Portfolio optimization in an uncertain world

Generalized Modern Portfolio Theory

Marielle de Jong

Head of Fixed-Income Quant Research at Amundi

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In this speech we

- 1. generalize the Markowitz optimization problem
- 2. solve it
- 3. link it to risk parity and similar methods

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Modern Portfolio Theory

Markowitz' (1952) mean-variance portfolio optimization problem

- Asset returns (R) follow a multivariate normal distribution

$$R \sim N(\mu, V)$$

- Objective is to maximize portfolio utility (U_x) defined on first two moments

$$U_x = x^T \mu \; - \; \lambda \cdot x^T V \, x$$

- Underlying hypothesis: parameters μ V are known with certainty.

Generalize Modern Portfolio Theory

We relax the certainty hypothesis,

asset returns don't strictly obey to predefined laws

accord the definition of utility,

investors seek to minimize loss due to unfavourable price shocks, which can be

foreseen – within the sphere defined by the Hessian
unforeseen – unframed

and accord the optimization objective

minimize loss due to

foreseeable shocks – via variance minimization
unforeseeable shocks – via <u>diversification maximization</u>

Diversification

Diversification: stake, participation, footprint, span, ...

- Concrete measures of diversification

Rao's (1982) squared entropy	$-x^T x$
Shannon (information theory)	$-x^T ln(x)$
Roncalli (2013)	$-e^T ln(x)$

- Measures that depend on risk estimates

Meucci (2009)	eigenvectors of *V*
Choueifaty & Coignard (2008)	$x^T \sigma \,/\, \sigma_p$

Generalized Modern Portfolio Theory

Generalized portfolio optimization

- We define a level of order θ (entropy)

  order - multivariate normal $R \sim N(\mu, V_\theta)$

  disorder - uniform distribution $R \sim U[0,1]$

- Maximize utility

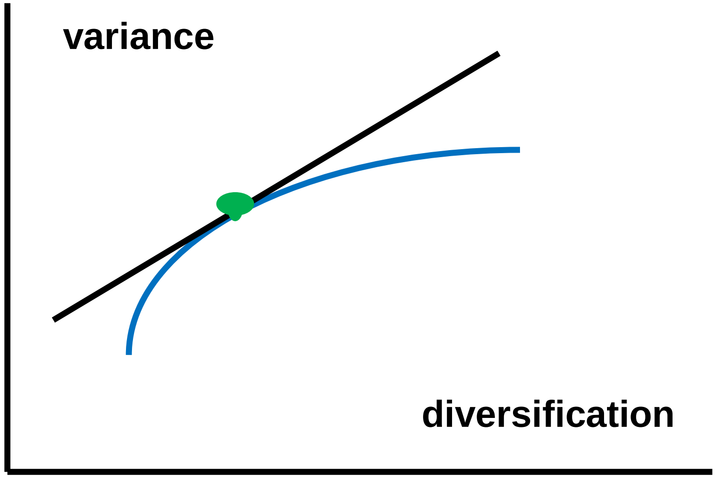
  with return objective $U_x = x^T\mu - \lambda \cdot x^T V_\theta\, x - \theta \cdot e^T ln(x)$

  without return objective $U_x = -x^T V_\theta\, x - \theta \cdot e^T ln(x)$

Solution to the generalized problem

Without return objective

variance-to-diversification frontier



- Min. $x^T V_\theta \, x + \theta \cdot e^T ln(x)$

- $\frac{\delta x^T V x}{\delta x} = \frac{\delta e^T ln(x)}{\delta x} \quad \leftrightarrow \quad Vx = \frac{1}{x} \quad \leftrightarrow \quad (Vx) \cdot (x)$ equal

- **Risk parity is optimal !!**

Proof by Roncalli (2013)

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Risk parity is optimal

in an uncertain world

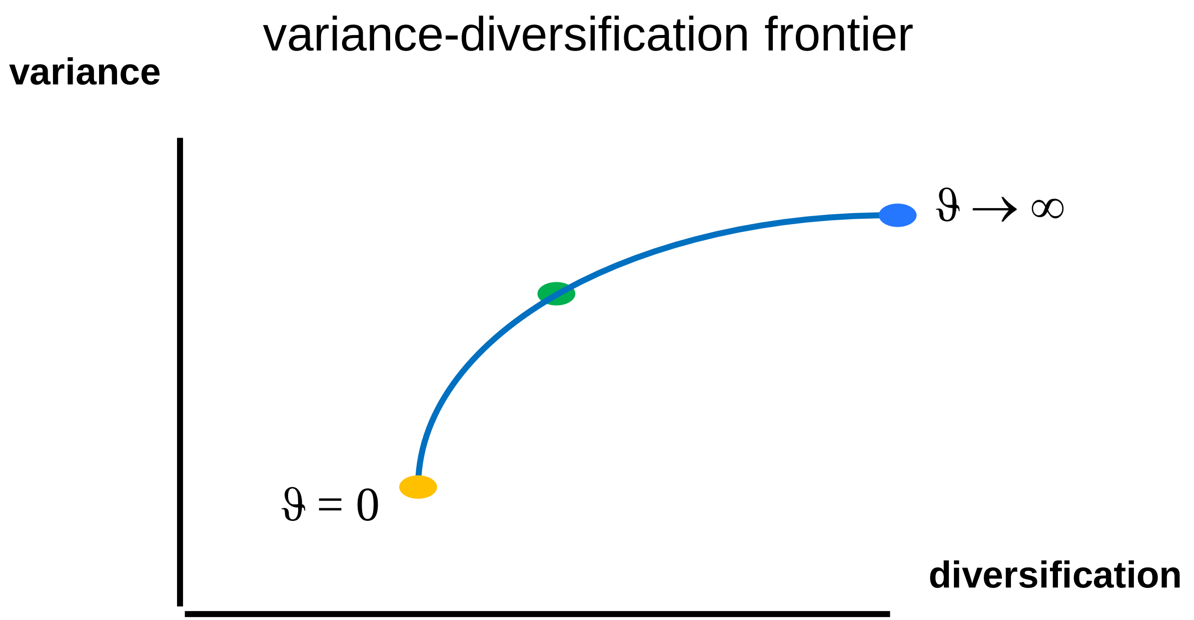
- By admitting uncertainty one acknowledges the need to protect against the unknown.
- Portfolio is optimal, i.e. loss is minimized, when portfolio variance and diversification optimized.
- Risk parity when variance-over-diversification at maximum.

- Risk parity solution not unique.

  frontier of solutions depending on θ entropy (disorder/uncertainty)

  several measures of diversification

Optimal solutions depending on θ



● (blue)	High entropy – optimal portfolio	1/N
● (green)	Intermediate – optimal portfolio	1/ risk contribution
● (yellow)	Low entropy – optimal portfolio	Markowitz

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Choice of diversification measure

If Rao's measure adopted

- Min. $x^T V_\theta\, x + \theta \cdot x^T x \qquad \longleftrightarrow \qquad$ Min. $x^T (V_\theta + \theta \cdot I)\, x$
- $(V_\theta + \theta \cdot I)$ variance increased with respect to covariance
- This is what happens in

  Ledoit & Wolf's (2003) covariance shrinkage method

  Kritzman's (2016) stability-adjustment method

- **Covariance shrinkage and stability adjusting are optimal !!**

Choice of diversification measure

If Kullback-Leibler (cross entropy) measure adopted

- Min. $\sum_i p_i ln(^{p_i}/_{q_i})$ s.t. performance target
- If $q = (^1/_N, \ldots, ^1/_N)$ the measure is equivalent to Shannon
- Method introduced by

  Bera & Park (2008) "Maximum entropy principle"

- **Entropy-based methods optimal !!**

Bayesian optimization is optimal

Brown (1976) "Optimal portfolio choice under uncertainty"

- Min. $x^T V_B \, x$

  V_B *is estimated such that the loss is minimized due to sub-optimality that would arise if the estimates turn out to be wrong.*

- A less circumvent formulation:

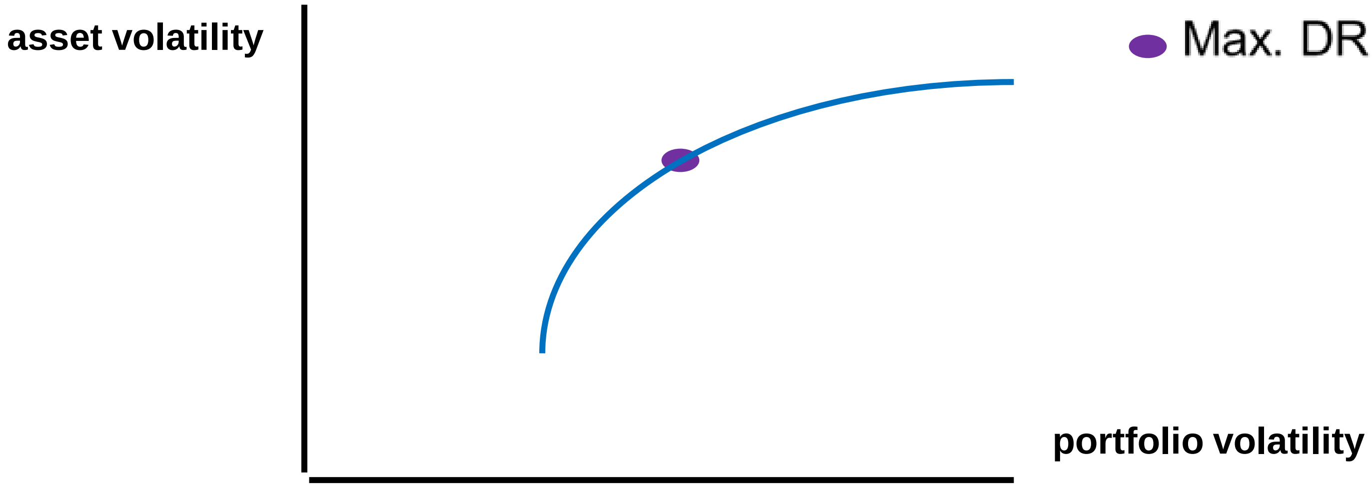
  Loss is minimized which is partly foreseeable, partly not.

- **Bayesian portfolio optimization is optimal !!**

Diversification measure based on risk estimates

Diversification Ratio

asset volatility / portfolio volatility



● Max. DR = $x^T\sigma \,/\, \sigma_p$

If estimates are wrong, diversification is wrong and <u>inefficient</u> as a result.

- **Maximum diversification is not in the same list**

Recap

entropy θ intermediate | **θ extreme**

without return objective

optimal strategies

entropy θ intermediate		θ extreme
risk parity	$\theta \to \infty$	1 / N
covariance shrinkage	$\theta = 0$	minimum variance
stability adjusting		
Bayes		

with return objective

investment settings

Strategic asset allocation

parity approach

high-conviction investing

Markowitz

Black-Litterman

Investment example - *equity-bond allocation problem*

without return objective

Let σ_{eq} = 15% **σ_{bo}= 5%** **ρ = 0.2** ***high entropy***

risk parity

$$\begin{bmatrix} eq \\ bo \end{bmatrix} = \begin{bmatrix} 25 \\ 75 \end{bmatrix} \qquad x_i = 1/\sigma_i$$

Markowitz

$$\begin{bmatrix} eq \\ bo \end{bmatrix} = \begin{bmatrix} 5 \\ 95 \end{bmatrix} \qquad x_i \approx 1/\sigma_i^2$$

with return objective

Let μ_{eq} = 6% **μ_{bo} = 2%**

risk parity

$$\begin{bmatrix} eq \\ bo \end{bmatrix} = \begin{bmatrix} 40 \\ 60 \end{bmatrix}$$

Markowitz

$$\begin{bmatrix} eq \\ bo \end{bmatrix} = \begin{bmatrix} 25 \\ 75 \end{bmatrix}$$

Example put forward by Qian (2011)

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Strategies coherent with generalized MPT

1. Top-down approach* (not full optimization)

As covariance more predictable within asset classes than between, Markowitz optimization is deployed within- and risk budgeting between.

2. Bridgewater® All Weather Funds

Four plausible economic scenarios are weighted by their respective volatility levels.

3. Global Macro

Asset class weights set inversely proportional to volatility levels

* Discussed by Scherer (2007) Portfolio Construction & Risk Budgeting

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A bit of philosophy

- Main critic on MPT: "*Markowitz optimization vulnerable to estimation error.*"

  This statement is a misconception. The flaw is the certainty hypothesis.

- Generalized MPT $U_x = x^T\mu - \lambda \cdot x^T V_\theta \, x - \theta \cdot e^T ln(x)$

  λ risk aversion is a subjective parameter

  θ entropy is an objective parameter

- A risk model that defines order differs from a risk model that defines beliefs.

  i.e. $V_\theta \neq V_0$

Risk model that defines order

- Whether to adopt a risk factor, assess if it more damaging

  to ignore it – so that risk efficiency will be foregone

  to adopt it – so that diversification will be foregone

Statistical significance may not be an effective criterion.

- We explore two models

Model 1. one notch down from darkness

only volatility levels predictable

Model 2. two notches down from darkness:

Capital Asset Pricing Model

Risk model that defines order

Model 1. Volatility levels (σ) predictable

optimal portfolio $x_i = {}^{1}/_{\sigma_i}$

Model 2. CAPM – market betas (β) and specific volatilities (σ) predictable

optimal portfolio* $x_i \propto {}^{1}/_{\sigma_i} \; and \; x_i \propto {}^{1}/_{\beta_i}$

Model 2[bis]. constrained CAPM*

$$\forall_{i \in industry\ j}: \quad R_{it} = \beta_j \cdot {R_t}^M + \varepsilon_{it}$$

optimal portfolio when $x_i = {}^{1}/_{\sigma_i}$ between individual assets

$x_j = {}^{1}/_{\beta_j}$ between industry groups

* Clarke et al. (2013) derive the near closed-form solution

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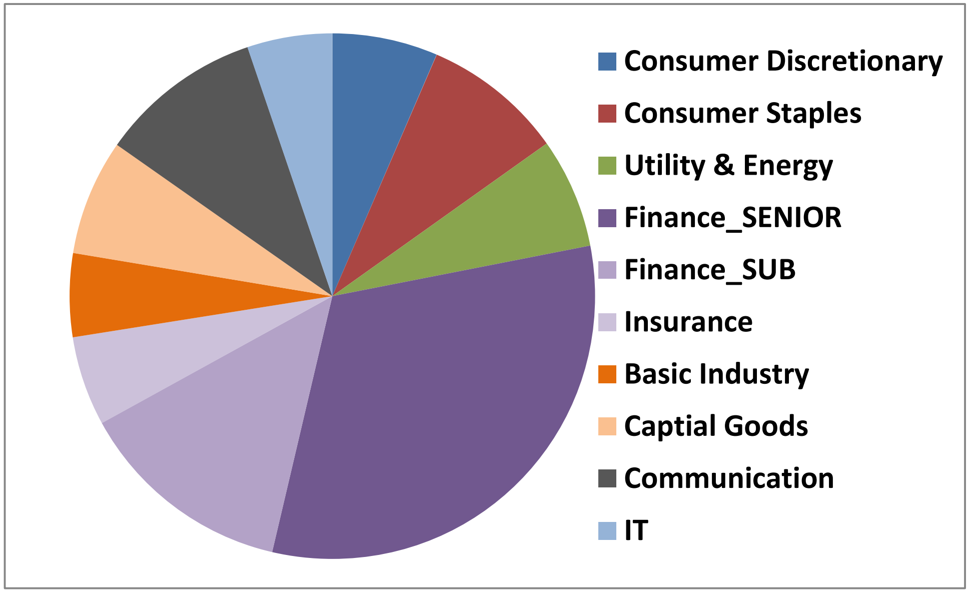
Back-test Model 2bis on corporate bonds

Testbed

- Region: Eurozone
- Period: 2007 to 2016
- Industries: Barclays level 3 sectors aggregated to ten groups

- Universe: Barclays Euro Corporate
- Monthly data frequency



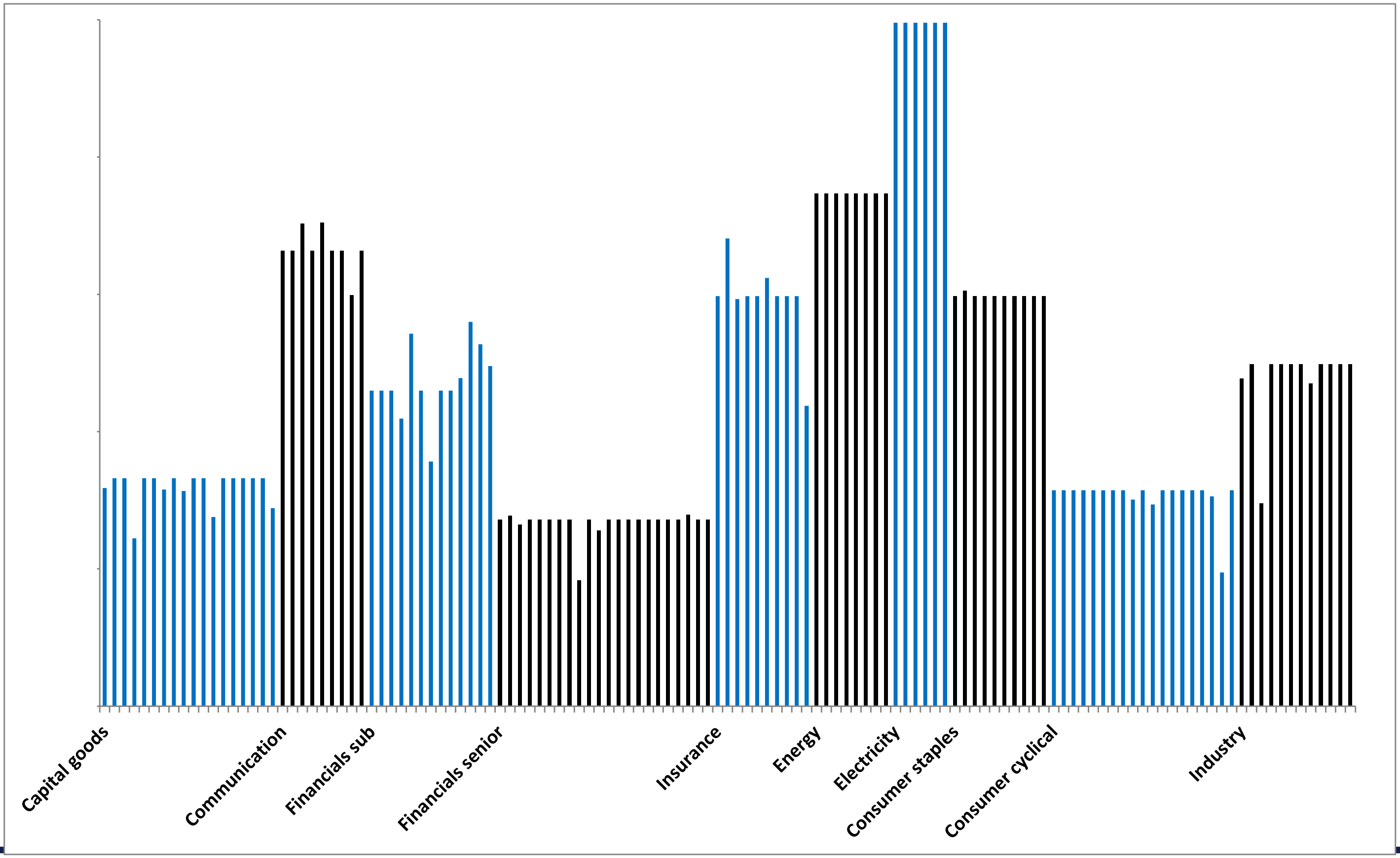
Test setup

- Portfolio rebalanced at regular intervals (no foresight)
- *Ex post* performance over test period compared with standard benchmark

Parity portfolio

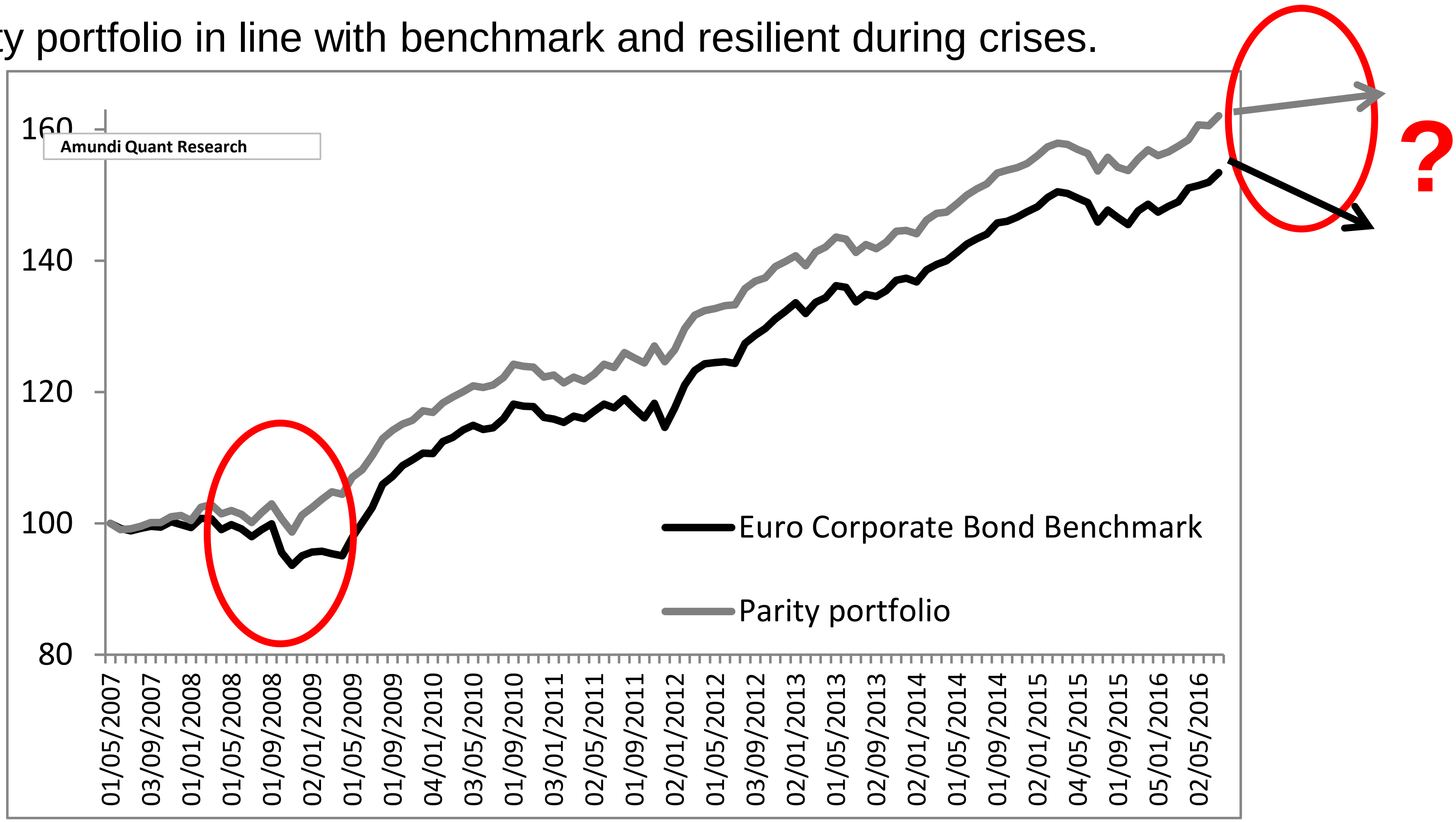
Ten sectors contribute equally to overall portfolio risk (DTS)

Within sectors firms (bond issuers) contribute equally



Back-test: return

Parity portfolio in line with benchmark and resilient during crises.



Underlying figures on next page

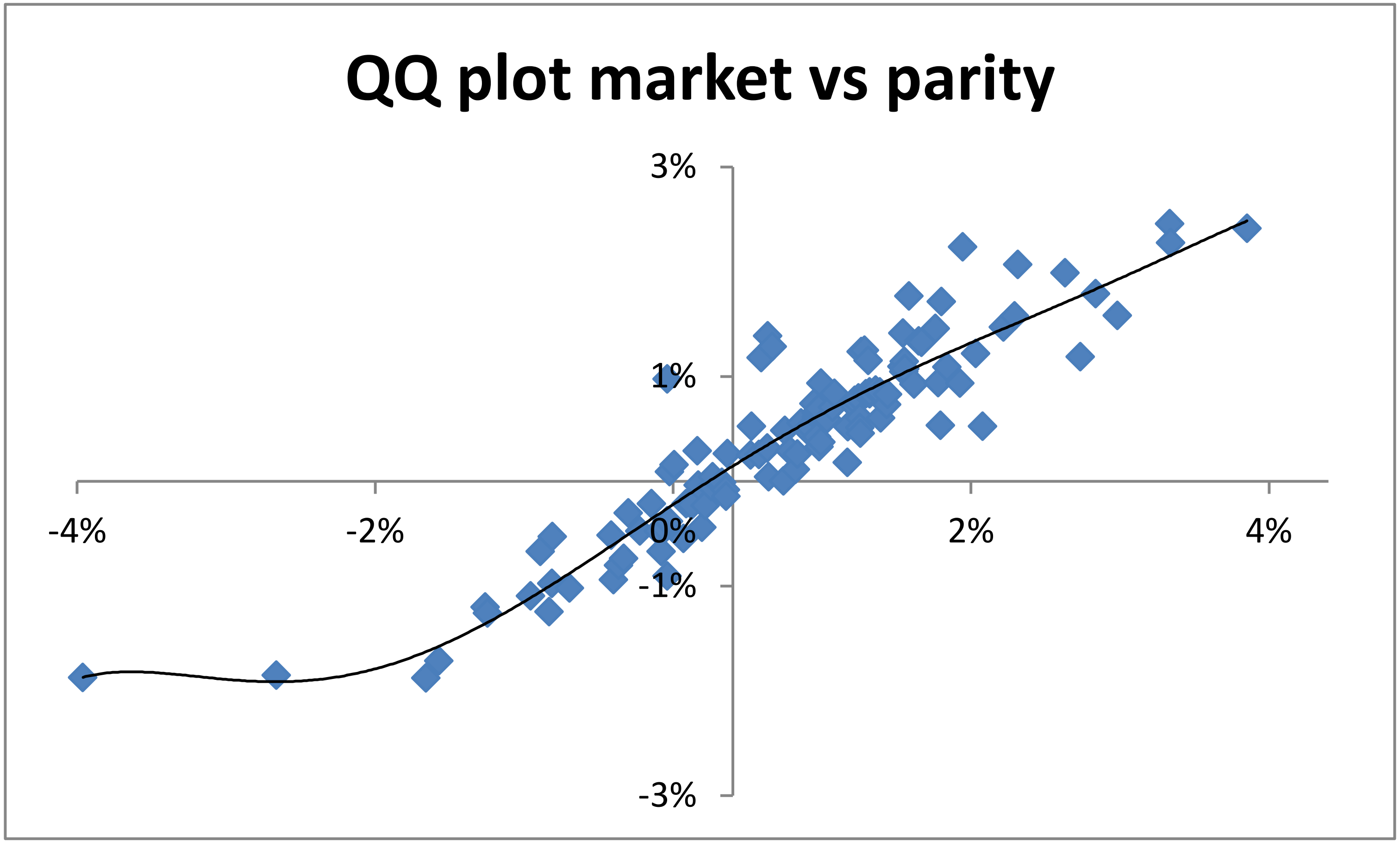
Key figures

	benchmark	Parity index	Parity portfolio
return	4.8%	5.4%	5.4%
volatility	4.1%	3.4%	3.4%
Sharpe ratio	1.2	1.6	1.6
max drawdown	-6.3%	-4.3%	-4.1%
TE	-	1.64%	0.11% (from target)
# bonds	1843	1843	≤ 233
# issuers	400	400	≤ 127
turnover per month #	30	30	10
weight	0.3%	-	1.6%

All figures are annual if not specified

Convex risk profile

Risk parity amortizes drawdowns

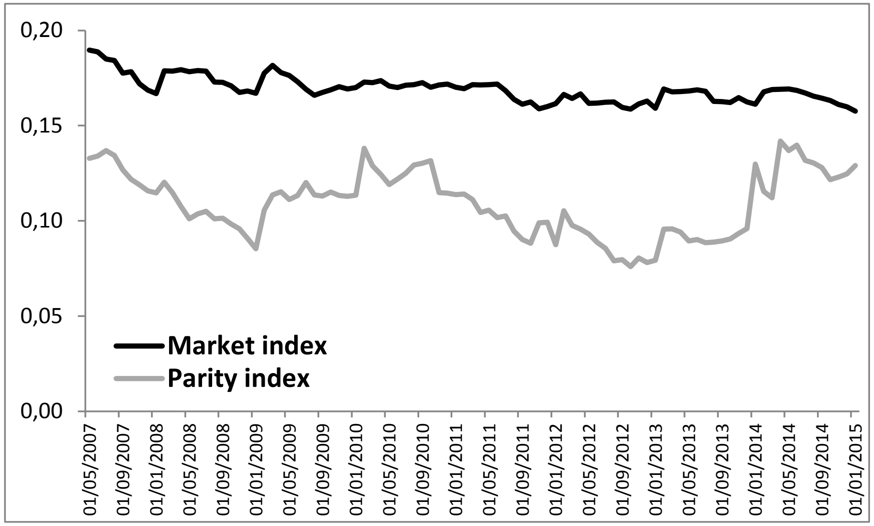


Induced quality bias

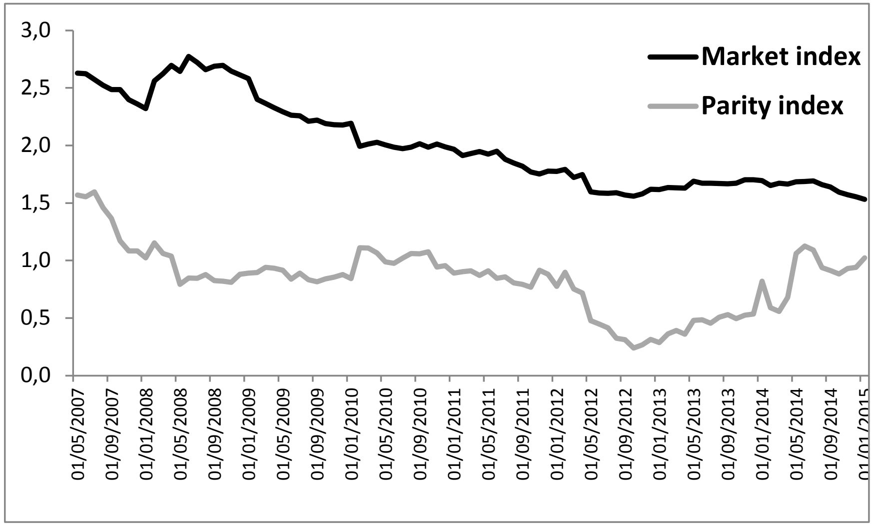
The parity index has a structural bias towards low-indebted firms

As measured by two debt ratios.

Debt to assets



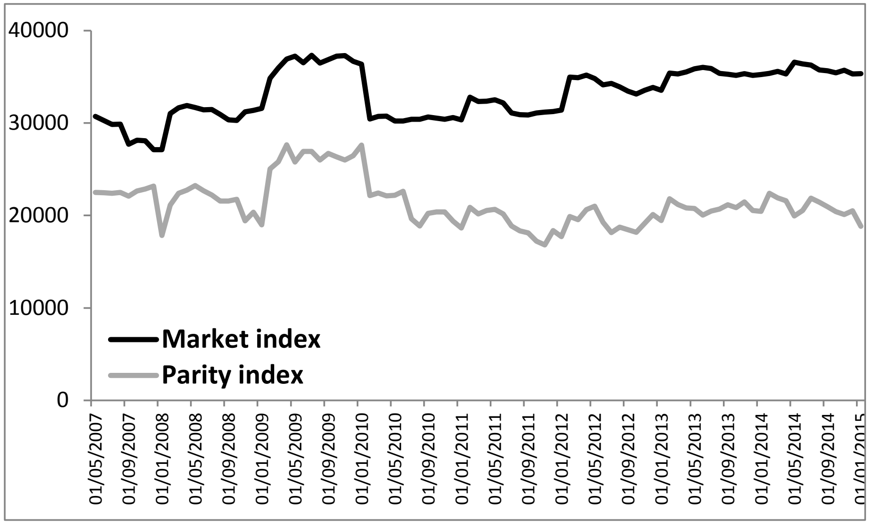
Debt to equity



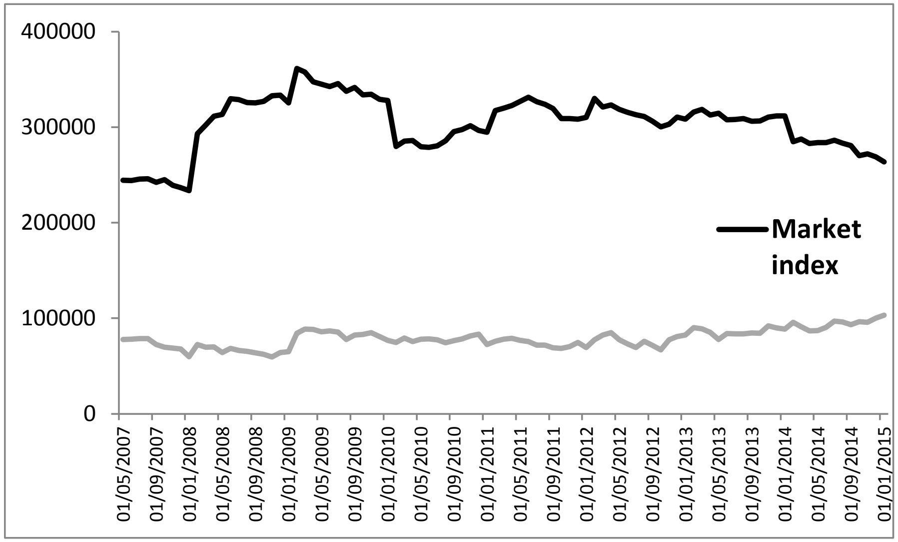
Induced size bias

The parity index has a structural bias towards small firms as measured by two firm fundamentals, sales revenues and book value

Sales



Assets



Conclusion

Incorporating uncertainty

and consequently

generalizing the Markowitz optimization problem makes sense.

It corresponds to investment practice,

with a list a portfolio construction methods

with alternative philosophies, e.g. regret theory

Incorporating uncertainty doesn’t suit active high-conviction investment.