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Active Mismanagement: Defining Optimal Portfolio Turnover

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Introduction

- In the debate among active, passive and quasi-passive (e.g. smart beta) approaches to portfolio management, there are two concepts of what *active* means.
  - This term is sometimes used to mean "differences from benchmark" while in other contexts it refers to the frequency of trading of the securities in the portfolio.
  - Studies such as Elton, Gruber and Blake (1996, 2004) suggest that active managers exhibit risk-adjusted positive relative returns before transaction costs and expenses, but lag passive indices after costs.
  - Paradoxically, these studies show that alpha is *decreasing function of turnover.*
  - We assert that active managers trade too frequently as a result of an improper conception of how turnover levels vary across securities *within a portfolio.*

A Motivating Example

- Consider an actively managed equity portfolio of $100M
  - During a given year, the portfolio does $30M in buy transactions and $30M in sell transactions.
  - In the way most people think of turnover, 30% of the portfolio positions have been sold and replaced in one year, suggesting a 30% level of annual "one way" turnover.
  - At 30% per annum turnover, the average holding period of each portfolio position is 3.33 years.
  - If we are considering rebalancing the portfolio, it would seem at first glance that any transaction costs incurred in this event would have to be offset by portfolio improvement (higher returns or lower risk) over the horizon of 3.33 years.
  - In reality, the horizon over which transaction costs must be recaptured is *usually a lot shorter.*

Decomposition with the Johnson J Statistic

- Let's consider the following decomposition. Our $100M active portfolio can be broken into two portfolios:
  - A $100M index portfolio holding the benchmark
  - A long/short portfolio of smaller dollar exposure (total value = 0). The long position are the positive active weights and the short positions are the negative active weights.
  - The relative absolute magnitude of the long/short portfolio as compared to the total portfolio value is often referred to as "active share".
  - We prefer the more formal definition: J-statistic (see Johnson . 1974)

$$J = 1 - (\Sigma \text{ (i=1 to n) abs } (P_i - B_i)) / 2$$

Finishing Up the Example

- Let's assume that turnover in the benchmark due to corporate actions is 4% per annum ($8 Million total transactions).
- If the long/short portfolio is $20M per side, this active portion of the portfolio must account for the other $52M in transaction volume.
  - $52M in transactions for $40M in positions implies an annual one way turnover rate of 26/40 = 65%
- The implied holding period for active positions is 1.54 years, far less than the 3.33 years for the whole portfolio.
  - *Transaction costs arising from efforts to improve the portfolio utility must be recovered over this shorter horizon.*

More Thoughts on Single Period

- In the real world, things change and our parameter estimates for return and risk (even if initially exactly correct) are likely to change as well.

- If transaction costs are zero, we can simply adjust our portfolio composition to optimally reflect our new beliefs whenever they change.

- If transaction costs are not free, the single period assumption is a serious problem.

- If transaction costs are large (e.g. capital gain taxes), the single period assumption is wholly unrealistic. Tax authorities also seem to be interested in things like weeks, months and especially "tax years."

Single Period Optimization: Unlike Units

- In trying to trade off between expected return, risk and transaction costs, over one time period we can't combine these items unless they are in the same units.
  - Transaction costs occur at a moment in time while risk and return are experienced over time
  - Common practice is extend the objective function to include transaction costs (C) that are linearly amortized at a periodic rate "A" that reflects the expected economic life of the benefits of the transaction

$$U = R - (S^2 / T) - (C \times A)$$

  - The expected average holding period for the positions resulting from a transaction is just the reciprocal of the expected one-way turnover

Multi-period Optimization

- Mossin (1968) suggests an explicit multi-period formulation for portfolio optimization.
- Cargill and Meyer (1987) focus on the risk side of the multi-period problem.
- Merton (1990) introduces continuous time analog to MVO
- Pliska (1997) provides a discrete time analog to MVO
- Li and Ng (2000) provide a framework for multi-period MVO using dynamic programming
- Multi-period optimization is rarely employed because you need *period by period expected values* for the return and risk inputs. Parameters typically have very high estimation error which limits real world use.

Smart Rebalancing

- Numerous "smart" rebalancing rules have been proposed to avoid trading costs when the expected improvement is not significant.
- Rubenstein (1991) examines the efficiency of continuous rebalancing and proposes a rule for avoiding spurious turnover.
- Kroner and Sultan (1993) propose a "hurdle" rule for rebalancing currency hedges when return distributions are time varying.
- Engle, Mezrich and Yu (1998) propose a hurdle on alpha improvement as the trigger for rebalancing.

Smarter Rebalancing

- Bey, Burgess, Cook (1990) use bootstrap resampling to identify "indifference" regions, along a fuzzy efficient frontier.
- Michaud (1998) uses resampling to measure the confidence interval on portfolio return and risk to form a "when to trade rule". Elaborated upon in Michaud and Michaud (2002) and patented.
- Markowitz and Van Dijk (2003) propose a rebalancing rule designed to approximate multi-period optimization, but argue it is mathematically intractable (at least in closed form).
- Kritzman, Mygren and Paige (2007) confirm the effectiveness of MvD(2003) to be similar to full dynamic programming up to five assets. They show that MvD can be used up to one hundred assets.

The Fundamental Law and Turnover

- In Grinold (1989), the "Fundamental Law of Active Management" describes one of the properties of a strategy as "breadth".
  - Breadth is often described as the "number of independent bets in a portfolio strategy" times annual turnover.
  - Per the Fundamental Law, Grinold and Stuckelman (1993) show that the value added by an investment strategy is approximately a *square root function of turnover, while dollar transaction costs are roughly a linear function of turnover, so an optimal level of turnover must exist for a given strategy.*
  - Quantifying the number of "independent bets" in a portfolio is problematic which has led to ad hoc choices of turnover levels.

Geometric Versus Linear Tradeoffs

- For small transaction costs, arithmetic amortization is sufficient, but if costs are large we need to consider compounding.

- Assume a trade with 20% trading cost and an expected holding period of one year.
  - We can get an expected alpha improvement of 20%. But if we give up 20% of our money now, and invest at 20%, we only end up with 96% of the money we have now.
  - Solution is to adjust the amortization rate to reflect the correct geometric amortization rate.

One Way to Think About Independent Bets

- If all securities were uncorrelated and of equal volatility, the risk of an equally weighted portfolio would be inversely proportional to the number of portfolio holdings.

- Given the estimated volatility of a portfolio, and the average of the estimated volatility values for individual securities, we can solve for the implied number of equal weight "uncorrelated portfolio positions".

- This value acts as a proxy for the "number of independent bets".

- At any rebalancing, the optimal level of turnover will get us to the best compromise between the zero cost optimal portfolio, and the initial portfolio, conditional on the alpha decay rate. See Sneddon (2005, 2008).

Diamond Are Forever But Alpha Isn't

- Once we've imposed a finite time horizon on a portfolio strategy, we also have to consider that probability that our optimal portfolio might underperform our current portfolios.
- Consider an optimal portfolio that is better than my current portfolio by *annual utility increment positive D.*
  - If I knew that the increment D would be fixed forever, we should be willing to pay a lot of trading costs now to get to the optimal portfolio. In the long run we know the optimal portfolio will provide more terminal wealth.
  - However, if we knew that increment D would only last 30 seconds, then we would not want to spend material trading costs now for a benefit that might not be realized during the horizon.

Probability of Realization

- We need to adjust the rate at which return/risk and trading costs are traded off to incorporate the likelihood that a positive increment in utility will be realized as better risk-adjusted returns during the expected active holding period.

- We define the probability of realization, P, like a one-tailed T test

$$P = N\left(\left(\left(U_o - U_i\right) / TE_{io}\right) * \left(1/A\right)^{.5}\right)$$

N(x) is the cumulative normal function

The Algebra of Realization Probability

- The numerator is the improvement in utility (risk adjusted return) between the optimal and initial portfolios.

- The denominator is the tracking error between the optimal and initial portfolios. Essentially it's the standard error on the expected improvement in utility.
  - If there is no tracking error between the initial and optimal portfolios, P approaches 100%. Consider "optimizing a portfolio" by getting a manager to cut fees. The improvement in utility is certain no matter how short the time horizon. Not something to which we investors paid attention until recently.
  - If turnover is very low, A will approach zero, so P will approach 100%. For long time horizons, we have the classical case that assumes certainty

Recursion and What's Left?

- If we assume trading costs are small, so we don't have to worry about the geometric issues, we get that the active amortization constant should be divided by the probability of realization.
  - Optimal turnover in high frequency strategies could be cut by half.
- Unfortunately, the probability of realization is a function of the optimal portfolio, which itself is a function of the amortization constant.
- *The problem is recursive.*
  - For manual cases, we can either do a lot really complex algebra, or rely on a little trial and error. The Northfield optimizer already handles this.

Conclusions

- A simple decomposition of a typical active portfolio into an index fund and a long short portfolio illustrates the relationship of active share to the proper estimation of the time horizon over which the cost of a rebalancing transaction must recaptured.
  - Typically, active turnover is much higher than overall turnover so this time horizon is much shorter than conventionally assumed.
- Making optimal tradeoffs between return, risk and small trading costs also involve consideration of alpha decay and the breadth of the active strategy.
- Obtaining optimal active time horizon (or turnover rate) is a recursive calculation once the probability of realization is included.
- For very large trading costs (e.g. taxes) the amortization rate should be adjusted for geometric effects.