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# Return and Risk in Endogenous Time

**Dan diBartolomeo**
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# Introduction

- This presentation will explore one of the most basic yet least understood concepts in all of investing, the nature investment of return.
- We define the return on an investment as the rate of change in the value of the investment per unit time, which ought be understood as a simple arithmetic ratio.
- To a nearly complete extent the attention of both investment practitioners and finance academics have been focused on the numerator of the ratio, the change in the investment value (price).
- It has always been the custom of the industry to take the concept of units of time as a given and immutable quantity.

# Industry Practice

- It is quite common for equity traders to think of time as being measured more in trading volume, rather than the standard units of seconds, minutes and hours.
  - This reflects knowledge that some times of the trading are routinely very busy (around the open and close) while other parts of the day typically slow, with reduced trading activity.

- In terms of asset valuation, hedonic models are very common in which an asset may be described as "overvalued" or "undervalued" without regard to how quickly the market price and estimated value are likely to converge.

- Within the realm of technical analysis there is the "point and figure chart" in which entirely eliminates the concept of time in describing price change patterns.

# Background Literature

- Our *Risk Systems That Read®* method for improving risk estimation treats the passage of time as varying according to the amount and importance of information coming to investors.
  - http://www.northinfo.com/Documents/356.pdf

- The idea of time itself speeding up and slowing down has been previously explored in a small number of papers
  - Derman (2002)
  - Haug (2004)
  - Kyle, Obizhaeva, Sinha and Tuzun (2012).

- One important strand in this literature is the process by which speculative "bubbles" in prices may arise.

# Borrowing from Physics

- In many areas of physics, complex or "noisy" processes are often analyzed using methods of *intrinsic time-scale decomposition (ITD)*

- This is a well developed literature in mathematics and signal processing. There are even "ready to use" analytical routines in MATLAB and similar systems.

- One excellent example of the parallels between this world and the interests of investment practitioners are two papers on finding trends in time series using ITD
  - Restrepo, Comeau, Flaschka and Ventakaramani, *New Journal of Physics*, 2014.
  - A closely related paper by the same authors appeared in *Statistics*, 2014.

# The Arithmetic of Endogenous Time

- The concept of an investment return represents a change in the value of an investment in a known of interval of time.
  - Return = (Final Price- Initial Price) / Initial Price) / 1 unit of time
- We've adopted this convention due to the existence of opportunity costs. We can always choose to make a different investment than the one we are making, and at least some of the available alternatives have a known return (e.g. the risk free rate)
- While it is a human convention that we define time in terms of days, hours, minutes, and seconds, there is nothing that requires this.
  - Plants do not start their growing season based on the calendar. They act in response to soil temperature, so "spring" starts when the temperature rather than the "time" is right.
  - Maybe investors should think more like plants

# The Trade Scheduling Problem

- An example of how endogenous time is used in asset management is the problem of how to optimally break a large order (e.g. BUY 1 million shares of IBM) into a sequence of small orders that can be executed over time.
  – Bertsimas and Lo (1998)
  – Almgren and Chriss (2000)
- The problem is even more complex when multiple simultaneous orders are involved.
  – Northfield, http://www.northinfo.com/documents/408.pdf
- The problem is a multiperiod optimization in discrete time:
  – We break the trading day into short periods that are K minutes long on average, but where the clock time length of each period is adjusted so that the expected trading volume in each period is equal. *In effect, we make trading volume the real measure of time.*

# Hedonic Models

- Hedonic models of economic value are extremely common in some areas of finance. A real estate "appraisal" is perhaps the most familiar.

- In such models, the dependent variable represents the "value" of an asset as a function of various independent variables.

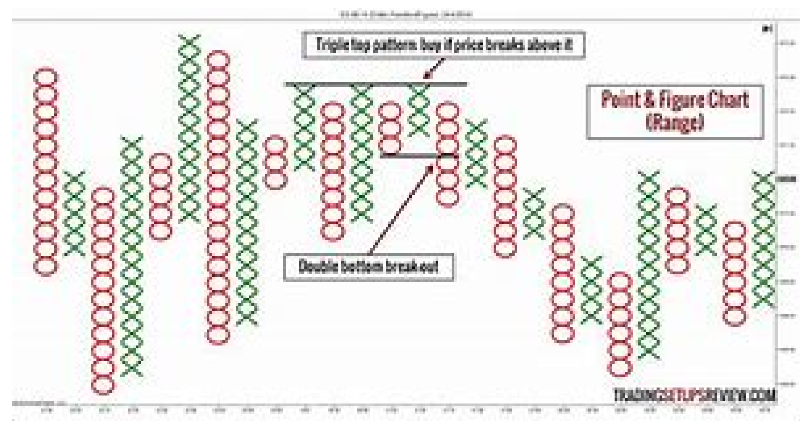
- If the value of an asset is deemed to be sufficiently different from the "market price" that a transaction can be profitable after related costs we *assume* that a positive return will be generated.
  - However, we cannot explicitly estimate the magnitude of the return because the time scale for when the market price will change to our valuation is unknown.

# Hedonic Investors

- One could reasonably argue that large scale, fundamental investors (e.g. Warren Buffet) are really carrying out hedonic modelling processes.
- They buy financial assets they believe are materially undervalued but without an explicit time estimate of when "price" and "value" will converge.
- One way to think of this is as a Poisson process in which we know that the probability of convergence is one within a finite time period (e.g. 10 years) but the timing of the event is random within in the interval.
- Ang and Bollen (2008) model investor "lock ups" for private equity and hedge funds using this construct. They estimate that large return premiums are required to compensate investors for the uncertainty as to the timing of realization.

# Technical Analysis

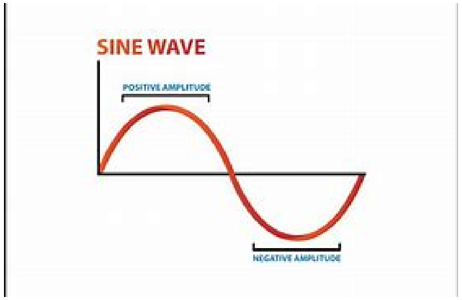
- A staple of technical analysis of price trends is the “point and figure” chart in which asset price movements are described while removing the time element.

# Risk Systems That Read®

In December 2017, Northfield introduced daily updating of our risk models based on defining the rate of the passage of time by the rate of information flow to investors on a particular topic (company, industry, country). The rate of information flow is defined by analyzing over 5000 financial news articles per day in terms of normalized frequency and *perceived importance.*

The easiest way to understand the process is to think of the volatility of a stock as being like a sine wave. The risk is the area under the curve. If the time scale is compressed the amplitude must increase. If the time scale is stretched, the amplitude must decrease.



# Speculative Bubbles

- Derman (2002) uses the concept of endogenous time to consider how investment markets would behave if investor utility was defined in terms of the *number of profitable trades* they could make, rather than the accumulation of wealth.
  - In this framework, investor utility is maximized under volatile conditions. In essence, the greater the volatility (compression of time) the greater the likelihood that the investor will have a chance to make a profitable trade. If there are no price movements, there are no opportunities.
  - This framework also suggests how "speculative" bubbles could naturally form. The greater the distortion of prices away from fundamental value, the greater the risk, the greater the volatility and the greater the potential for traders to profit.
- Recent examples include the broad expansion of retail "day traders" into Internet stocks in the late 1990s, and current retail interest in cryptocurrencies.

# Finance and Special Relativity

- In Space-Time Finance (2004) by E.G. Haug, the author draws the explicit financial parallel to concept in Einstein's Theory of Special Relativity that time itself slows down when we are travelling close to the speed of light.

- The implications of applying the mathematics of this parallel are interesting:
  - Estimates of risk and volatility will be different for different market participants given the exact same input data and methods.
  - The assumption that volatility scales with the square root of time is invalid.
  - The existence of "fat tails" in return distributions and stochastic volatility are a natural outcome of the true relationship with time.
  - Consistent with Mandelbrot (1960) who assumed stable Paretian distribution for asset returns

# Information Time

- Kyle, Obizhaeva, Sinha and Tuzun (2012) presents a new twist on the Kyle "Invariance Theorem". The Invariance Theorem argues that the when trading volumes vary they do so in a predictable way.
  - About two thirds of volume changes arise from the number of transactions About one third of volume change is related to the size of transactions. This relationship seems to hold across all asset types and geographic locations.

- This relationship can be used to model several aspects of financial market microstructure such as asset volatility, trading volumes, bid/asked spreads, and the market impact of large trades.

- In the 2012 paper, they define the rate of the passage of time as being related to the frequency of news articles on a particular company rather than clock time. Empirical testing against a large news data set from Reuters confirmed the theoretical relationship.

# Conclusions

- In considering the concept of an investment return, the financial literature has concentrated almost entirely on the numerator of the ratio (the change in price) to the exclusion of the denominator (defining the unit time).
- We find numerous examples of investors using the concept of endogenous time to better articulate the realities of financial market behavior.
- Northfield has made two important advances that take advantage of defining time itself in a more flexible fashion.
  - Our Risk Systems That Read® technique for improving short horizon risk estimates is framed so variations in asset volatility can be neatly described as arising from "investor" time speeding up or slowing down.
  - Our trade scheduling techniques frame the optimal execution problem as a multiperiod optimization problem in discrete time, but where time "slices" are formulated to represent equal expectations of trading volume.