A Unified Model of Equity Risk, Credit Risk & Convertibility Using Dual Binomial Trees"

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Structure

We need to answer the following questions

- What is a convertible bond?
- How can we sensibly price it in a consistent fashion?
- How can we decompose it into “atomic” risks?
- How does this fit into our “Everything Everywhere” model?

What is a convertible bond?

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What Are Convertible Bonds?

Simple yet Complex!

A bond that may be converted at some time in the future at the bond holders' discretion into N shares of stock X at a price P.

Key Common Features:

- call schedule
- put schedule
- sinking fund or other redemption features
- convertible into issuing (or another) companies stock
  - may be convertible at any time
  - may be convertible at a fixed price or a time-dependent price

A Bond plus a Warrant plus Embedded Options?

How Can We Price A Convertible Bond?

Break it into pieces and price those...

- Choose an approach for bonds with embedded options
- Choose a *consistent* approach for stock options
- Add the pieces together

Important: the equity model must be consistent with the bond model

Risks of Convertible Bonds

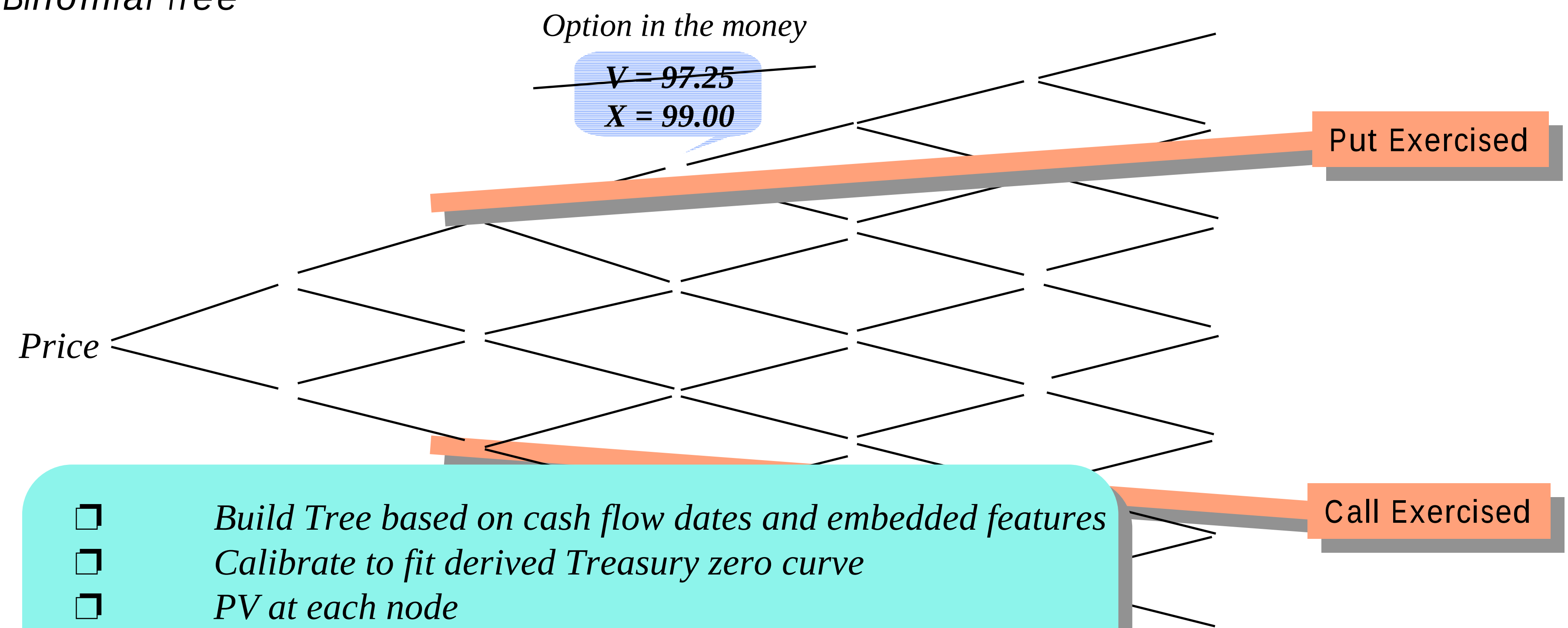
Break it into pieces and think about "atomic" risks...

- Interest Rate Risk
- Credit Risk (default risk)
- Equity Risk
- Embedded Options
- Currency Risk

We must consider *all* the risks associated with the convertible

Interest Rate Risk: How Do We Price a Bond?

Binomial Tree



- *Build Tree based on cash flow dates and embedded features*
- *Calibrate to fit derived Treasury zero curve*
- *PV at each node*
- *If PV>Option strike, assume exercised => Value = Strike*
- *Continue back down tree*
- *PV at time 0 (now)*

Tree accommodates Calls, Puts and Sinking Funds

How Could We Price a Stock Option or Warrant?

It must be fast, useable in production environment, and consistent with bond pricing model

Black-Scholes	-	easy to use, but assumes interest rates are constant!
Finite Difference Methods	-	hard to set up constraints, intractable in production system
Binomial Tree	**-**	**easy and fast to use but not very accurate?**
Trinomial Tree	-	much slower and more difficult but better accuracy?

Choose the Right Model – *consistent* with Bond Pricing Model

How to Connect Them and Price Convert in One Go?

Is there an easy way?

Standard Practitioner Choice as Proposed by Many Textbooks:

- ❑ Black-Scholes for stock option part of convertible
  (ignores inconvenient BS assumptions about constant interest rates)

Other Academic or Textbook Answers:

- ❑ Finite Difference Methods to Solve PDE directly
  (difficult to run as large-scale production process)
- ❑ Binomial tree for stock prices, determine at each node whether instrument is stock or bond. Use risk-free rate (fixed) to discount stock, use risky rate (fixed) to discount bond.*
  (ignores term-structure of interest rates entirely)

** Suggested by Goldman Sachs: "Valuing Convertible Bonds as Derivatives," Quantitative Strategy Research Notes, November 1994*

No easy way unless you make horrible assumptions

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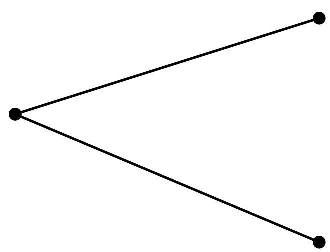
Binomial Tree Approach – More Detail

Could We Adapt This Approach?

Use a binomial tree for stock prices, determine at each node whether instrument is stock or bond. Use risk-free rate (fixed) to discount stock, use risky rate (fixed) to discount bond.



At this node the convert value > bond value
The bond is converted. Since it is now an equity we should discount it at the risk-free rate.

At this node the convert value < bond value
The bond is not converted. Since it is still a risky bond we should discount it at the risky rate.

*The value at the target node is reached by discounting the value at the two future nodes by the **average** of the risky and risk-free rates.*

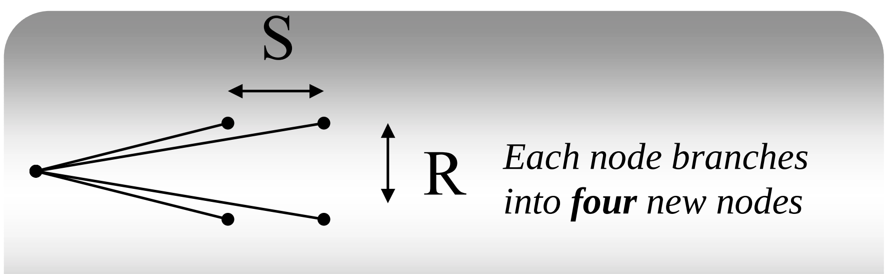
*With this approach the **same** two rates (one risky, one risk-free) are used throughout the tree.*

Can we adapt this to include stochastic interest rates?

Our Approach

Two trees – two stochastic processes

- We use ***two combined*** trees at the same time.
- The interest rate tree allows us to model the short-rate diffusion
- The stock price tree allows us to model the stock price diffusion
- We discount value back at each node based on the average rate
  - if it's still a bond at some node we should use risky rate
  - if it's an equity at some node we should use risk-free rate
- Given what we determine it to be at each future node we can discount the value back to the preceding level.

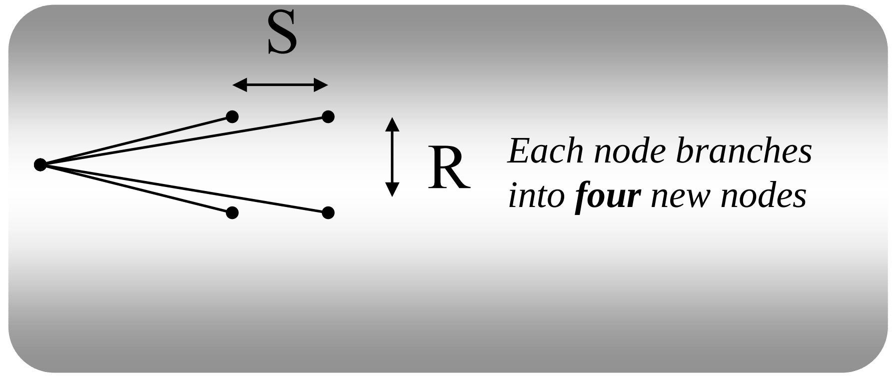


Diffusion processes for both stock and interest rates

Branching and Dividends

Other Features of the Stock Diffusion Tree

- Most stocks pay dividends
- What about volatility?
- What about accuracy?



- Branching for stock tree **includes** dividends based on constant annual dividend rate of q

  $u = \exp\{ (r-q-\sigma^2/2)\Delta t + \sigma \sqrt{(\Delta t)} \}$

  $d = \exp\{ (r-q-\sigma^2/2)\Delta t - \sigma \sqrt{(\Delta t)} \}$

- Stock tree branching based on volatility – can be either **one** volatility or a **term-structure** of volatility. (Ours is one for now…)
- Add many extra nodes at the short end to ensure accuracy – 50 additional levels added over first five years.

Diffusion processes for both stock and interest rates

Valuation

An attempt at clarity…

For each of Four Nodes Branching from Target Node:

- Compare Value V_N with convert value
- Compare (if not converted) with Call / Put strike
- If called, compare *again* with convert value (so-called “forced” convert)
- Final result: a new value V_N^*, and a status (Bond or Equity)
- Final final result: a discount rate D_N:
  - appropriate *risky* rate if Bond
  - or appropriate *risk free* rate if Equity

Use the Average Rate (average of D_1 to D_4) to discount V_1^* to V_4^* back to Target Node

Continue Rolling Back through Tree

Semi-Final Result: Price of Instrument

Stochastic Processes and Forced Conversion features Modeled Fully

More Thinking

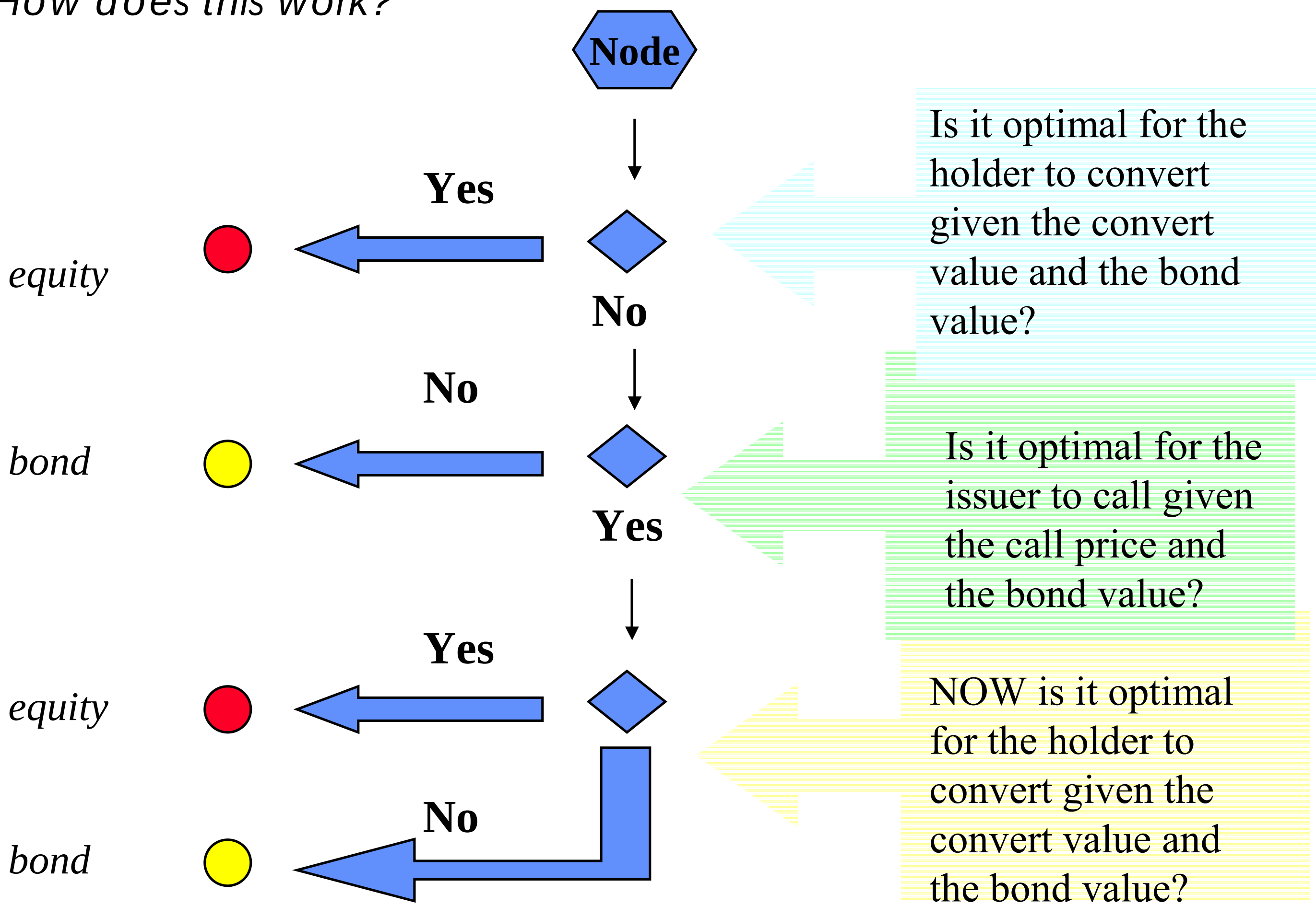
What falls through the cracks?

- Forced Conversion
- Correlation between stock return and interest rates
- Credit Risk & Equity Risk

We need to focus on these key attributes

Forced Conversion Flowchart

How does this work?



Forced Convert is Treated in a Cascade Process

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Correlation Between Stock Price Process and Interest Rates

How can we rationally treat this correlation?

- *Stock prices are correlated to interest rates. So we have to compute the state price densities for evaluating the equity warrant have to be computed conditionally on the covariance matrix of the equity factors and the term structure factors. We "bend" the stock pricing tree to fit expected returns, given the term structure state*

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The transformed tree explicitly captures the correlated relationship

Equity Risk - Capturing Equity Related Covariance

How can we rationally treat this source of risk?

- Idea 1: observe the fraction of the total number of nodes in the combined tree that reflect conversion into equity. We could then say that the instrument has, say, (20/100)×equity security exposure.

- (Better) Idea 2: treat the equity factor exposure *today* as the present value of the *future* equity factor exposure. This captures two effects:
  - *the time aspect of convertibility*
  - *the interest rate environment at, or path toward, convertibility*

- Using the tree pricing procedure captures the interaction of multiple option features
  - *Conversion to equity effectively reduces bond maturity*
  - *Call or put options on the bond effectively shorten the expiration date of the warrant*

A more sophisticated version of a "delta equivalent" equity risk

Risk in the "Everything Everywhere" Model

Can we model converts with the same risk factors?

- 19 Factors, plus currency covariance matrix
  - *5 geographic regions*
  - *6 aggregate industry-sectors*
  - *Interest rates*
  - *Energy cost (an inflation proxy)*
  - *Investor confidence #1: large cap – small cap spread*
  - *Investor confidence #2: emerging - developed spread*
  - *Dividend yield: a proxy for growth / value in equities*
  - *Three-Factor Model of Term Structure Movements*
  - *Currencies*

Sources of Risk in Convertible Bonds can be modeled by EE Factor Set

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How Can We Treat These in Our EE Risk Model?

Can we model converts with the same risk factors?

- We capture interest-rate risk by varying our three *term-structure* factors and re-pricing under term-structure changes
- We capture *credit risk* of the bond (the risky bond part of the convertible) as a duration-weighted exposure to a credit-synthetic
- We capture *embedded options* (calls, puts etc) *explicitly* in the pricing process
- We capture the *equity portion of the convert* risk using a version of a delta-neutral underlying exposure to the equity.
- We capture *currency risk* explicitly as factors in the risk model

Sources of Risk in Convertible Bonds can be modeled by EE Factor Set

Musings

What other fun things could we do?

- Use *micro-economic* factors, such as those in our Fundamental Model for equity risk, and repeat the same process linking credit risk to those.
  - (see "Credit Risk Management", Nick Wade, Newport 1999)
  [http://www.northinfo.com/papers/pdf/19990607_credit_risk.pdf]

- (following Merton) Treat the stock price at node N in our stock price tree as a *call option on the assets* of the firm, reverse out the implied expiration date of the option, and compare that to the maturity of the debt issues. Compare to industry / sector averages and make inferences about the health of the company.

- *Adjust our OAS on a per-node basis* within the tree to take into account the implied expectations about the risk of the firm given the stock price. (i.e. if the stock price is 80, our OAS should be less than if it's 50).

- Final thought that given our effort building this tree approach to what is essentially option pricing, we can do better than Black-Scholes for estimating the implied expiration date of the option on the assets of the firm.

We can neatly join many mutually-conditional effects in one model

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Conclusions

Can we model converts with the same risk factors?

- We can both *price* and *evaluate the risks* of convertible bonds using a three dimension tree approach adapted from our EE risk model
- We believe this model is a substantial advance over methods that rely on seriously flawed simplifying assumptions.
- Empirical evidence is encouraging. Our model prices are in excellent agreement with reported trading prices

Sources of Risk in Convertible Bonds can be modeled by EE Factor Set