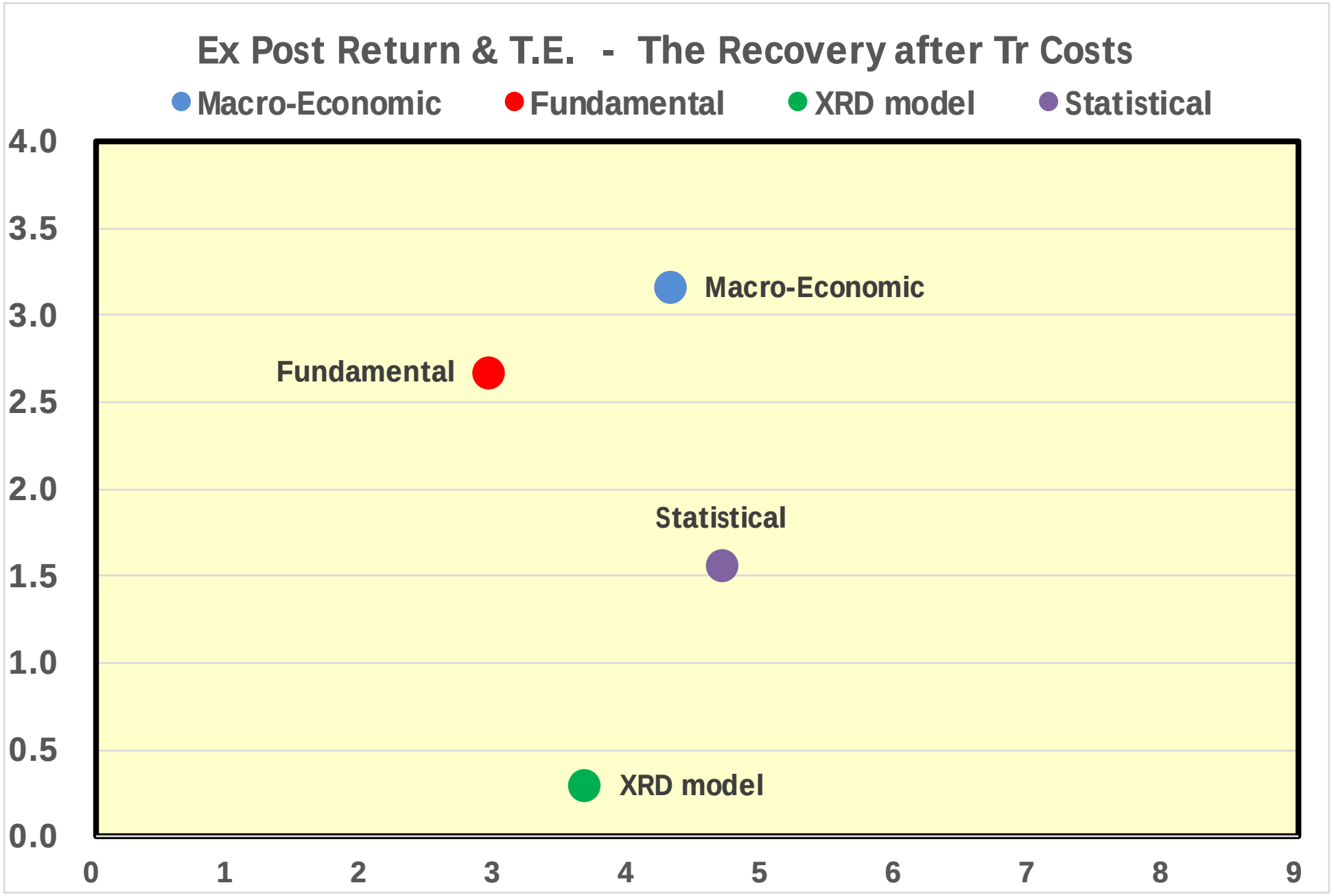
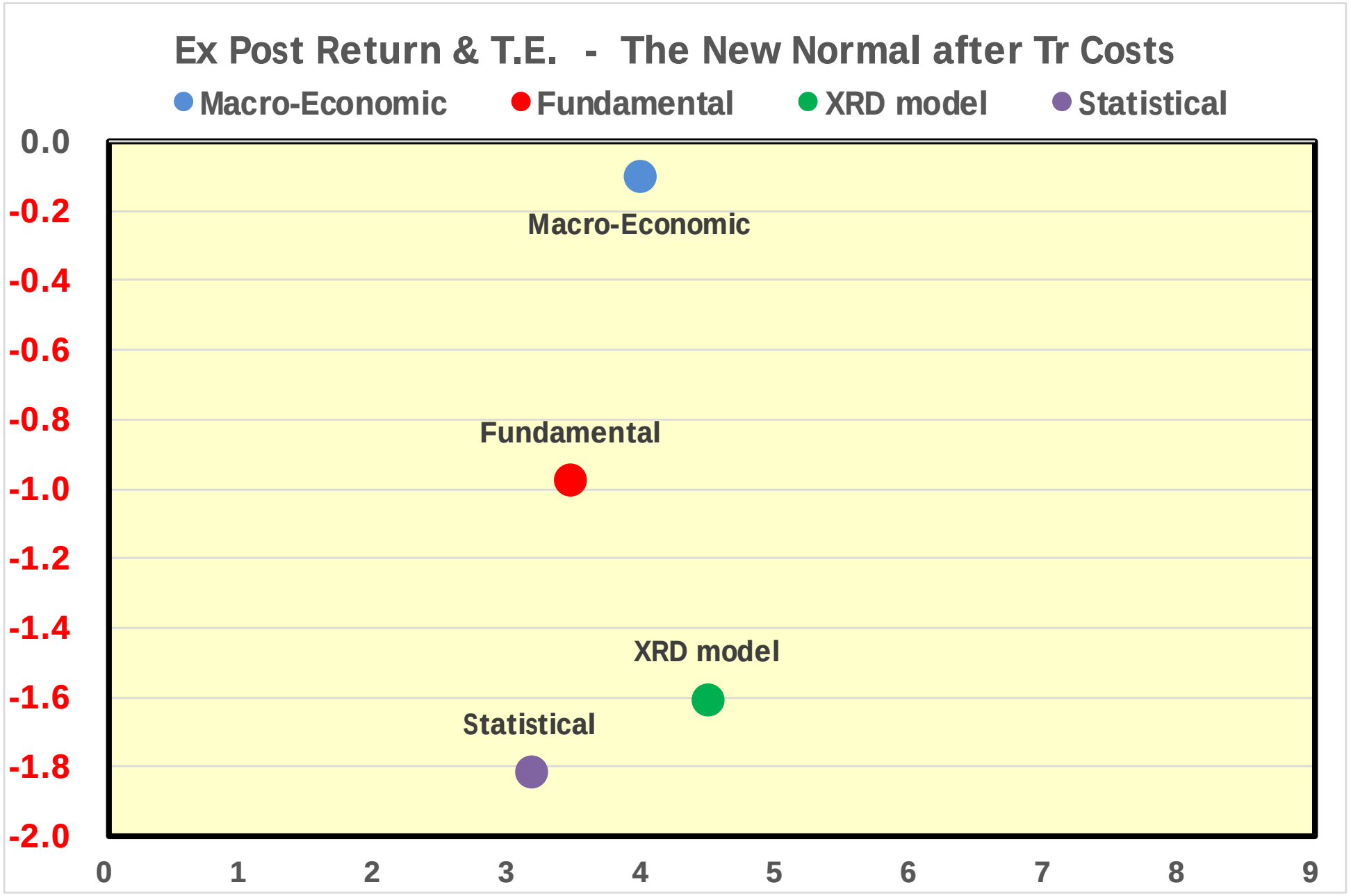
XRD model
Statistical
Macro-Economic

Ex Post Return & T.E. - The Recovery after Tr Costs



Ex Post Return & T.E. - The New Normal after Tr Costs



Summary of Minimum T.E. Portfolio results - 2

- In case you are wondering why the optimiser didn't simply give us a full-replication index fund with zero Tracking Error at every quarterly rebalance, remember that we have a minimum holding size of 0.5%, and a maximum of 5%
- However, each risk model is equally handicapped in this respect, so we can usefully compare the results
- Clearly, none of the models did a very good job during the GFC period
- During the other periods, the Fundamental and XRD models tended to achieve lower Tracking Errors than the Statistical and Macro-Economic models

Conclusion - 1

- We have deliberately not mentioned the returns achieved by either the MVP or MTE portfolios, as the point of the exercise was to see how effective the risk models were at building low Risk or low Tracking Error portfolios
- Putting it slightly differently, since there was no Skill involved in the stock selection, the returns were simply due to Luck
- As might be expected, there was no overall ‘best’ model, although all the models did a reasonably good job given the limitations of the exercise

Conclusion - 2

- As we have said before, most sensible multi-factor risk models will give a reasonably good estimate of a portfolio's risk or tracking error, its beta to the benchmark, and the split between systematic (factor-related) and residual (stock specific) risk
- One model only really becomes preferable to another when you consider the factor decomposition it gives for a particular portfolio
- Such a decomposition is more or less useful only insofar as the factor risk breakdown corresponds to the bets the manager was trying to make in the portfolio
- Thank you