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Use of Factor Models in the Presence of Higher Moments

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Introduction

- Usually Implicit in the design of linear models is that concept that the distribution of individual asset returns is at least symmetric if not Gaussian.
- *Many wrongly believe that it not possible for factor risk models to do a good job of dealing with risk for securities where there is a structural expectation of skew or kurtosis in the asset return distribution,* as would arise from assets such as options or "structured products". There is a belief that portfolio risk of portfolios containing assets with non-linear behavior must be evaluated through "simulation and repricing".
- We will show that repricing is not necessary, *or even desirable under the vast majority of cases.*
- We will show that a factor representation of higher moments is sufficient to allow for portfolio optimization in the presence of non-zero estimation error of the parameters.

Solvency Risk is Divorced from Investing

- Many risk systems that were created for commercial banks focus on the *current solvency of the entity.* The current position of a financial entity is summarized in the Balance Sheet as a moment in time.
- If the assets of the entity would have to be liquidated in order to pay liabilities, the value of the assets would subject to market fluctuations during the process.
- Value at Risk, and Conditional Value at Risk are really ways of expressing a confidence interval on the balance sheet asset value.
- This conception of risk may be of great interest to organizations like a highly geared hedge that wishes to avoid risk levels that would potentially expose the organization to non-survival.
- The vast majority of institutional investors do not actually face economically material solvency risk, so systems focused on solvency risk are the proverbial "square peg in a round hole."

Risk in Investing

- The purpose of risk assessment and risk management in investing is not about avoiding risks. If it was, an investor could just hide their wealth under their mattress and declare themselves successful.

- The purpose of risk assessment and risk management in investing is to allow investment in risky assets that are likely (but not guaranteed) to produce a greater multi-period return than risk free assets, while confining the dispersion of cumulative investment performance over time to a range acceptable to the investor.

- Like any business activity or project, the risks of investing are about the future profitability of the investment activity. From a financial statement perspective, this is manifested in the Income Statement not the Balance Sheet.

Solvency Risks May Be of Opposite Sign

- Consider a sovereign wealth fund with a portfolio of bonds with effective duration 15, priced at par with an 8% yield.
  - Interest rates instantaneously rise to 10%
  - The value of the portfolio falls by 30%, which most investors would see as a very painful loss.
- However, consider the long run investment outcome over a 50 year horizon
  - The original 8% yield would produce $46.91 for every dollar of current value
  - Losing 30% of the value but then investing at 10% would produce $82.17 per dollar of current value.
  - Why should we fear making almost twice as much money?

Higher Moments and Portfolio Formation

- At each moment in time investors have to form their portfolio. When must they be concerned if their portfolio contains securities where higher moments are present in the return distribution?
  - Markowitz (1952) says he would have preferred to use semi-variance as the risk measure instead of variance. Too computationally difficult for the 1950s. He later argued that just mean and variance were sufficient for practical cases.
  - Samuelson (1970) argues that investors should define their choices to maximize utility over all moments.
  - Hlawitscka and Stern (1995) show the simulated performance of mean-variance portfolios is nearly indistinguishable from the utility maximizing portfolio.

Should Investors Care About Higher Moments?

- Wilcox (2000) shows that the importance of higher moments is an increasing function of investor gearing.

- Cremers, Kritzman and Paige (2003) Use extensive simulations to measure the loss of utility associated with ignoring higher moments in portfolio construction
  - They find that the loss of utility is negligible except for the special cases of concentrated portfolios or "kinked" utility functions (i.e. when there is risk of non-survival though bankruptcy).
  - For an agent manager being fired may be the equivalent of non-survival. Ethical issue: Whose risk are we worried about?

- Satchell (2004) Describes the diversification of skew and kurtosis
  - Illustrates that plausible utility functions will favor positive skew and dislike kurtosis.

A Key Advancement

- "Repricing" based risk engines do some form of simulation (historical, Monte Carlo) to estimate the distribution of possible dollar values for a portfolio of K assets over N observations.
  - Computational burden is N*K, which is material for large N and K
- If we have estimates of the four moments of the return distribution for each individual asset, we can *calculate the estimated four moments of the portfolio return distribution in closed form.*
  - Satchell and Hall (2013)
- Even for highly non-linear instruments the computational burden is reduced to what is necessary to estimate the skew and kurtosis of the each asset.
  - If we are interested in a particular confidence interval (e.g. 95% Value at Risk) we reduce the computational burden to 2 * K

Extension to Factor Models

- For any portfolio viewed in a factor model, there exists a numerically equivalent full covariance matrix. See diBartolomeo (1999)
- In a factor model context, any asset portfolio can be replicated as combination of factor mimicking portfolios plus some amount of idiosyncratic risk.
- This feature allows us to easily capture a second source of higher moments: the skew and kurtosis observable in the factor returns.
  - If the returns to a factor (FMPs) have skew and kurtosis, we can now readily include those effects in the closed form computation of portfolio level skew and kurtosis.
- We can also capture this second source of higher moments via bootstrap simulations of factor returns with fixed factor exposures
  - This analysis is *already available* in the **Optimized Scenario** function within the Northfield PRISM application.

Building the Intuition

- The key to understanding the incorporation of higher moments that we cannot say that a particular security has more or less risk.
  - We need to know the sign of the position weight, or the active weight against a benchmark

- Consider a simple "out of the money" call option
  - The distribution of return has a large positive skew with a large positive return (at low probability) and maximum loss truncated at -100% (at higher probability).
  - Compared to a symmetric distribution with the *same standard deviation,* I have less risk if I have a long position, but more risk if I have a short position.

Combining Two Ideas into A Short Cut

- Satchell (2004) Illustrates that plausible utility functions will favor positive skew and dislike kurtosis.
  - It is therefore very natural to consider skew an increment to return and kurtosis an increment to volatility.
- For an asset with positive skew, compared to a symmetric distribution with the *same standard deviation,* I have less risk if I have a long position, but more risk if I have a short position.
  - For positive skew, a long position has a smaller *effective volatility* and a short position has a larger *effective volatility.* For positive excess kurtosis, effective volatility is larger and vice versa.
- For any confidence interval, the magnitudes of the adjustments to both return and volatility are given by Cornish and Fisher (1937)
  - Practitioners have long used this kind of adjustment such as "convexity adjusted yield", a standard bond metric.
  - We can convert the four moment problem to mean-variance

A Bit More "Lateral Thinking" Combinations

- For investors not accustomed to thinking in terms of a specific utility function for their portfolio, even knowing the four moments of their portfolio and related metric (e.g. VaR) may be unintuitive.
- If we know (or can infer) mean-variance risk aversion for the investor we can convert the Cornish Fisher adjustment to expected return into another "utility equivalent" adjustment to volatility. Expected volatility is adjusted three ways (skew, kurtosis, and the incremental impact on return).
- Conversely, we could adjust post CF expected returns for the change in volatility expectations. This difference may be an intuitive way to describe the "multi-period, long term cost of risk".
- Wilcox (2003) provides an approach to inferring optimal risk aversion (or risk tolerance) for an investor.
  - In Northfield terminology, a simple rule of thumb is RAP = 6 times volatility (after adjustment for higher moments)

Intuition for the Factor Conscious

- To the extent we aggregate the impact of higher moments as adjustments to the expected volatility of a security or portfolio, we could also represent these adjustments as different magnitudes of factor exposure.
- For example, it is common for practitioners to think of "market timing alpha" as having a low "beta" (market factor exposure) in down markets and a high "beta" in up markets.
  - This concept goes back to Treynor and Mauzy (1966) where beta was subdivided into sensitivity to market returns, and sensitivity to market returns squared (implicit tail events).
- Volatility adjustments arising from kurtosis represent changes in the factor exposures and idiosyncratic risk. Volatility adjustments arising from skew represent changes in factor exposures conditional on the sign of the position weight.

Optimization Example: Catastrophe Bonds

- If there is skew in the return distribution, a four moment optimization is computationally messy. Converting to mean-variance via CF is convenient.
- Our basic framework is to consider a catastrophe bond as a portfolio which is long a Treasury bond and short a lottery ticket (i.e. a weather lottery). We represent the lottery ticket as a synthetic security whose only property is a unit amount of idiosyncratic risk.
- Given the yield on the CAT bond, and the expected loss from the term sheet (typically 2%), we can back into the notional value of the lottery ticket that gives the correct absolute volatility for the CAT bond.
- If you own more than one CAT bond such that the lottery risks are correlated (all Florida hurricane risk), you can define the same synthetic security as part of multiple CAT bonds, thereby creating a correlation effect among the lottery payouts.

Catastrophe Bond Attributes

- If a catastrophe bond has a 2% chance of losing everything, there is a 98% probability that the volatility will be equivalent to the related Treasury bond with a 2% likelihood of 100% loss
- Calculate the standard deviation, skew and kurtosis associated with the lottery ticket(s) from a binomial distribution
- Add in the volatility of the Treasury bond without covariance
- Convert to a two moment distribution as described
- 95% VaR (the most popular P value) is zero, since 95% of the time your loss due to catastrophe will be zero. 95% CVaR is 40%

Catastrophe Bond Optimization

- Run a four moment optimization on a portfolio of N independent catastrophe bonds. Return estimate is the yield to maturity minus the 2% loss expectation.
- Obtain the efficient frontier of optimal portfolios
- Convert the return distribution of each bond to two moments using just the volatility adjustment, with no change in return estimate
- Obtain the efficient frontier of optimal portfolios
- The two efficient frontiers are not statistically significantly different under the test from Jobson (1991) until N is very small

Conclusions: Putting Things Together

- There are tractable methods for estimating the higher moments of a portfolio return distribution within a factor model framework.
  - The source of higher moments can be the structural aspects of the securities (e.g. options) or higher moments in the return distribution of underlying factors.
  - Even in the presence of assets with material higher moments we can dramatically reduce the computational intensity with a combination of minimal repricing, closed form computation and in some applications our **Optimal Scenario** method.
  - Any commonly used risk metric (volatility, variance, VaR, CVaR, M-squared, drawdown probability) is available.
  - The economic impact of higher moments can be transformed into units of either risk or return, so as to move the focus of the process away from solvency risk *to the more relevant framework of investment risk and optimization.*